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DOCTORAL THESIS

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Aspects of baryon and lepton number non-conservation in the Standard model of particle interactions and beyond

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In: *Hodkovice n. M.* date: *17 Jan 2021* *Matěj Hudec*

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Abstract:

This work is devoted to baryon and lepton number violation and flavour physics in theories beyond the Standard Model of particle interactions.

First, the relations between the accidental and imposed symmetries are discussed. For example, we argue that the Levi-Civita tensors in the color space may be understood as a unique spurion carrying the baryon number and hence its absence in the Lagrangian of a specific model indicates baryon number conservation.

Afterwards, the two minimal scenarios of quark-lepton unification are analysed in detail: the Minimal quark-lepton symmetry model and its extension by the inverse seesaw mechanism. We investigate if the observed anomalies in the B-meson decays could be to some extent accommodated within these models.

Finally, possible low-energy effects of gauge leptoquarks are studied in the theory of quark-lepton unification and its simple extensions by vector-like leptons. Taking fully into account the freedom in the quark-lepton mixing, a catalogue of measurements currently forming the border of the excluded parameter space is found. We argue that, within the considered class of models, the gauge leptoquark can account for the discrepancies in the neutral-current B decays if and only if at least two extra generations of weak-isosinglet leptons exist.

Keywords: physics beyond the Standard Model, baryon and lepton number non-conservation, extended gauge symmetries, leptoquarks, B-meson anomalies

Contents

Notation and abbreviations

Indices:

$i, j, \ldots$	$\in \{1, 2\}$	fundamental $SU(2)_L$ indices
$\alpha, \beta, \ldots$	$\in \{1, 2, 3\}$	fundamental $SU(3)_C$ indices
$a, b, \ldots$	$\in \{1, \ldots 4 \text{ or } 5\}$	fundamental indices of $SU(4)_C$ or $SU(5)$
$I, J, \ldots$	$\in \{1, 2, 3\}$	indices in adjoint representation of $SU(2)_L$
$A, B, \ldots$	$\in \{1, \ldots, 8 \text{ or } 15\}$	indices in adjoint of $SU(3)_C$ or $SU(4)_C$
$\mu, \nu, \ldots$	$\in \{0, 1, 2, 3\}$	Lorentz timespace indices
$M, N, \ldots$	$\in \{1, 2, 3, 4\}$	Dirac 4-spinor indices
l, l'	$\in \{e, \mu, \tau\}$ or $\{1, 2, 3\}$	lepton flavour index
q, q'	$\in \{d, s, b\}$ or $\{1, 2, 3\}$	down-type quark

Matrices:

$q = \begin{bmatrix} u \\ d \end{bmatrix}$	$SU(2)_L$ or $SU(2)_R$ structures
$\hat{e} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$	flavour structures
$\begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}, \begin{pmatrix} q \\ \ell \end{pmatrix}$	other matrix structures: $SU(3)_C$, 3+1 structure of $SU(4)_C$, ...

Other notation and conventions:

$Q = T_L^3 + Y$	normalization of the weak hypercharge
$\mathrm{Y}_2, Y_e, \ldots$	Yukawa couplings are always labeled by an index
σ^I, λ^A	Pauli and Gell-Mann matrices
$\lambda_1, \lambda_2, \ldots$	quartic scalar coupling constants
$\mathcal{L}_0^\uparrow$	proper ortochronal Lorentz group
$\mathcal{L}$	Lagrangian
$G_\mu, W_\mu, B_\mu, A_\mu$	gauge fields usually carry the Lorentz index
G_2	color-octet *scalar* field
G_{SM}	Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$
G_{421}	compact notation for $SU(4) \times SU(2) \times U(1)$
B	beauty meson
$\mathcal{B}, \mathcal{F}, \mathcal{L}, \mathcal{M}, \mathcal{W}, \mathcal{Z}$	baryon, fermion, lepton and other global numbers
$\mathcal{C}$	Fermion conjugation matrix
C, P, T	discrete transformations
$(\psi_L \psi'_L)$	shorthand for $\psi_L{}^{\mathrm{T}} \mathcal{C} \psi'_L$

