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Referee report on the PhD thesis of M. Walterová

The topic of the PhD thesis of Mgr. M. Walterová is “Orbital and internal dynamics of terrestrial planets”. Its goal is to develop a complex model which would account for orbital, rotational, tidal, as well as thermal evolution. This requires a multi-disciplinary approach, connecting celestial mechanics, geophysics and exoplanet science. It is a logical continuation of works by Kaula (1964), Ferraz-Mello (2013), or Boué & Efroimsky (2019). First, a useful review is presented in Chapters 1 and 2. Second, an analytical model of the coupled orbital/thermal evolution based on *realistic* rheological models ($J(\omega)$) is developed in Chapters 3, 4 and 5. This is quite important, because long-term evolution results in different equilibrium states, in particular, spin-orbit resonances of higher orders (3:2, 2:1) than synchronous (1:1). Another analytical model for planet-planet tides is described in Chapter 6. Finally, there is a numerical model of tides (from Chapter 7) which has a big potential -- it uses the FDM in r, spherical harmonics in θ, φ, it’s implicit in t. It was used to compute the torque, heating, Love number k_{22} or phase lag ε_{22}, and served as a verification of analytical models (p. 172).

On the other hand, there seem to be several weak points. For example, in Chapter 3, the material rigidity μ and viscosity η are treated as free parameters -- spanning 12 orders of magnitude. Unfortunately, they are not constrained by observations very well. Consequently, the

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respective discussion is a bit uncertain.

Moreover, in Chapter 4, where an application to GJ 625 b, GJ 411 b, and Proxima Centauri b exoplanets is discussed, none of the observed eccentricities is significantly *non-zero* (see below). Because the initial eccentricities of exoplanets are generally unknown (and they might have been also close to zero), the interpretation of their tidal evolution is very uncertain.

The analytical model from Chapter 6 suffers from slow convergence, which is likely due to “localized” conjunctions described by the Kaula expansion, i.e., as a Fourier in the orbital longitudes. Possibly, some computational optimisations may solve this problem, or as mentioned by the author, a fully numerical model (as the one in Chapter 7) may be more practical.

Finally, let us mention a few individual issues:

page 15, paragraph 1, line 1
“... described by a Keplerian ellipse.” <- This isn’t accurate, perturbed trajectories aren’t ellipses. The assumption of small perturbations is not needed (at this point). One can still use the Lagrange planetary equations (and integrate them numerically). The only “problem” is that osculating elements -- or their geometrical representation, respectively -- will be very different from the actual trajectory (cf. β-meteoroids).

16, Eq. 1.10f
There is a mistake in the 2nd term, which should read + \sqrt{1-e^2}... (cf. Murray & Dermott 1999, p. 251).

79, Tab. 3.1
How do you combine the 3 layers? Is it a sum of 3 torques, or only 1? What do you expect about the interfaces? Is a separate rotation of core/mantle possible, similarly as in stars? (cf. $\Omega(r, \theta)$ for the Sun)

80, Fig. 3.3-3.5
How this can be possibly observed (e.g., as the heat flux, volcanic activity, surface magma ocean, distribution of eccentricity, or spin rates of exoplanets)? Are η, μ parameters constrained? This is probably the weakest point of the work...

Note: There is 1 paragraph on observations on p. 120.

93, Sec. 4.4

Is the surface temperature also affected by the atmosphere (opacity)?

95, Tab. 4.3

Why the surface boundary condition is T_s = 500 K = const.? Consequently, there is no response to the computed tidal heat flux...

For comparison, the equilibrium T_s = 234 K on Proxima Centauri b.

97, Sec. 4.5

Some information on your numerical model is missing: the numerical method (FDM?), discretisation in space, discretisation in time (Euler?), time step limitation (adaptive?), discretisation error, accumulated error, etc.

99, 5, 3

Why the geometric mean of $\eta(r)$ is used instead of volumetric- or mass-weighted mean?

102, 2, 3

"e = 0.04, 0.13, and 0.25" <- This does *not* correspond to observations of GJ 625 b. Please, see the posterior distribution $p(e)$ in Suares-Mascareno etal. (2017), Fig. 17, where e = 0 has the same probability as 0.13. It's really necessary to understand the observations...

103, 1, 6

"e = 0.22 ± 0.13" <- Again, this does *not* correspond to observations of GJ 411 b. Please, see Diaz etal. (2019), p. 14, where they write: "... a significant eccentricity is not detected". Moreover, the SOPHIE spectrograph suffers from zero-point offsets ± 7 m s^{-1}, while the RV amplitude is only 2 m s^{-1}.

