REPORT ON RADEK HONZIK'S HABILITATION THESIS

This thesis explores the behavior of the continuum function $\kappa \mapsto 2^\kappa$ as it relates to large cardinals, singular combinatorics, and compactness properties such as the tree property.

The thesis is well written and gives a clear presentation of the author's papers. There is a nice unifying narrative motivating all of these results. The larger context is "how much compactness can be obtained in the universe?" A key question (as pointed out in Section 6 of the thesis) is whether the tree property can consistently hold at every regular cardinal greater that ω_1. Another key question is what Easton functions can be consistently realized while preserving compactness type properties. This latter question is the main focus of the thesis. The more specific context is proving consistency results and analyzing bounds on obtaining various values of $\kappa \mapsto 2^\kappa$ together with specific large cardinals, and together with compactness type combinatorial principles, such as the tree property, and failure of weak square.

The thesis is divided into three main topics. The first one is on which Easton functions are consistent with certain large cardinals (here the context is 2^κ for regular κ). More precisely, given a large cardinal property, which Easton functions can be realized by a cofinality preserving forcing, so that cardinals retain that large cardinal property. The ones explored here are measurables, strong cardinals, supercompact tall cardinals. For example, the author has proven, jointly with S. Friedman, that for any Easton function F, starting from some large cardinals assumptions, there is a generic extension where F is realized and all closure points of F that satisfy said large cardinal property in V, remain measurable. In this and other results, the strategy is using Sacks forcing, which is nicely amenable to lifting embeddings.

This line of research fits into the broader context of analyzing the effect of of large cardinals on the continuum function at regulars. The results presented can be viewed as a generalization of Easton's theorem that $\kappa \mapsto 2^\kappa$ for regular κ can have any reasonable behavior. The author's work is a nice contribution, and he is likely to make good progress on remaining open questions regarding the cases of other large cardinals, such as strongly compact, subcompact, among others.

The second part of the thesis deals with the continuum function at singular cardinals, an old project in set theory, often referred to as "the singular cardinal problem". Consistency results here involve failures of the singular cardinal hypothesis (SCH) and Prikry type forcings. For example, take the generic extension of the theorem mentioned above. I.e. given any Easton function F, all closure points that also belong to a certain class Γ remain measurable. The author shows that forcing further with a Prikry iteration, any cardinal κ in Γ is singularized and SCH fails at κ according to the Easton function ($2^\kappa = F(\kappa)$). Honzik also discusses results on the failure of the SCH at $\aleph_\omega$.

The singular cardinal problem is both an interesting and challenging one. Honzik's results described in this section are a natural continuation of his work in part one.

The last part of the thesis deals with the tree property. This is a compactness type combinatorial property that follows from large cardinals but can hold at successors. The author continues his investigation of the behavior of the Easton function in the presence of the tree property. For example, jointly with S. Stejskalova, they show that the tree property simultaneously at $\aleph_{2n}$ for $n > 0$ is consistent with any Easton function, modulo Specker's theorem and $\aleph_\omega$ strong limit. In a different paper, with S. Friedman, they obtain the tree property simultaneously at $\aleph_{2n}$ for $n > 0$ together with failure of SCH at $\aleph_\omega$. Let us remark here that Unger showed that the tree property can hold at all of the $\aleph_n$, $n > 1$ together with failure of SCH, from a much stronger large cardinal hypothesis of course. Violating SCH is required to get the tree property at $\aleph_{\omega+2}$. Then the author discusses some more results on the relationship of the tree property at the double successor of a singular and the value of the powerset of that singular cardinal.

In addition to the motivation connected with the behavior of $\kappa \mapsto 2^\kappa$, these are also incremental results towards the old question of whether the tree property can consistently hold at every regular cardinal greater than ω_1. A positive answer would require many failures of SCH. To truly address this question, one has to analyze models where the tree property holds at each $\aleph_n$, for $n > 1$, and also tackle the problem of the tree property at the first successor of a singular cardinal. While the presented results do not directly deal with theses issues, the latter are natural questions for the author to pursue.

This is a solid, well written Habilitation thesis. Most of the papers that make up the results in this thesis have been peer reviewed (except only for two that are currently submitted). They have appeared in reputable and in strong journals, for example the Israel Journal of Mathematics, the Journal of Symbolic Logic. The theorems are part of a clear research program, and Dr. Honzik looks poised to continue and be productive in the future.

Dima Sinapova 04 -05- 2018

Dima Sinapova

Associate Professor of Mathematics
University of Illinois at Chicago