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REPORT on habilitation thesis
"On the interplay of Continuum Theory, Topological Dynamics and Descriptive Set Theory" by Dr. Benjamin Vejnar
submitted at Charles University in Prague

The aim of this report is to evaluate scientific level of the results of the candidate, measured through the scientific level of the thesis (quality of publications, international recognition and reputation of the candidate, etc.) . The habilitation was submitted at the Faculty of Mathematics and Physics, Charles University in Prague. Since the University is largest and oldest in Czech Republic and is among best universities worldwide, I will apply the highest international measures for the evaluation. I will also focus mainly on candidate's achievements in the thesis, which seems most important factor, but with a broader view on his career as a whole.

The thesis, which is main object of this evaluation consist of the following papers:

[A] J. Bobok, P. Pyrih, B. Vejnar; *Non-cut, shore and non-block points in continua*, Glasnik Matematicki **51** (2016), 237-253.
doi: 10.3336/gm.51.1.14

[B] J. Bobok, P. Pyrih, B. Vejnar; *On blockers in continua*, Topology and its Applications **202** (2016), 346-355.
doi: 10.1016/j.topol.2016.01.013

[C] A. Bartoš, R. Marcinã, P. Pyrih, B. Vejnar; *Incomparable compactifications of a ray with Peano remainder*, Topology and its Applications **208** (2016), 93-105.
doi: 10.1016/j.topol.2016.05.008

[D] A. Bartoš, J. Bobok, J. van Mill, P. Pyrih, B. Vejnar; *Compactifiable classes of compacta*, Topology and its Applications **266** (2019), article id: 106836.
doi: 10.1016/j.topol.2019.106836

[E] B. Vejnar; *Every continuous action of a compact group on a uniquely arc- wise connected continuum has a fixed point*, Journal of Fixed Point Theory and Applications **20** (2018), article id: 69.
doi: 10.1007/s11784-018-0552-3

[F] J. Bobok, P. Pyrih, B. Vejnar; *On minimal homeomorphisms on Peano continua*, Topology and its Applications **210** (2016), 263-268.
doi: 10.1016/j.topol.2016.07.022

[G] A. Bartoš, J. Bobok, P. Pyrih, S. Roth, B. Vejnar; *Constant slope, entropy and horseshoes for a map on a tame dendrite*, Ergodic Theory and Dynamical Systems (2019), on-line first view.
doi: 10.1017/etds.2019.29

[H] M. Doležal, M. Rmoutil, B. Vejnar, V. Vlasák; *Haar meager sets revisited*, Journal of Mathematical Analysis and Applications **440** (2016), 922-939. doi: 10.1016/j.jmaa.2016.03.065

[I] M. Doležal, B. Vejnar; *Classification of the spaces $C_p^*(X)$ within the Borel-Wadge hierarchy for a projective space X*, Topology and its Applications **183** (2015), 11-17. doi: 10.1016/j.topol.2014.12.021

[J] P. Krupski, B. Vejnar; *The complexity of homeomorphism relations on some classes of compacta*, arXiv e-prints:1808.08760

Before I comment more on details of Vejnar's research in the thesis, let me present bibliometric data. While by no means it is necessary to look into details of papers to judge quality of research, such measures can give a preliminary view of the candidate and his recognition in the mathematical community. Since his first publication in 2010, Vejnar is the author of 20 research articles indexed in WoS. The list of authors of these publications vary, including many papers with mathematicians from local group in Prague (but several different universities), many with Vejnar as the sole author, and a few with colleagues from abroad. This shows that he can work independently, but also does not avoid research co-operation. Papers with mathematicians from outside of Czech Republic are more recent, including one in 2019 which is part of the thesis (and another preprint mentioned in the thesis, co-authored with foreigner). This is good sign, which can be understood as increase of international recognition, reputations and research contacts.

Looking at citations in Web of Science we get the following numbers: 61 citations (51 without self citations) and h-index 5. These numbers are not impressive, however acceptable for mathematician at this level of career (personally I would expect h-index 6 for candidate for habilitation, however 5 is still good score). Analyzing 10 papers in the habilitation, most of them were published in Topology and its Application which is respected journal in the field of topology and related topics (such as topological dynamics). It is not top journal in neither of these fields, but has good reputation and long history. In my personal opinion, the best dynamical systems journal in the list is Ergodic Theory and Dynamical Systems.

All the above measures are good prerequisites to expect solid mathematical work. Let me now comment in more detail on content of papers in the thesis. They cover broad research directions such as continuum theory, dynamical systems and set theory, with connecting ingredient provided by focus on topological aspects. Since my personal research interest is in topological dynamics and continuum theory I start by comment on these results and then move to other papers.

In [G] Vejnar with his co-authors studies dynamics on so-called tame graphs. These are generalizations of topological graphs, by the assumption that the closure of branch and endpoints is at most countable. They start with definition of constant slope, which is not completely obvious in this context. Then they focus on mixing maps on tame graphs with countably infinite Markov partitions. This way they generalize notion of piecewise monotone map, since considered maps are monotone on pieces of the partition. They develop necessary and sufficient conditions for such maps to be conjugate with a map with constant slope, and fully characterize existence of such conjugacy in the case of locally eventually onto maps. The condition is expressed in terms of eigenvectors (eigenvalues) of associated transition matrix of the Markov partition. While the condition looks standard, it is worth emphasizing that even in the case of interval maps such conjugacy may not exist, when the partition is infinite. And tame graphs make it even harder, since loops are allowed in the space. The authors of [G] connect entropy with Gurevich entropy of associated infinite graphs and provide some isomorphisms, as well as to some extent standard (natural for intervals; on tame graphs rather unexpected) relation between entropy and horseshoes. Clearly special structure of the space (size of the set of endpoints) and

the map provided by Markov partition are essential here, since even on dendrites entropy can be positive, while there is no horseshoe. Finally they show that for locally eventually onto maps, standard upper bound of entropy for Lipschitz maps is in fact as tight as we want, up to change of the metric defining the same topology.

