UNIVERZITA KARLOVA
CERGE

# ADAPTIVE LEARNING IN MONETARY MODELS

HABILITAČNÍ PRÁCE

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# Contents

# Preface

This compilation of published papers forms my habilitation work. The three papers presented here have benefited from useful comments by the referees and editors, and were supported by several funding agencies.

# Introduction

This habilitation work presents my research in the field of adaptive learning, which is one of the ways of modeling boundedly rational agents, and thus a deviation from the rational expectations (RE) hypothesis. For the last 40 years, REH was the cornerstone of modelling in macroeconomics. Under RE, the expectations are formed consistently with the underlying model and the policy environment, and all available information is used efficiently by the agents. The expectations are crucial for the macroeconomic models, because modern literature insists on the agents' behavior being 'micro-founded', that is, optimal, given well defined preferences and budget constraints. Absolute majority of modern macro models are defined as a series of intertemporal optimization problems, and the solution to these problems depends crucially on how agents form their expectations about future variables that inform their action.

REH is a very useful and powerful assumption. It tightens the link between theory and estimation, allows for an efficient estimation of the deep parameters of the model by exploiting all the cross-equation restrictions that are imposed through the model-consistent expectations hypothesis, and often results in existence of a unique equilibrium. However, REH does not provide a description of the information problem that agents have to solve to discover systematic relations between current and future values of the relevant variables. RE might be thought of as a result of some asymptotic process whereby the agents, having lived for an infinite time in a stationary environment, were able to learn exactly all the relevant relations and distributions.

In reality, households and firms have limited knowledge and diffused information about the correct form of the underlying model, about the exact value of the model parameters or the state vector of variables, and especially about the exogenous and latent disturbances that hit the economy. Agents, like econometricians, need to find out the dynamic structure of the economy using the data available in real time. As processing information is costly, it is more realistic to assume that they will concentrate on a limited amount of information and that they update their beliefs about the underlying economic relations as new data becomes available, in order to capture possible changes in the stochastic structure or in the policy environment. If expectations are allowed to deviate from the RE solution, the model dynamics changes as well and expectations become, potentially, an important additional source of business cycle fluctuations. Such beliefs are called *mis-specified*.

Real time economic environment is characterized by frequent changes, ranging from the seemingly one-off events such as the Great Depression to the constant churning of market leaders and modes of production caused by the technological progress. It is, therefore, realistic to assume that the agents, unsure of the parameters of their environment and thus learning, also make allowance for the possibility that their world could be non-stationary. In non-stationary environments, using of the so-called 'perpetual learning' becomes optimal. Such learning, however, introduces its own stochastic component to the agents' behavior. If the economic environment is, indeed, stationary, then usage of tracking algorithms for learning could become the source of non-trivial economic fluctuations, even asymptotally.

In my research, I concentrate consequences of AL agents using perpertual learning algorithms and mis-specified beliefs on the economic dynamics, both from theoretical and practical point of view. In the first presented paper, "Escape Dynamics: A Continuous Time Approximation", joint with D. Kolyuzhnov and A. Bogomolova, published in the *Journal of Economic Dynamics and Control* 2014, **38**, 161-183, we investigate theoretically and by the way of computer simulations the most prominent way mis-specified perpetual learning could affect the economic dynamics: so-called 'escapes' , whereby the agents' (Central Banks and the private sector's) beliefs leave a neighborhood of the Nash Equilibrium and move towards beliefs consistent with another equilibrium. The escapes occur purely by chance, due to a sequence of shocks which lead the Central Bank to believe in existence of an exploitable inflation-unemployment tradeoff and thus deviate from the Nash equilibrium. Attempts to exploit such a tradeoff, if accompanied by several shocks reinforcing the original deviation, force the beliefs outside of the small neighborhood of the NE, where the so-called mean dynamics (the averaged driving factor in the beliefs updating process) is pushing them further away form the NE. In contrast to the earlier literature, starting with Cho, Williams, and Sargent (2002), utilizing the discrete-time large deviations theory approach to the escapes, we apply the continuous-time approximation of the original discrete-time process, resulting in a continuous-time diffusion. These earlier papers worked directly with discrete-time learning dynamics and used the earlier results of Williams (2001), who derived numerically the action functional for a linear-quadratic case when the state variable process is autoregressive with Gaussian noise. The basic problem associated with this approach is that characterizing escape dynamics for the discrete-time process as proposed by Cho *et al.* implies numerical calculation of a functional in a calculus-of-variation problem that leads to a system of non-linear differential equations with numerically derived right hand side functions. For complicated problems (with many lags, and/or high dimensionality), this approach can become numerically intractable. An analytical solution for escape dynamics of a discrete-time process can be derived only for a restrictive class of learning processes, such as recursive least squares or stochastic gradient learning with a constant gain with Gaussian shocks.

