Evaluation of the doctoral thesis "*Orbital and internal dynamics of terrestrial planets*" by Michaela Walterová of the Faculty of Mathematics and Physics at the Charles University in Prague

This is a very well-thought and well-written dissertation whose author definitely deserves the title of a Doctor.

The work addresses several complex questions posed by the rich and complex dynamics of close-in terrestrial planets. These are the planets mostly affected by tides, a phenomenon which influences the planets' orbits and spin and also heats up the planets, thus adding to their internal evolution. The interplay between the orbital and rotational dynamics, on the one side, and the geophysical evolution, on the other side, makes this research highly complex and requires combination of analytical and numerical tools.

The author has carefully chosen the questions to explore, and has demonstrated a professional knowledge of both celestial mechanics and geophysics, along with a solid proficiency in numerics. The conception and execution of the modeling is sufficiently broad as to allow application to a wide variety of terrestrial planets. The author has made a considerable contribution in the planetary sciences, and the principal results of her research have already been published in peer-reviewed scientific journals. This contribution safely covers the requirements for a Ph.D. degree.

Chapter 1 offers a concise but deep review of the orbital and rotational mechanics methods employed in the project. On one point, the author's story goes beyond a mere review, though. It has been known for some time that certain inclination functions provided in Kaula's works have signs opposite to those given in other sources. Veras et al (2019) suggested that Kaula's signs were in error. I am glad to see that the author of this Thesis has put an end to this controversy by establishing, in Section 1.3.3, that the difference in signs had resulted simply from different conventions, not from an error. This is a small but *bona fide* contribution.

Chapter 2 serves as a geophysical exordium. It provides a squeezed explanation on how our knowledge about rocky exoplanets is gleaned, and how the observed data and numerical modeling set constraints on the planets' composition and structure. The chapter also presents the necessary information on rheological models, as well as on the mechanisms determining the heating and cooling of a terrestrial planet.

Chapter 3 explains the basics of tidal torques and tidal heating. This chapter also presents the author's study of the sensitivity of the tidal heating rate to the eccentricity, to the core mass fraction (CMF), and to the values of the mantle's average rigidity and viscosity.

Chapter 4 addresses coupled thermal and orbital evolution of model bodies exemplified with three low–mass exoplanets with nonzero eccentricity: Proxima Centauri b, GJ 625 b, and GJ 411 b. The study is carried out through the medium of a semianalytical model of a spin-orbit evolution of a layered planet with emerging subsurface magma ocean. The model includes a self–consistent calculation of the tidal dissipation rate, and a simplified parameterised model of the mantle convection in the stagnant–lid regime. The dependence of the tidal dissipation rate and the highest stable spin state on the rheological parameters is explored. Coupled thermal-orbital evolution of each of the three planets is investigated.

Chapter 5 is devoted to consideration of the secular effect from a perturber on an outer orbit. The setting implies large mutual inclinations of the two orbits, which may give rise to the Lidov-Kozai cycles. A combined calculation of the tidally– and externally–induced orbital and spin evolution of a planet is carried out.

Chapter 6 deals with planet-planet tides in tightly packed systems. The author derives the tidal potential due to planet–planet loading, transformed to a planetocentric coordinate system and expressed via the Keplerian elements, in the spirit of the Darwin-Kaula series. Therefrom, the author naturally derives an expression for the tidal torque. I am very impressed with this piece of work. It is destined to be in textbooks.

Chapter 7 is dedicated to developing an alternative, numerical, approach to tides. The author presents a numerical scheme to compute the tidal torque and the tidal heating rate in a possibly nonhomogeneous mantle. The approach employs a spherical harmonic decomposition and the method of finite differences.

Comments and criticisms

My comments and criticisms vary in their scale from cosmetic to minor, and none of them will influence my very positive impression of this thesis.

- Section 2.2 On the right-hand side of equation (2.4), $\sin\epsilon$ must be changed to $\sin|\epsilon|$, to keep Q positive definite. Also, to avoid confusion between formulae (2.3) and (2.4), it is necessary to point out that (2.3) renders the nominal (“seismic”) lag, while (2.4) gives us the tidal lag. Generally, the two are different. Equation (2.3) entails $1/Q = \sin|\delta|$ where δ is the phase lag between the strain and the stress. Generally, there is a difference between δ and ϵ, and this difference becomes noticeable at the lowest frequencies.

  The same pertains to formulae (1.34) and (1.35) in Section 1.3.2.

- In Section 1.3.1, we read:

  "*To remain in the domain of the perturbation theory, we also require that the disturbing function for each of the planets is much smaller than the Newtonian potential due to the primary, < ……> In other words, the trajectories of both planets can be at each time t described by a Keplerian orbit*."