Selected abbreviations:

BNV / LNV	baryon/lepton number violation
(B)SM	(beyond the) Standard Model of particle interactions
EW(SB)	electroweak (symmetry breaking)
FCNC	flavour-changing neutral current
FPW model	Fileviez Pérez – Wise model
(G/V)LQ	(gauge/vector) leptoquark
GUT	grand unified theory
irrep	irreducible representation
LF[U](V)	lepton flavour [universality] (violation)
MQLS(M)	Minimal quark-lepton symmetry (model)
NP	new physics (i.e., physics BSM)
QCD / QED	quantum chromo-/electro-dynamics
QLU	quark-lepton unification
QFT	quantum field theory
RG(E)	renormalization group (equations)
(SM)EFT	(Standard Model) effective field theory
SSB	spontaneous symmetry breaking
SUSY	sypersymmetry
WC	Wilson coefficient
WET	Weak effective theory

Introduction

Symmetries of laws of Nature are one of the most fruitful concepts in the modern fundamental physics. Since the beginning of the 20^{th} century, many of the important new paradigms in theoretical physics relied on postulating the existence of new symmetries – consider theory of relativity, gauge theories such as the *Standard Model* (SM) or grand unified theories, supersymmetry (SUSY) or conformal field theory – or questioning their exactness (Galilean spacetime symmetry, parity violation).

On the experimental side, testing various predictions based on symmetries forms a large part of the past and contemporary research in high-energy physics. Corresponding to the fundamental concepts mentioned, a variety of tests of Lorentz invariance have been performed [1] as well as searches for CPT violation [2] which test the paradigm of local Lorentz-invariant quantum field theories. Furthermore, various consequences of non-Abelian gauge symmetries are now fairly established for both cornerstones of the SM – the quantum chromodynamics (QCD) and the theory of electroweak interactions. One can mention the well known evidence for color with absence of fine structures in hadronic spectra, which points towards the unbroken nature of the $SU(3)_C$ symmetry, confirmation of the very existence of the self-interacting gauge fields and their universal interaction strengths, or the evidence for more complex predictions of gauge theories such as asymptotic freedom. Needless to say, the discovery of the Higgs boson [3, 4] was nothing but confirmation of the electroweak-symmetry breaking mechanism. Going beyond the SM, a substantial part of the Atlas and CMS experimental focus in the first two runs was SUSY motivated (e.g. [5]), though lacking any significant signal.

Apart from the fundamental symmetries mentioned so far, equally important are the (exact or approximate) *accidental* symmetries of the physical laws – the invariances which are not contained in the basic assumptions of the theory but emerge anyway when the theory is built. Accidental symmetries of the SM provide crucial probes of theories *beyond the Standard Model* (BSM) which do not respect them.

In context of the *exact* accidental symmetries of the SM, let us mention testing the most important implication of continuous symmetries – the conservation laws. First, the limits on proton partial lifetimes like $\Gamma^{-1}_{p\to e^+\pi^0} > 1.6\times10^{34}$ years [6] form an incredibly strong test of *baryon number conservation*, probing energy scales many orders of magnitude above the energies of particle colliders. Secondly, *lepton number violation* (LNV) is being searched for in a variety of different modes, especially in tens of experiments looking for neutrinoless double beta decay [7, 8], and furthermore in subnuclear particle decays (e.g. [9, 10, 11]) and in high-energy collisions [12, 13]. Thirdly, the discovery of neutrino oscillations [14, 15, 16, 17] provided the evidence for non-conservation of *lepton flavour*, another symmetry predicted by the SM. On the other hand, lepton flavour violation (LFV) in the charged lepton sector has not been observed so far (see, e.g., the reviews [18, 19]).