The continuous-time approximation proposed in our paper contributes to a partial resolution of this problem. Our approximation around the REE is a linear diffusion with constant coefficients. In large deviations theory, all escape dynamics characteristics such as the expected time until the beliefs escape any given neighborhood $D$ of the REE, the point through which this escape is most likely, and the probability of leaving $D$ within a given amount of time, are obtained by minimizing a so-called action functional on the boundary of the neighborhood, $\partial D$. Given our choice of the approximating diffusion, this task is a standard linear control theory problem: minimizing the action functional is equivalent to finding a minimum of a quadratic form on $\partial D$, where a closed form solution for many geometric forms of boundaries exists. We argue that our approach allows the construction of an approximation to the true characteristics of escape dynamics, which would be hard to derive otherwise, investigate this approximation using numerical simulations, and further show that the escape process is well described by the Central Limit Theorem, while large deviations approach becomes applicable only for the much smaller value of the learning gain than the one used in the macroeconomic literature.

The other two papers, included into this habilitation work, deal with consequences allowing the agents, populating a medium-scale DSGE model similar to that of Smets and Wouters (2007), to be adaptive learners holding potentially mis-specified beliefs and engaging in perpertual learning. In "Learning in an estimated medium-scale DSGE model", joint with Raf Wouters, published in the *Journal of Economic Dynamics and Control* (2012), **36**, 26-46, we evaluate empirically the fit of a DSGE model while allowing the agents to form their expectations as linear functions of past model variables. Coefficients of these linear functions, commonly known as beliefs, are re-estimated every period using a constant-gain (perpetual) learning algorithm. The beliefs about the relationship between expectations and current and past variables adapt to the patterns recently observed in the data. Several authors have suggested that adaptive learning can enhance the propagation mechanism of the DSGE models and generate the persistence that is otherwise caused by these models' frictions or by the dynamics in the exogenous stochastic processes. For instance, Orphanides and Williams (2003-2005a) illustrated how adaptive learning can lead to inflation scares or to increased inflation persistence. Milani (2007) estimated a small-scale model both under RE and learning and showed that the learning reduces the scale of structural frictions and results in an improved marginal likelihood relative to the RE model. We extended this previous work by estimating the learning process in a *medium-scale* DSGE model. We investigated systematically the role of initial beliefs and the information set in our learning models. The initial beliefs are hard to discover, because they depend on historical observations that are not part of the likelihood function. We investigated initial beliefs based on pre-sample data information, the beliefs that maximise the likelihood of the in-sample data, and the initial beliefs consistent with the final estimated model. We showed that if the agents are allowed to use the same information set under AL as under RE, there is no much difference between model dynamics, and the adaptive agents' beliefs stay close to their RE counterparts. However, if the information set for adaptive learners included only the observable variables and was smaller than the RE set, several interesting features could be observed: First, the model fit, measured using the log data density, improved; Second, there was a clear drift in the agents' beliefs, clearly showing that the initial beliefs (selected to be consistent with the RE equilibrium, corresponding to the estimated model parameters), were not providing the best possible description of the model dynamics. If the initial beliefs were estimated together with the other model parameters, then the model fit was improving dramatically. The latter finding raises the question of selecting the initial beliefs under AL, as it could significantly affect the estimation results. Finally, we found that the learning models that fit the data better than the model with rational expectations tend to add some additional persistence to the DSGE model, in particular following a monetary policy shock, that reduces the gap between the IRFs of the DSGE model and the more data-driven DSGE-VAR approach. We also observed that the additional dynamics introduced by the learning process did not systematically alter the estimated structural parameters related to the nominal and real frictions in the DSGE model.

