Report on the Habilitation Thesis "Mathematical analysis of nonlinear systems describing flows of incompressible fluids" by Miroslav Bulicek

Bulicek's habilitation thesis is based on six joint research papers published, or accepted for publication, in international mathematical journals. They deal with various fundamental aspects of the mathematical theory of flows of incompressible fluids. The results obtained in these papers are collected in three separate sections of his thesis, which are preceded by an introduction devoted to a general presentation of the mathematical models under consideration. Their basic formulation amounts to the system of partial differential equations

$$\begin{cases} \varrho(\partial_t v + \operatorname{div}(v \otimes v)) - \operatorname{div} S = -\nabla p + \varrho b \\ \operatorname{div} v = 0, \end{cases} \tag{1}$$

where v denotes the velocity of the fluid, ϱ its density, b the density of the external forces, p the mean normal stress and S the part of the Cauchy stress that characterizes the material properties of the fluid.

Section 2 deals with those presented papers that deal with models involving constitutive laws linking S with the symmetric gradient $D(v)$ of v via an implicit relation of quite general nature. In particular, laws governed by growths of non-necessarily polynomial type, namely of Orlicz type, are included in the discussion. Various deep results on the existence and regularity of solutions are obtained. Some of the conclusions also improve and complement results available in the literature in the classical case of power type growths.

In Section 3, system (1) is coupled with the equation for the internal energy e, which takes the form

$$\partial_e + \operatorname{div}(ev) - \operatorname{div}(\kappa(e, |\nabla e|)\nabla e) = S \cdot D(v), \tag{2}$$

where κ stands for heat conductivity. Diverse results, both in dimension 2 and 3 are exibited on the maximal regularity of solutions, under specific additional strucure assumpions on the equations.

Finally, in Section 4 fluids with possible turbolent behaviour come into play. Specifically, a mathematical model of turbolence introduced by Kolmogorov is adopted, which amounts to a couple of additional equations to be combined with system (1). The classical velocity v has now to be interpreted as an averaged velocity. The novelty consists in a non-constant effective kinematic viscosity of the fluid. A delicate existence result under suitable assumptions on the intial and boundary data is established for the resultant system.

Altogether, the material of this thesis is a remarkable contribution to its area. The results of the presented papers definitely constitute a substantial progress in the theory of systems of partial differential equations arising in fluid dynamics.

The exposition of the thesis is extremely effective. In each section, the physical setting is carefully described and the mathematical model explained and justified. The state of the art on the matter is exaustively outlined and the difficulties to be faced in approaching the problems under consideration are clearly summarized. The original results are eventually stated, sometimes not in the most general form, in order to avoid unnecessary complications and make them accessible even to non-experts. Several useful comments are also included about novelties in the proofs, that made the advances of this habilitation thesis possible.

The work of M.Bulicek demonstrates that he has an excellent konwledge of the status of the art and of methods of the mathematical theory of fluid dynamics, to which he has contributed with original ideas and techniques. I expect further innovative results from Bulicek's research projects. He is definitely an outstanding candidate for an habilitation at the Faculty of Mathematics and Physics at Charles University, as well as in any other prestigious academic institution worldwide.

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