Furthermore, on-going direct tests of *approximate* symmetries of the SM are being performed. One of the long-term priorities of the contemporary particle physics is the focus on the CP violation, study of which is also highly motivated by cosmology. In particular, studying the CP properties of the B-meson decays is the main raison d'etre of the B-factories as well as of the LHCb experiment. Historically, the *isospin symmetry* (or, more generally, the $SU(N_q)$ symmetry) of strong interactions is actually what made the theory of group representations the basic mathematical tool in particle physics. This approximate quark flavour symmetry of the SM is also continuously being tested [20]. Similarly in the lepton sector, the *lepton flavour universality* (LFU) is being probed. In the last decade, several hints of signals of new sources of violation of this approximate symmetry of the SM in the B-meson decays (the so-called *B-meson anomalies*) have been reported [21, 22, 23]. Finally, let us also mention the the famous $\rho = m_W^2/(m_Z^2 \cos_W^\theta)$ parameter, tree-level value of which is fixed by the custodial symmetry [24] of the SM scalar potential.

In this work, we touch upon several aspects of various symmetries mentioned. In both demonstrative examples and seriously meant phenomenological analyses a lot of our attention is devoted to leptoquarks (LQs). These BSM fields provide an excellent material to study since they necessarily break the quark and lepton flavour symmetries, some of them may even induce the lepton and baryon number violation, and they naturally arise in the theories with extended gauge symmetries.

In Chapter 1 we review the status of baryon and lepton numbers in the perturbative regime of quantum field theories like the SM and its extensions. In doing so, we adopt a slightly nontraditional perspective, focusing on relations between the gauge and accidental symmetries. Being a mere angle of view, this approach does not bring new phenomenological results; nevertheless, thanks to its intuitiveness it has a certain pedagogical value. For example, we point out that in a broad class of BSM models the color Levi-Civita symbol can be understood as a baryon-number-charged spurion and thus easily indicate baryon number violating interactions. The main idea has been shortly described in the appendix of author's first publication [25].

Chapter 2 inspects the theory of quark-lepton unification of the Pati-Salam type. We apply the approach from Chapter 1 and clarify the status of baryon and lepton numbers in this class of models, as one can be easily confused by the famous but sometimes misleading motto *"lepton number as the fourth color"* [26] which is widely used to characterize the main new idea of this theory. Furthermore, we present a detailed tree-level analysis of the two minimal models of quark-lepton unification [27, 28], based on the $SU(4)_C \times SU(2)_L \times U(1)_R$ gauge symmetry group. The original parts of this analysis consist of identification and a detailed investigation of the scalar potential (published in Refs. [25, 29]) and a careful parameter counting, including the discussion of number of physical phases in the mixing matrices between leptons and quarks.

In Chapter 3, various model-independent prerequisites for the phenomenological studies in later chapters are provided. We formally introduce the lepton flavour group and its important subgroups and describe in detail how this approximate symmetry of the SM would be further violated in the presence of leptoquarks. After a brief introduction to low-energy effective theories we point out some subtleties of this framework on an example of leptonic pseudoscalar decays $P^0 \to l_1^+ l_2^-$. Furthermore, we overview the current status of the signals of LFU violation in the semileptonic B-meson decays.

The two distinct topics in Chapters 2 and 3 get interconnected in the 4$^{\text{th}}$ chapter which studies the abilities of the quark-lepton symmetry models to accommodate the B-meson anomalies. Original author's results are presented, most of which have been published in Refs. [25, 29, 30].

Finally, Chapter 5 contains a yet unpublished phenomenological study [31] of possible first signals of the gauge leptoquark in theories of quark-lepton unification, taking fully into account the freedom in quark-lepton mixing and considering a large number of relevant processes.

Chapter / Appendix	1	2	3	4	5	A	B	C
Baryon and lepton numbers, $B-L$	✓	✓			5.2.2	✓	✓	
Leptoquarks	1.2.2	✓	3.5	✓	✓	✓		✓
Flavour physics		2.8	✓	✓	✓			✓
Quark-lepton $SU(4)_C$ unification		✓		✓	✓			✓
B-meson anomalies			✓	✓	5.2			

Table 1: Coverage of various topics in different chapters and appendices of this thesis.