  This popular belief is incorrect. If we examine, step by step, the derivation of the Lagrange- or Delaunay-type systems of planetary equations, we will find that, embarrassingly enough, these derivations do NOT require the perturbation to be small. In principle, it can be larger than the fiducial, Newtonian, part of the interaction. Consequently, the resulting perturbed orbits can, in principle, differ very considerably from the fiducial conics.

  I certainly don't hold this minor oversight against the defendant, because it in no way influences her calculations. Also, the topic has never been presented well in textbooks.

- In Section 2.3.2, it might be good to point out that, for expression (2.43) for the tidal dissipation rate to come out so short and elegant, we have to average not only over the orbital period but also over the period of apsidal precession, under the assumption that (a) apsidal precession is uniform and (b) its period is shorter than the geophysical timescales for which the said expression is relevant. This may be a minefield if you attempt to consider faster geophysical changes.

  This pertains also to equation (4.9) in Section 4.3.2.

- After equation (A1) in Appendix A, it might be good to say that the vector $\mathbf{u}$ is lacking a longitudinal (toroidal) component because the toroidal force is neglected.

As I said above, none of these points is considerable.

Now I would like to ask the author several questions.

- Although I have already asked this question in my correspondence with the author – and have received from her a perfectly clear answer – I would repeat it here, to keep our discussion in the open, and also in case other involved colleagues get interested in this topic. So here it goes: why in Kaula's expansion for the potential do we have a product of two G functions in the tidal problem and a product of G by H in the orbital case (eqn 1.14)?

- Regarding the author's expression (2.34) for the Andrade creep function. I used to think that the second term in this expression should read as

$$-\ \frac{i}{\eta\ \omega}\ =\ -\ \frac{i\ J_U}{\omega\ \tau_M}$$

where $\tau_M = \eta/J_U$. Why does the author have in (2.34) $\delta J = J_R - J_U$ instead of simply J_U ? Is this a misprint? Or does the author define the timescale as $\eta/\delta J$?

- The opening paragraph of Section 3.2.2 says that to reduce the truncation error to 10^{-4} for e = 0.5, the terms with $q_{max} = 7$ were sufficient. This is surprising. In the paper by Makarov et al. (2012),[1] when dealing with e ≈ 0.27, we had to take into account the terms from q = −1 through q = 6. It is also known that with each decimal step in the value of e the number of terms required is growing very rapidly.[2] Getting a reasonable precision with $q_{max} = 7$ for e = 0.5 looks miraculous. Did the author, perhaps, imply e = 0.05 ? Or was such a small number of terms sufficient in Section 3.2.2 because no orbit integration was involved there? (Orbit and spin propagation would imply higher precision at every step.)

- Regarding the PSR regime depicted in Figure 3.5, the author very rightly warns us that the PSR is available only for very low mantle viscosities. What are the actual surface temperatures corresponding to the PSR in that figure? Are they really high enough to ensure the absence of permanent triaxiality? Or is the PSR shown here for purely illustrative purposes and should not be taken close to heart?

- How was equation (3.7) derived from (3.6)?

Recommendation for the future research:

- There is a topic virtually neglected in the literature hitherto. So far, I have seen it addressed only passingly, on one or two occasions only. This is the sensitivity of the combined thermal-orbital and spin evolution to the rheological parameter ζ. If the author wants, she may, in the future, consider looking into this issue.

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[1] Makarov, V.V.; Berghea, C.; and Efroimsky, M. 2012. ``Dynamical evolution and spin-orbit resonances of potentially habitable exoplanets. The case of GJ 581d.'' *The Astrophysical Journal* 761:83

[2] Renaud, J. P.; Henning, W.G..; Saxena, P.; Neveu, M.; Bagheri, A.; Mandell, A.; and Hurford, T. 2020. ``Tidal Dissipation in Dual-Body, Highly Eccentric, and Non-synchronously Rotating Systems: Applications to Pluto-Charon and the Exoplanet TRAPPIST-1e.'' *The Planetary Science Journal* - In Press. https://arxiv.org/abs/2010.11801

- I am glad that the author has mentioned, in Section 5.5, both the J_2-generated and relativistic additions to the apsidal-precession rate. In fact, there also exists a J_2-generated addition to the nodal precession. Now, let us recall that both precession rates enter the expressions for the Fourier tidal model. For extremely tight configurations, this effect becomes noticeable, as was recently demonstrated by Santiago Luna in his PhD Thesis; also see Figure 2a in his A&A paper.[3] The presence of these additions in the expressions for the tidal modes will be working to render small deviations of the stable states from the exact spin-orbit resonances. These deviations, however, will be close to zero in the presence of an additional torque generated by the permanent triaxiality. I am wondering if J_2 will manage to entail any considerable effect aside from that described in *Ibid*.

I recommend that this work be accepted as a doctoral thesis.

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[3] Luna, S., et al. 2020. ``The dynamical evolution of close-in binary systems formed by a super-Earth and its host star.” *Astronomy & Astrophysics*, 641 : A109