Paper [F] deals with minimal homeomorphisms on Peano continua. The motivation for this work is an old result by Mañe, who proved that minimal and expansive homeomorphism can exist only on totally disconnected set. It was proved later by Artigue that if X is of non-zero dimensional, and (X,T) is minimal, then for every $\eta > 0$ there is a continuum $C \subset X$ whose diameter never exceeds η under iteration of T (so-called η-stable continuum). A point x belongs to a stable continua, if for every $\eta > 0$ there is a continuum $C \ni x$ which is η-stable. Classical example of Floyd provides example of minimal system with several points belonging to stable continua (points there are either in singleton components or in arc I satisfying $\lim_{|n|\to\infty} \operatorname{diam} T^n(I) = 0$, therefore belongs to a stable continua in the later case). Motivated by this example, Artigue asked question whether each point in minimal homeomorphism on a continnum belongs to a stable continua. Authors of [F] construct an example providing a negative answer to this question. The construction starts with a blow up of a dense orbit in a minimal rotation on the torus, creating a null sequence of holes (the authors use Aarts-Oversteegen technique by special inverse limits). Dynamical system obtained this way is minimal. The main observation is that each point outside blown up orbit (not belonging to inserted circle) cannot belong to a stable continua. A clear idea why it must happen is visible on Fig. 2 in [F].

Paper [E] is devoted to fixed point property on some continua. There are numerous results for different classes of actions (homeomorphism, continuous maps, open maps, compact groups, amenable groups) and continua (dendrites, dendroids, tree-like, etc.). The paper deals with groups contained in compact subgroups and compact commutative groups. The space is uniquely arcwise connected and the group acts on it continuously. The main result states that in this context there is always a fixed point. The paper is a short, elegant addition to existing literature with a nice survey on previous results.

In [B] the author study so-called blockers in continua. For two compact sets $A, B \subset X$ we say that B does not block A if $A \cap B = \emptyset$ and the union of all subcontinua of X intersecting A and contained in $X \setminus B$ is dense in X. If we denote by $\mathcal{B}(H)$ the family of B that block any element of $H \subset 2^X$ then result of Illanes and Krupski states that $\mathcal{B}(F(X)) = \mathcal{B}(2^X)$ in all locally connected continua. Opposite implication is not true however, by a nice example of λ-dendroid satisfying $\mathcal{B}(F(X)) = \mathcal{B}(2^X)$ constructed in [B]. The authors provide in [B] also characterization of the cases when $\mathcal{B}(F(X)) = \mathcal{B}(2^X)$ in some classes continua, in particular smooth dendroids or hereditarily decomposable chainable continua. Some open questions are posed there as well. This paper is closely related to [A], where non-blocking points are analyzed from topological point of view. It is proven there, among other things, that every continuum contains at least two non-blocking points, while all points in circle-like continua are non-blocking.

Recall that two continua are called incomparable if none of them can be mapped onto the other by a continuous mapping. Main result of [C] shows that if X is a nondegenerate Peano continuum, then there exists a family of size continuum of incomparable compactifications of the ray with remainder X. This fits very well into long and intensive studies of compactifications of continua and their topological "similarities". The result states that there exists huge set of different ways of presentation of a ray accumulating at Peano continuum. In [H] yet another approach to compactifications is considered. The authors consider so-called compactificable and Polishable classes together with "strong" versions of these notions. They systematically study these notions, providing some characterizations of these classes, e.g. using hyperspace descriptions. Paper [H] brings also several

open questions for further research. Finally, papers [I] and [J] are devoted to set-theoretic characterizations of various spaces.

Summary. Dr. Vejnar's research is at good international level. He publish a few papers each year, all the time in respected journals on topology and related fields. He has papers both with and without collaborators. He disseminate his research at meaningful meetings in the field each year, and was also invited speaker at some of them. He has experience in organizing conferences, and encourage younger colleagues in his local community to joint research. As disadvantage can be regarded his relatively small recognition in terms of citations and h-index, but also here some increasing trend can be observed (in WoS). Another disadvantage can be lack of papers in top international journals of general interest. Vejnar's broad research interest can be regarded as a plus, however he is lacking longer systematic study on one topic, "jumping" between problems he can solve by techniques he know which can be regarded as minus. After all, I find these disadvantages as a minor, since they do not influence directly research quality, which in my opinion is at good level.

Taking all the above factors and comments into account, and considering all the usual requirements with respect to candidates for docent degree, without hesitation I **support** the application of dr. Vejnar and **recommend** admission to further stages of the habilitation process.

At the end, let me also mention, that I have gone through the check of originality of the thesis done by the system Turnitin and without doubt I find that the thesis represents an original work with minimum overlap with the existing literature.