1. Baryon and lepton number in the SM and beyond

The baryon and lepton numbers, $\mathcal{B}$ and $\mathcal{L}$, are important quantities which seem to be conserved in all measurements performed so far. This is in harmony with the perturbative regime of the SM in which $\mathcal{B}$ and $\mathcal{L}$ are protected by global symmetries of the classical action.

Nevertheless, there are strong cosmological motivations to believe that $\mathcal{B}$- and $\mathcal{L}$-violating processes do exist. The initial observation is the apparent nonzero baryon number density in the visible Universe, augmented by the electron density which neutralizes the electric charge of our world to an extreme level.[1] On the other hand, the today's baryon to photon number density ratio $(n_B - n_{\bar{B}})/n_\gamma \sim 10^{-10}$ indicates that the relative difference between baryon and antibaryon abundance $(n_B - n_{\bar{B}})/(n_B + n_{\bar{B}})$ had been extremely tiny in the early hot Universe at temperatures above 1 GeV, when the production of baryon-antibaryon pairs was essentially as easy as their annihilation. Such a scenario seems more and more contrived as times closer and closer to the Big Bang are considered, which suggests that $\mathcal{B} \propto (n_B - n_{\bar{B}}) \neq 0$ has evolved in physical processes from symmetric initial conditions.

The necessary conditions for baryogenesis are $\mathcal{B}$, C and CP violation in the relevant effective physical laws and an out-of-equilibrium state [32]. As is well known, C and CP are violated in the SM; however, the size of CP violation is too tiny [33]. In fact, also processes violating baryon number are predicted even within the SM: since both $\mathcal{B}$ and $\mathcal{L}$ are anomalous, they may be violated in the non-perturbative "sphaleronic" phenomena which should have occurred in the hot Universe, keeping only $\mathcal{B} - \mathcal{L}$ untouched. However, in the thermal equilibrium at temperatures around $T \sim 1$ TeV, these processes are more likely to smear any previously cooked $\Delta\mathcal{B} = \Delta\mathcal{L}$ departure from zero than to create one. [34] Departure from thermal equilibrium at the electroweak (EW) scale could have emerged as a first-order phase transition inhering in scalar field bubble nucleation. For such scenarios, mechanisms of electroweak baryogenesis are known. Nevertheless, this scenario is not applicable in the SM since the SM Higgs potential with the measured Higgs mass leads to the phase transition which is not of the 1st order.

Thus, the electroweak baryogenesis calls for physics BSM which would improve the conditions for the phase transition as well as bring additional sources of CP violation. A more prosaic option, studied in this work, is a perturbative baryon number violation (BNV) or lepton number violation (LNV) in the New Physics sector, which would open many other doors also in the cosmological considerations if $\Delta\mathcal{B} \neq \Delta\mathcal{L}$ can be created at even earlier times. Especially, even creating only $\mathcal{L} \neq 0$ might be sufficient as its would be later partially transformed into an excess of baryons via the electroweak sphalerons [35, 36].

Baryon and/or lepton number violation at the level of classical action is not uncommon in the BSM theories: LNV is present in the models employing some kind of the seesaw mechanism; both $\mathcal{B}$ and $\mathcal{L}$ non-conservation is predicted by

[1] Notice that the lepton number density of the today's Universe is not known due to our ignorance of the (anti)neutrino abundances.

grand unified theories (GUTs) [37]. For the prominent role of $\mathcal{B}$ and $\mathcal{L}$, LNV and especially BNV usually become one of the most important characteristics of the models in which they occur. Hence, even though systematic study of global symmetries of a given model is a straightforward task, it is useful to be able to recognize the $\mathcal{B}$ and $\mathcal{L}$ violation ”at first sight”.