103, 3, 6

"e = 0.08-0.06+0.07" <- Once again, e = 0 is fully compatible with observations of Proxima Centauri b. Please, see the RV curve in Jenkins etal. (2019), Fig. 3. Moreover, 4 data sets are combined, and one can expect some (remaining) systematics.

111, 2, 5

Despite the fact that you consider it beyond the scope, could you comment on the subsurface magma ocean, please?

114, Sec. 4.8.2

Among the sources of eccentricity, you may want to mention giant collisions (not only close encounters), or the hot-trail effect (Chrenko etal. 2017). Note Ragusa etal. consider a very massive eccentric disk as well as planets (10 M_J).

126, 3, 1b

"... was tested against the symplectic N-body integrator Rebound (ref.)." <- This is not sufficient information -- results have to be different, because the models are different (secular vs. N-body).

153, Fig. 6.1

Regarding the slow convergence -- could be the evaluation faster on multiple cores (using OpenMP or MPI code)?

170, 1, 11

A comparison of tidal torques with Correia etal. (2014) is only qualitative. Would it be possible to do it quantitatively, please?

Nevertheless, none of the issues is 'critical' -- I believe most of them can be clarified during the defense. On contrary, I think it is the very nature of scientific work that some issues always remain, even though a lot of issues must have been solved. It is clear that the PhD candidate is capable of doing it, use state-of-the-art models, and discuss up-to-date problems, which is confirmed by her 2 first-author publications.

Therefore, I fully support the application of M. Walterová and I recommend her PhD thesis for defense.

With kind regards

doc. Mgr. Miroslav Brož, Ph.D.

In Prague, Feb 12th 2021

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Minor points:

English language is good, only a few words could be better in the given context, e.g.: inhabited, cooperation, introduction -> definition, signifies, symbolises -> denotes, endowed, journey, in place -> instead, neglected -> suppressed, potentially -> possibly, unfolded -> described, applied to -> in, mention -> attention, grounds, empirically.

Regarding a/the articles, the rule isn't that simple as 1st a, 2nd the.

6, 2
It would be useful to explain exactly, *where* the dissipation occurs.

7, 2
It would be useful to mention the actual heat flux is 0.09 W m^{-2}.

8, 1
The discussion of exoplanets is qualitative; a *quantitative* plot (period vs. mass) would be useful. Eccentricities are also increased by close encounters, collisions, secular perturbations, gas dynamics, pebble accretion etc. (as explained later on p. 67).

11, Eq. 1.1
The law is surely vectorial.

11, 3, 1b
"substitution" is actually a coordinate transformation, inertial -> non-inertial
"one-body problem" is misleading, it's a *relative* motion

12, 1, 3
This approximation is not needed (at least at this point).

12, 2
the Kepler eq. $E(M)$ is not included

14, 2
definition of L, G, H momenta is missing

15, footnote

“vanishing of the perturbation” <- but keeping the velocities

18, 1, 3

“...described by a Keplerian orbit.” <- *dtto*

19, 4, 2b

“\psi represents the unique configuration...” <- Well, it’s only a linear combination; the configuration is rather expressed as coefficients.

22, 2, 2

“(studies)... over the entire age of the system... require... secular resonances.” <- no, even on much shorter time scales

22, 2, 7

“... secular resonances involving Saturn” <- no, it’s a fundamental frequency (determined by the configuration of the *whole* planetary system)

22, 3, 3

“centrifugal force” <- first, one should introduce a non-inertial reference frame

22, footnote

“... infinite period.” <- which is called deterministic chaos

23, 1, 2

“energy dissipation” -> transversal acceleration; dissipation is responsible for the misalignment

26, 2, 2b

“two model satellites” <- I understand, but would be much easier to understand it, if the notation is like “1, 2, 3”, or “Earth, Moon, test particle”.

47

again, a reprinted mass-radius Fig. would be useful

58

again, Figs. would be useful (to understand dislocations, boundaries)

67, 2

additional heat sources might be a nebula (cf. its viscous heating), irradiation, or climate?

68, Eq. 2.41

this formula isn’t valid for radioactive daughter elements, right?

96, Eqs. 4.21, 4.22

Can we use them safely up to the core-mantle boundary pressure of ~140 GPa?

107, Fig. 4.5

panels are too small, although it is the main result

117, Sec. 4.8.4

I agree the concept of the habitable zone is problematic.

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