The final paper of this work, "Learning in a medium-scale DSGE model with expectations based on small forecasting models", also with Raf Wouters, published in the *American Economic Journal: Macroeconomics* (2012), **4**, 65-101, developed further the themes discovered in the previous paper. We allowed the adaptively learning agents to use an even smaller information set than previously — typically, only two lags of a forward-looking variable being forecasted and a constant. In contrast to much of the earlier literature, both theoretical and empirical, we let our agents update their beliefs using Kalman filter (KF) rather than constant gain least squares (CG LS). We observed that Kalman filter learning is more efficient and adjusts more quickly than the constant gain learning, a finding that is in line with Sargent and Williams (2005). We documented more extensively the macroeconomic implication of the learning dynamics: the impact is mainly concentrated in the inflation dynamics, and contrary to Milani (2007), we did not observe an important effect on the role of real frictions in households' and firms' decision problems.

We allowed the agents to experiment with different forecasting rules and combine their predictions using either simple averaging or Bayesian Model Averaging (BMA) techniques, and showed that using small forecasting models further improves the models fit, that using several forecasting models leads to almost the same result as using only the best one and that simple averaging works better than BMA methods.

Assuming that agents use only a limited information set in forming expectations may be criticized for being largely arbitrary. Therefore, we conducted an extensive robustness exercise to underline that our results do not depend on a specific choice of the small forecasting model or of the initial beliefs. We also documented that the out-of-sample forecast performance of the DSGE model with adaptively learning agents using small forecasting models is competitive with a RE DSGE model where expectations are formed using a much larger forecasting model. The use of small forecasting models is important for the learning dynamics to adjust in a flexible, fast, and stable way. The empirical performance of the learning model depends on three properties: the specification of the forecasting model, the initial beliefs and the efficient updating procedure. All three aspects are contributing to a successful fit, but nevertheless the results were robust for relatively minor changes on each of the three properties.

We also showed that under adaptive learning the transmission mechanism of the model changes significantly, with very persistent mark-up processes which are needed to explain the data under RE becoming *iid* processes under AL. Finally, we demonstrated that RE and AL models deliver rather different time series for inflation expectations, and that these expectations are rather different from the ones measured in the Survey of Professional Forecasting, with AL model's expectations closer to the SPF data than RE expectations. The latter finding served as the beginning of my current work, also with Raf Wouters, on empirical DSGE models with AL agents.

# Literature

Adam, K. (2005). “Learning to Forecast and Cyclical Behavior of Output and Inflation,” *Macroeconomic Dynamics* 9, 1–27.

Altig, D., Christiano L., Eichenbaum M., and J. Linde (2005). “Firm-Specific Capital, Nominal Rigidities and the Business Cycle,” NBER Working Paper Series no. 11034.

Atkeson, Andrew, and Lee E. Ohanian (2001). “Are Phillips Curves Useful for Forecasting Inflation?”, *Federal Reserve Bank of Minneapolis Quarterly Review*, 25 (1): 2-11.

Barnett, A., and M. Ellison (2012): “Learning by Disin‡ating,” Bank of Finland Research Discussion Paper 10.

Barro, R. J., and D. B. Gordon (1983): “Rules, Discretion and Reputation in a Model of Monetary Policy,” *Journal of Monetary Economics*, 12, 101–121.

Berardi, M. (2013): “Escape Dynamics and Policy Specification,” *Macroeconomic Dynamics*, 17, 123–142.

Binmore, K., and L. Samuelson (1997): “Muddling Through: Noisy Equilibrium Selection,” *Journal of Economic Theory*, 74, 235–265.

Borio, Claudio, and Andrew Filardo. (2007). “Globalisation and Inflation: New Cross-Country Evidence on the Global Determinants of Domestic Inflation”, BIS Working Paper 227.

Boyd, S. (2004): “Lecture Notes for EE263,” Stanford lecture notes.

Branch, W. and G. Evans (2006). “A Simple Recursive Forecasting Model,” *Economics Letters* 127, 264 – 295.

Bray, M. M. (1982): “Learning , Estimation, and The Stability of Rational Expectations Equilibria,” *Journal of Economic Theory*, 26, 318–339.

Bray, M. M., and N. E. Savin (1986): “Rational Expectations Equilibria, Learning, and Model Specification,” *Econometrica*, 54(5), 1129–60.

Bullard, J. B., and I.-K. Cho (2005): “Escapist Policy Rules,” *Journal of Economic Dynamics and Control*, 29(11), 1841–1865.

Bullard, J., and K. Mitra (2002): “Learning About Monetary Policy Rules,” *Journal of Monetary Economics*, 49(6), 1105–29.

Bullard, James B., and Aarti Singh (2009). “Learning and the Great Moderation,” FRB of St. Louis Working Paper 2007-027A.