This chapter is devoted to certain aspects of baryon and lepton numbers in the SM, SMEFT and various renormalizable BSM models. We focus mainly on the identification of perturbative BNV and LNV in the theory, putting aside the experimental signals. In Section 1.1 we overview the status of $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$ in the SM and remind how these accidental symmetries may be broken in the simplest neutrino extension of the SM or by the effective operators of higher mass dimension. Then, we focus on formal relations between the accidental and imposed symmetries in Section 1.2. During the whole chapter, the flavour structure is made as implicit as possible; discussion of this aspect is postponed to later chapters of this work.

1.1 Standard Model and first steps beyond

This introductory section serves mostly to set up the conventions. We review here the Lagrangian of the SM as well as a few simple examples of lepton or baryon number violating scenarios that do not go beyond the SM gauge group

$$G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y \,. \tag{1.1}$$

1.1.1 Symmetries of the Standard Model

Let us begin fixing the conventions by casting the SM Lagrangian in the following form:

$$\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = &-\frac{1}{2} G^{\alpha}_{\mu\nu\beta} G^{\beta\mu\nu}_{\alpha} - \frac{1}{2} W^{i}_{\mu\nu j} W^{\mu\nu j}_{i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
&+ \overline{\ell_L}_i i(\not{D}\ell_L)^i + \overline{e_R} i\not{D} e_R + \overline{q_L}_{\alpha i} i(\not{D} q_L)^{\alpha i} + \overline{u_R}_{\alpha} i(\not{D} u_R)^{\alpha} + \overline{d_R}_{\alpha} i(\not{D} d_R)^{\alpha} \\
&+ (D_\mu \phi)^\dagger_i (D^\mu \phi)^i + \mu^2 \phi^\dagger_i \phi^i - \lambda \left(\phi^\dagger_i \phi^i\right)^2 \\
&- \left(\overline{q_L}_{\alpha i} Y_d \, {d_R}^{\alpha} \phi^i + \overline{q_L}_{\alpha i} Y_u {u_R}^{\alpha} \varepsilon^{ij} \phi^\dagger_j + \overline{\ell_L}_i Y_e e_R \, \phi^i + \text{h.c.}\right)
\end{aligned} \tag{1.2}$$

Here $\alpha, \beta \in \{1,2,3\}$ stand for the $SU(3)_C$ fundamental indices, whereas $i, j \in \{1,2\}$ refer to $SU(2)_L$. The field-strength tensors $G_{\mu\nu}, W_{\mu\nu}$ and $B_{\mu\nu}$ relate to the three factors in Eq. (1.1). The corresponding gauge fields appear in the covariant derivatives, e.g., $(Dq_L)^{i\alpha} = \partial_\mu {q_L}^{\alpha i} + i g_3 G^{\alpha}_{\mu\beta} {q_L}^{\beta i} + i g W^{i}_{\mu j} {q_L}^{\alpha j} + i\frac{1}{6} g' B_\mu {q_L}^{\alpha i}$. We represent them as traceless tensors in the fundamental representations of the relevant gauge groups, avoiding using other kinds of indices in Eq. (1.2). An alternative notation for the case of gluons would be

$$G^{\alpha}_{\mu\beta} = \frac{1}{2} \left(\lambda^A\right)^{\alpha}_{\beta} G^A_\mu \tag{1.3}$$

where the implicit sum runs over $A \in \{1, \ldots, 8\}$. The SM gauge group is spontaneously broken as

$$G_{\mathrm{SM}} \quad \to \quad G_{\mathrm{vac}} = SU(3)_C \times U(1)_Q \tag{1.4}$$

