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Report on the habilitation thesis "On the interplay of Continuum Theory, Topological Dynamics and Descriptive Set Theory" by Benjamin Vejnar

As it is mentioned in the title, Benjamin's work is related to three areas of General Topology and Set Theory: Continuum Theory, Topological Dynamics and Descriptive Set Theory. In the three areas Benjamin has made a relevant work. His achievements vary from clever and impressive counterexamples to the development of theories which provide important and deep mathematical tools.

I consider this thesis has a high quality and its mathematical content is original, creative and interesting. This thesis indeed shows that the author has the ability for creative scientific work and also shows that Benjamin is a mature researcher with a deep understanding of the mentioned fields. He and his coauthors have solved important published open problems. I personally (and some reputed colleagues of mine) tried to solve some of these problems without success.

I have had the chance to discuss with Benjamin and I have a very positive impression of his ability to work on difficult mathematical problems.

Next I will make some comments on three of the papers, the ones which are closets to my area of interest.

1. "Non-cut, shore and non-block points in continua", by J. Bobok, P. Pyrih and B. Vejnar.

A continuum is a compact connected metric space with more than one point. The letter X will denote a continuum.

Many properties of a continuum are related and can be better understood by using special types of points. In many cases, it is particularly useful to identify the "border" of the continuum. So in the literature one can find several ways to describe when a point is in the border. In this paper the authors work with 6 different definitions for a point p in X, in this direction, namely:

a. p is a point of colocal connectedness,
b. p is not a weak cut point,
c. p is a non-block point,
d. p is a shore point,
e. p is not a strong center, and
f. p is a non-cut point.

After observing that a => b => c => d => e => f and no reverse implication holds, the authors of this paper make a detailed and systematic study of the role of these notions on some specific families of continua such as chainable and circle-like continua.

The notion of shore point was introduced by Luis Montejano and Isabel Puga (1992); and the notion of block point was introduced by Alejandro Illanes and Pawel Krupski (2011). Continuum Theory researchers have worked and keep working with these types of points. We received this paper very well since the authors clarify and give us new tools to work several aspects on continua.

2. "On blockers in continua", by J. Bobok, P. Pyrih and B. Vejnar.

In 2011, A. Illanes and P. Krupski, introduced the concept of blockers in a continuum X. We proved that:

On locally connected continua the following implication holds:

"if a closed nonempty subset B blocks every finite set, then B blocks every nonempty closed subset of X."

We also asked if this implication holds only in locally continua.

Benjamin and his coauthors answered this question in the negative by finding a clever way to construct a large family of planar non-locally connected lambda dendroids for which the implication holds. In their paper they also study families of continua for which the implication holds.

They also included the question whether the implication holds for the very particular mysterious continuum called the pseudo-arc. The specialists are very interested in this paper and trying to solve this problem.

3. "Incomparable compactifications of the ray", by A. Bartos, R Marciña, P. Pyrih and B. Vejnar.

A ray is the space [0,1). In this paper the authors consider compactifications of the ray having a specific remainder Z. Their main result is the following:

"There exists a family of continuum many incomparable compactifications of the ray whose remainder is an arbitrary fixed non-degenerate Peano continuum."

This result is very interesting for several researchers since, my colleagues and I have considered some related problems. In (2004), Verónica Martínez de la Vega constructed an uncountable collection of non-homeomorphic metric compactifications of the ray with the pseudo-arc as remainder; in (2014), Verónica Martínez de la Vega and Piotr Minc, extended this result for any fixed continuum (not necessarily the pseudo-arc). Moreover, Alejandro Illanes, Piotr Minc and Frank Sturm, in (2015) showed that the in the above result it is not possible to change the word "non-homeomorphic" by "incomparable".

I have gone through the check of originality of the thesis done by the system Turnitin and it is absolutely clear that the thesis represents an original work with minimum overlap with the existing literature.

As you can see, some of Benjamin's work is closely related with the work I have done. So I have a good perspective of some aspects of his research.

Please, do not hesitate in contact me if something else is needed.

Sincerely yours,

Alejandro Illanes,
Full Professor.