Interdisciplinary Center for Scientific Computing (IWR)
Mathematical Methods of Simulation
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To whom it may concern

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Report on the doctoral thesis "Goal-oriented a posteriori error estimates and adaptivity for the numerical solution of partial differential equations"

The thesis is about error estimation and adaptive mesh refinement in context of higher order discretizations, which as such is a field somewhat neglected by the main stream adaptivity research. This by itself is a recommendable feature. It is a well-written overview over the state of the art and it extends available methodology in three directions.

1. **Anisotropic hp-adaptivity:** most research in adaptive methods focuses on bisection refinement. Few researchers include adaptation of shape and orientation of mesh cells, which on the other hand is most important for advection dominated problems as considered in this thesis. The author extends ideas originally developed by Vít Doleiší in a very original way to obtain discretizations with a much better ratio of accuracy and degrees of freedom than standard approaches. The way of combining anisotropic mesh cells with higher order polynomials and the selection between h- and p-refinement is quite original.
2. **Reconstruction of higher order approximations:** these are not only a way to obtain higher accuracy, they are also a a key ingredient to the error estimation technique using weighted residuals. Standard reconstruction techniques require regularity of the solution and the mesh. While regularity of the solution is unavoidable, the author found a way to produce a stable and accurate reconstruction without mesh regularity, and in particular on the anisotropic meshes needed for optimal representation of the solution. The reconstruction principle works for any polynomial degree and produces accuracy of one degree higher. Therefore, the data required for error estimation and adaptivity can be computed in a reliable and efficient manner without solving global problems with much increased accuracy and effort.
3. **Error estimates with algebraic error:** a posteriori error estimation is based on Galerkin orthogonality, which in turn assumes accurate solution of the discrete problem. In particular if no good preconditioner is available, this can cause large computational effort. Therefore, a posteriori error estimates which include the algebraic error after an insufficient number of steps of an iterative, discrete solver are developed. The treatment in this thesis is considerably more advanced than previous work by Meidner or Becker, Johnson, and Rannacher.

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The scientific work in this thesis improves the capability of adaptive methods for advection dominated problems considerably towards efficient approximation with higher order polynomials. It addresses important questions, and thus it will have impact on an important branch of mathemaics and scientific computation. The mathematical treatment in chapters 2 and 3 is solid and original. The results for inviscid compressible flow in chapter 4 show, that the methods are not only analyzable for toy problems, but are actually applicable to very complicated, nonlinear equations of high relevance in applications. In addition to this application, the method should transfer almost immediately to viscous flow problems. In addition to fluid mechanics, the work in this thesis is in principle applicable to solid mechanics, such that beyond applied mathematics, the thesis is of interest to engineering in general.

In addition to the scientific novelty discussed above, the thesis comprises of a good introduction into the state of the art of goal oriented error estimation. It is well-written throughout, such that I can confirm the author's strong ability for creative scientific work and its communication.

Yours sincerely,

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