Carceles-Poveda, E. and C. Giannitsarou (2007). “Adaptive Learning in Practice,” *Journal of Economic Dynamics and Control* 31, 2659-2697.

Carroll, Christopher D. (2001). "The Epidemiology of Macroeconomic Expectations", NBER Working Paper 8695.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. (2009). "New Keynesian Models: Not Yet Useful for Policy Analysis", *American Economic Journal: Macroeconomics*, 1(1): 242-66.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113(1): 1-45.

Cho, I.-K., and K. Kasa (2008): "Learning Dynamics and Endogenous Currency Crises," *Macroeconomic Dynamics*, 12, 257–285.

Cho, I.-K., and K. Kasa (2012): "Learning and Model Validation," Simon Fraser University Working Paper 12-07.

Cho, I.-K., N. Williams, and T. J. Sargent (2002): "Escaping Nash In‡ation," *Review of Economic Studies*, 69(1), 1–40.

Cogley, Timothy, Giorgio E. Primiceri, and Thomas J. Sargent. (2010). "Inflation-Gap Persistence in the U.S.", *American Economic Journal: Macroeconomics*, 2(1): 43-69.

Croushore, Dean. (2011). "Frontiers of Real-Time Data Analysis", *Journal of Economic Literature*, 49(1): 72-100.

Dahleh, M., M. A. Dahleh, and G. Verghese (2004): "Lectures on Dynamic Systems and Control," MIT lecture notes.

Del Negro, Marco, and Stefano Eusepi. (2010). "Fitting Observed Inflation Expectations," Federal Reserve Bank of New York Staff Reports 476.

Del Negro, M. and F. Schorfheide (2008). "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics* 55, 1191-1208.

Del Negro, M., F. Schorfheide, F. Smets and R. Wouters (2007). "On the Fit of New Keynesian Models," *Journal of Business & Economic Statistics* 25, 123-143.

Dembo, A., and O. Zeitouni (1998): "Large Deviations Techniques and Applications," Springer, New York Berlin Heidelberg.

Dupor, B., Jing Han and Yi-Chan Tsai (2009). "What do Technology Shocks Tell Us About the New Keynesian Paradigm?" *Journal of Monetary Economics* 56, 560-569.

Dupuis, P., and H. J. Kushner (1989): "Stochastic Approximation and Large Deviations: Upper Bounds and W.P.1 Convergence," *SIAM Journal on Control and Optimization*, 27, 1108–1135.

Ellison, M., and T. Yates (2007): "Escaping Volatile In‡ation," *Journal of Money, Credit and Banking*, 39, 981–993.

Eichenbaum, M. and J. Fisher (2007): "Estimating the Frequency of Re–optimization in Calvo-Style Models," *Journal of Monetary Economics* 54, 2032-2047.

Evans, G. W., and S. Honkapohja (1994a): "Learning, Convergence, and Stability with Multiple Rational Expectations Equilibria," *European Economic Review*, 38, 1071–1098.

Evans, G. W., and S. Honkapohja (1994b): "On the Local Stability of Sunspot Equilibria Under Adaptive Learning Rules," *Journal of Economic Theory*, 64, 142–161.

Evans, G. W., and S. Honkapohja (1995): "Local Convergence of Recursive Learning to Steady States and Cycles in Stochastic Nonlinear Models," *Econometrica*, 63, 195–206.

Evans, G. W., and S. Honkapohja (2001): "Learning and Expectations in Macroeconomics," Princeton University Press, Princeton, NJ.

Evans, G. W., and S. Honkapohja (2003): "Adaptive Learning and Monetary Policy Design," *Journal of Money, Credit, and Banking*, 35(6), 1045–72.

Fourgeaud, C., C. Gourieroux, and J. Pradel (1986): "Learning Procedures and Convergence to Rationality," *Econometrica*, 54(4), 845–68.

Freidlin, M. I., and A. D. Wentzell (1998): "Random Perturbations of Dynamical Systems," Springer-Verlag, New York, second edn.

Intrilligator, M. D. (2002): "Mathematical Optimization and Economic Theory," SIAM, Philadelphia.

Hamilton J. (1994). "Time Series Analysis," Princeton University Press.

Honkapohja, Seppo, Kaushik Mitra, and George W. Evans. (2002). "Notes on agents' behavioral rules under adaptive learning and recent studies of monetary policy", mimeo.

