CZECH TECHNICAL UNIVERSITY IN PRAGUE

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REVIEW OF PH.D. THESIS

Goal-oriented a posteriori error estimates and adaptivity for the numerical solution of partial differential equations
by

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1 Contents of the review

This review consists of the following sections:

- scope of the Ph.D. thesis, extent of new knowledge contained in this work,
- scientific relevance of the submitted thesis,
- quality, correctness, and originality of results achieved,
- quality of treatment of the topic,
- scientific qualities of the applicant,
- conclusions, and
- questions and comments for the thesis defense.

2 Scope of the thesis, extent of new knowledge contained in this work

In the submitted thesis, the candidate presents a comprehensive strategy for goal-oriented error control in the numerical solution of time-independent Partial Differential Equations (PDEs). The author relies on the Discontinuous Galerkin (DG) method (for the discretization of the underlying PDEs) and the Dual Weighted Residual (DWR) method (for error estimation), and extends the existing results in multiple ways. Considering a general scalar linear convection-diffusion equation, he presents its adjoint-consistent discretization, a unified treatment of discretization and algebraic errors, higher-order reconstruction techniques, and fully automated anisotropic hp-adaptation algorithms. In the final part of the thesis, some of these results are extended to non-linear problems, the compressible Euler equations in particular. All these topics represent a significant amount of new knowledge and original contributions to the field of the adaptive solution of PDEs, with particular emphasis on the DG method.

3 Scientific relevance of the submitted thesis

From a broader perspective, the thesis serves as an excellent example of challenges one faces when constructing reliable and accurate algorithms for solving complex PDEs. I would like to particularly highlight that the candidate succeeded in tackling all steps of this involved process, starting from structural consistency between primal and dual formulations or the inclusion of algebraic errors (for DWR estimates), up to the application of higher-order reconstruction or fully anisotropic mesh adaptation (for DG discretization). Besides, the last chapter openly admits some limitations of the developed framework when applied to the compressible Euler system, which is appreciated.

4 Quality, correctness, and originality of achieved results

I see the main results of the thesis in

1. very systematic, instructive, and self-contained summary of the state-of-the-art in the fields of DG/DWR-based goal-oriented error estimates (Chapter 1),
2. development of the complete theoretical framework for linear convection-diffusion problems, including local higher-order reconstructions of the solutions and the treatment of algebraic errors (Chapter 2),
3. extension of techniques from Chapter 2 to fully anisotropic hp-adaptation and systematic testing of the method for elliptic, hyperbolic-elliptic, and convection-dominated problems (Chapter 3), and
4. application of the framework to non-linear compressible Euler equations, including identification of sources for the divergence of the algorithm (Chapter 4).

The results of Chapters 2 and 3 have already been published in leading peer-reviewed journals in the fields of applied and computational mathematics; Chapter 4 provides an excellent basis for yet another high-quality journal publication, once the candidate resolves the discretization-related issues. These facts best demonstrate the originality of the achieved results.

5 Quality of treatment of the topic

I much enjoyed reading the thesis because of (at least) two reasons. First, the candidate has succeeded in explaining the highly complex and technical subject of DG/DWR-based goal-oriented error estimation in a systematic, comprehensible, logical, and accessible manner that is easy to follow. Second, the text itself is written carefully with a minimum number of typos, in good English, and all developments are illustrated with well-chosen numerical examples when appropriate.

6 Scientific qualities of the applicant

I consider this aspect of being one of the strongest points of the submitted thesis. The topic itself is very challenging — for the successful completion of the thesis, the candidate must have obtained in-depth knowledge in diverse fields such as qualitative properties of solutions to PDEs, duality methods in error quantification, estimation of discretization and algebraic errors, or optimization methods for solving the hp-adaptation problems, among others. Complementary to that, the candidate has already published seven papers in high-quality peer-reviewed journals, which is impressive given his career stage. Therefore, I have no doubt that Mr. Filip Roskovec has become a fully proficient independent researcher (at least) in the field of numerical and computational mathematics.

7 Conclusions

As evident from the phrasing of all previous sections, I am confident that the scientific work collected in the submitted doctoral thesis presents a significant, topical, and timely contribution to the field of goal-oriented error estimation with high-order discontinuous Galerkin methods. I am fully convinced that the submitted thesis contains a considerable amount of new scientific results and demonstrates the applicant's command of the research field.

For these reasons, I wish to support, in full confidence, the award of Ph.D. degree to Mr. Filip Roskovec in the Mathematics/Scientific and Technical Calculations program, after a successful thesis defense.

8 Questions and comments for the thesis defense

The candidate is kindly asked to address the following questions and remarks during the Ph.D. thesis defense.

1. On page 10, last paragraph, you introduce a function space W_h such that $u \in W_h$ and $V_h \subset W_h$. How can you ensure that this condition holds for the space W_h defined in (1.7) and the solution u specified in Definition 1.1?
2. On page 46 you introduce the reconstruction procedure inspired by the Zienkiewicz-Zhu procedure for continuous Galerkin methods. Is there a proof that the reconstruction process is well-posed and well-conditioned?
3. Please outline how the anisotropic refinement algorithm, explain in Section 3.2, would extend to three dimensions, or explain why this is impossible.
4. In Remark on page 76 you claim that you used "an overkill degree of numerical quadrature to suppress these errors". Could you provide the committee with additional details?
5. On page 97 you claim "Yet, we cannot simply set $\boldsymbol{w}_h^{(-)} := \boldsymbol{w}_{\mathrm{BC}}$ on Γ_{IO} since the system (4.4) is hyperbolic". Can you briefly illustrate and clarify the statement?
6. As, e.g., the list of publications on page 145 suggests, some results presented in the thesis were obtained in collaboration with other co-authors. Could you please clarify what precisely was your contribution?

Professor
URL: http://mech.fsv.cvut.cz/~zemanj

Prague, August 19, 2019