Janjgava, B., and S. Slobodyan (2011): "Duopoly Competition, Escape Dynamics and Non-Cooperative Collusion," CERGE-EI Working Paper 445.

Juillard, Michel. (1996). "Dynare : A Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm", CEPREMAP Working Paper 9602.

Kandori, M., G. J. Mailath, and R. Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, 61, 29–56.

Karatzas, I., and S. E. Shreve (1991): "Brownian Motion and Stochastic Calculus," Springer-Verlag, New York, Berlin and Heidelberg.

Kasa, K. (2004): "Learning, Large Deviations, And Recurrent Currency Crises," *International Economic Review*, 45, 141–173.

Kimball, M.S. (1995). "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking* 27, 1241–77.

Kolyuzhnov, D., A. Bogomolova, and S. Slobodyan (2006): "Escape Dynamics: A Continuous–Time Approximation," CERGE–EI Working Paper 285.

Kushner, H. J. (1984): "Robustness and Approximation of Escape Times and Large Deviations Estimates for Systems with Small Noise Effects," *SIAM Journal of Applied Mathematics*, 44, 160–182.

Kuttner, Kenneth Neil, and Tim Robinson. (2008). "Understanding the Flattening Phillips Curve", Reserve Bank of Australia, Research Discussion Paper 2008-05.

Kydland, F. E., and E. C. Prescott (1977): "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473–91.

Mankiw, N. Gregory, and Ricardo Reis. (2002). "Sticky Information Versus Sticky Prices: A Proposal To Replace The New Keynesian Phillips Curve", *The Quarterly Journal of Economics*, 117(4): 295-1328.

Marcet, A. and J.P. Nicolini (1993). Recurrent Hyperinflations and Learning. *American Economic Review* 93, 1476-1498.

Marcet, A., and T. J. Sargent (1989): "Convergence of Least Squares Learning Mechanisms in Self-Referential Linear Stochastic Models," *Journal of Economic Theory*, 48(2), 337–68.

Marimon, R. (1997): "Learning from Learning in Economics," in Advances in Economics and Econometrics: Theory and Applications., ed. by D. M. Kreps, and K. F.Wallis, vol. 1, chap. 9, pp. 278–315. Cambridge University Press, Cambridge.

McGough, B. (2006): "Shocking Escapes," *Economic Journal*, 116, 507–528.

Milani, Fabio. (2006). "A Bayesian DSGE Model with Infinite–Horizon Learning: Do 'Mechanical' Sources of Persistence Become Superfluous?", *International Journal of Central Banking*, 2(3).

Milani, Fabio. (2007). "Expectations, Learning and Macroeconomic Persistence", *Journal of Monetary Economics*, 54(7): 2065-2082.

Milani, Fabio. (2011). "Expectation Shocks and Learning as Drivers of the Business Cycle," *Economic Journal,* 121(552): 379-401.

Mil'shtein, G. N., and L. B. Ryashko (1995): "A First Approximation of the Quasipotential in Problems of the Stability of Systems with Random Non–Degenerate Perturbations," *Journal of Mathematics and Mechanics*, 59, 47–56.

Nunes, Ricardo. (2009). "Learning the Inflation Target", *Macroeconomic Dynamics*, 13(2): 167-188.

Ormeño, Arturo. (2010). "Disciplining Expectations: Using Survey Data in Learning Models," mimeo, UPF.

Orphanides, A., and J. C. Williams (2003). Imperfect Knowledge, Inflation Expectations, and Monetary Policy. NBER Working Paper No. W9884.

Orphanides, Athanasios., and John C. Williams. (2005a). "Inflation scares and forecast-based monetary policy", *Review of Economic Dynamics*, 8(2): 498-527.

Orphanides, Athanasios., and John C. Williams. (2005b). "The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations", *Journal of Economic Dynamics and Control*, 29(11): 1927-1950.

Paciello, L. (2009a). "Monetary Policy Activism and Price Responsiveness to Aggregate Shocks under Rational Inattention," Mimeo, Einaudi Institute for Economics and Finance, Rome.

Paciello, L. (2009b). "Does Inflation Adjust Faster to Aggregate Technology Shocks than to Monetary Policy Shocks?" Mimeo, Einaudi Institute for Economics and Finance, Rome.

Pearlman, Joseph, David Currie, and Paul Levine. (1986). "Rational Expectations Models with Partial Information", *Economic Modelling* 3(2): 90–105.

Pesaran, H., A. Pick, and M. Pranovich (2013): "Optimal Forecasts in the Presense of Structural Breaks," *Journal of Econometrics*, 177, 134–152.

Preston, Bruce. (2005). "Learning About Monetary Policy Rules When Long-Horizon Expectations Matter", *International Journal of Central Banking*, 1(2).

Puhalskii, A. A. (2004): "On Some Degenerate Large Deviations Problems," *Electronic Journal of Probability*, 9, 862–886.

Sargent, Thomas J. (1993). "Bounded Rationality in Macroeconomics," Clarendon Press, Oxford.

Sargent, T. J. (1999): "The Conquest of American Inflation," Princeton University Press.

Sargent, T. J., and N. Williams (2005): "Impacts of Priors on Convergence and Escapes from Nash In‡ation," *Review of Economic Dynamics*, 8(2), 360–391.

Sargent, T. J., N. Williams, and T. Zha (2006): "Shocks and Government Beliefs: The Rise and Fall of American In‡ation," *American Economic Review*, 96, 1193–1224.

Sims, Christopher A. (2003). "Implications of Rational Inattention", *Journal of Monetary Economics*, 50(3): 665–690.

Slobodyan, S. and R. Wouters (2009). "Learning in an Estimated Medium-Scale DSGE model," CERGEI-EI Working Paper No. 396.

Slobodyan, S. and R. Wouters (2010). "Estimating a Medium-Scale DSGE Model with Expectations based on Small Forecasting Models," Mimeo.

Slobodyan, Sergey, and Raf Wouters. (2012a). "Learning in an Estimated Medium-Size DSGE Model", *Journal of Economic Dynamics and Control*, 36(1), 26-46.

Slobodyan, Sergey, and Raf Wouters. (2012b). "Estimating a Medium-Scale DSGE Model with Expectations based on Small Forecasting Models," *American Economic Journal: Macroeconomics* 4 (2), 65-101.

Smets, F. and R. Wouters (2003). "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association* 1, 1123-75.

Smets, F. and R. Wouters (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97, 586-606.

Sontag, E. D. (1998): "Mathematical Control Theory: Deterministic Finite Dimensional Systems," Springer, New York.

Stock, James H., and Mark W. Watson. (2006). "Why Has U.S. Inflation Become Harder to Forecast?", NBER Working Paper 12324.

Svensson, Lars E. O., and Michael Woodford, (2003), "Indicator Variables for Optimal Policy", *Journal of Monetary Economics*, 50 (3): 691-720.

Timmerman, Allan G. (2006). "Forecast Combinations". Ch. 4 in "*Handbook of Economic Forecasting, Volume 1*", Elliott, Graham, Clive W. J. Granger, and Allan Timmermann, eds. Elseiver, North Holland.

Watson, Mark W., and James H. Stock. (2004). "Combination forecasts of output growth in a seven-country data set", *Journal of Forecasting*, 23(6): 405-430.

Vilagi, B. (2007). "Adaptive Learning and Macroeconomic Persistence: Comparing DSGE models of the Euro Area," Mimeo, Magyar Nemzeti Bank.

Williams, N. (2001): "Escape Dynamics in Learning Models," Ph.D. thesis, University of Chicago.

Williams, N.(2002): “Stability and Long Run Equilibrium in Stochastic Fictitious Play,” Manuscript, Princeton University.

Williams, N. (2003): “Adaptive Learning and Business Cycles,” Manuscript, Princeton University.

Williams, N. (2004): “Small Noise Asymptotics for a Stochastic Growth Model,” *Journal of Economic Theory*, 119(2), 271–298.

Williams, N. (2009): “Escape Dynamics in Learning Models,” mimeo.

Kolyuzhnov, D., Bogomolova, A., Slobodyan, S. Escape dynamics: a continuous-time approximation. *Journal of Economic Dynamics & Control.* 2014, **38**(January), 161-183. ISSN 0165-1889.

Slobodyan, S., Wouters, R. Learning in an estimated medium-scale DSGE model. *Journal of Economic Dynamics & Control.* 2012, **36**(1), 26-46. ISSN 0165-1889.

Slobodyan, S., Wouters, R. Learning in a medium-scale DSGE model with expectations based on small forecasting models. *American Economic Journal: Macroeconomics.* 2012, **4**(2), 65-101. ISSN 1945-7707.