G_{SM}	$\to$	G_{vac}	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{F}$
Field content of the Standard Model					
Fermions:					
$\ell_{L(1,2,-1/2)}$	$=$	$\begin{bmatrix} \nu_{L(1,0)} \\ e_{L(1,-1)} \end{bmatrix}$	0	1	1
$e_{R(1,1,-1)}$	$=$	$e_{R(1,-1)}$	0	1	1
$q_{L(1,2,-1/2)}$	$=$	$\begin{bmatrix} u_{L(3,+2/3)} \\ d_{L(3,-1/3)} \end{bmatrix}$	$1/3$	0	1
$u_{R(3,1,+2/3)}$	$=$	$u_{R(3,+2/3)}$	$1/3$	0	1
$d_{R(3,1,-1/3)}$	$=$	$d_{R(3,-1/3)}$	$1/3$	0	1
Scalar bosons:					
$\phi_{(1,2,+1/2)}$	$=$	$\begin{bmatrix} w^+ \\ (v_{\text{ew}} + H^0 + iz^0)/\sqrt{2} \end{bmatrix}$	0	0	0
Gauge bosons:					
$G_{\mu(8,1,0)}$			0	0	0
$W_{\mu(1,3,1)}$	$=$	$\frac{1}{2}\begin{bmatrix} W^3_\mu & W^+_\mu\sqrt{2} \\ W^-_\mu\sqrt{2} & W^3_\mu \end{bmatrix}$	0	0	0
$B_{\mu(1,1,0)}$			0	0	0
Extension of the SM by right-handed neutrinos					
Fermions:					
$\nu_{R(1,1,0)}$	$=$	$\nu_{R(1,0)}$	0	1	1

Table 1.1: Fields in the Standard Model and their baryon, lepton and fermion numbers.

by the vacuum expectation value (VEV) of the Higgs doublet:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ v_{\text{ew}} \end{bmatrix}. \tag{1.5}$$

It is well known that the Standard Model features a $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$ global symmetry, implying conservation of the baryon number $\mathcal{B}$ and the lepton number $\mathcal{L}$ which are assigned to the fields as in Table 1.1. In the current paradigm, this is an *accidental* symmetry – it has not been imposed but has resulted as a consequence of the specific field content when writing down the *most general renormalizable Lagrangian invariant with respect to* G_{SM} *and the Lorentz group* $\mathcal{L}_0^\uparrow$.

1.1.2 Neutrino masses and lepton number violation

A well known shortcoming of the SM is the absence of neutrino masses. One way of improving this consists in including the right-handed neutrinos $\nu_R \sim (1,1,0)$ in the theory and extending the Lagrangian accordingly by

$$\mathcal{L}^{\nu_R} = \overline{\nu_R}\,\partial\!\!\!/\,\nu_R + \left(\overline{\ell_L}_i Y_\nu \nu_R\, \varepsilon^{ij}\phi_j^\dagger + \text{h.c.}\right), \tag{1.6}$$

generating the neutrino masses via the same Higgs mechanism that gives masses to the other SM fermions. However, the theoretical demand to include every term which is allowed, together with the attitude of *not imposing* the lepton number conservation calls for yet another part: the Majorana mass term

$$\mathcal{L}^{\nu_R\,-\,\text{Maj.}} = {\nu_R}^{\mathrm{T}} M_R\, \mathcal{C}\, \nu_R + \text{h.c.}\,, \tag{1.7}$$

which in the case $M_R \gg v_{\text{ew}}$ leads to the famous *type-I seesaw* mechanism of generating parametrically smaller neutrino masses of order $Y_\nu^2 v_{\text{ew}}^2 / M_R$ [38, 39].

As the interaction in Eq. (1.6) requires that ν_R carries $\mathcal{L} = +1$, the Majorana mass term breaks the lepton number by two units, which might be observable as a neutrinoless double beta decay. This is the basic illustration that the conservation of $\mathcal{B}$ and $\mathcal{L}$ depends strongly on the field content of the model.

1.1.3 SMEFT

The *Standard Model effective field theory* (SMEFT) is the framework for systematic description of high energy new physics effects in terms of operators with the mass dimension $d > 4$ built from the SM fields only. The SMEFT Lagrangian reads

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_A \left(C_A \mathcal{O}_A + \text{h.c. where necessary} \right) \tag{1.8}$$

where $\mathcal{O}_A$'s form a basis of all effective operators respecting the $G_{\text{SM}} \times \mathcal{L}_0^{\uparrow}$ invariance, up to a given mass dimension d. Specific scenarios are defined by the values of all the Wilson coefficients C_A. These dimensionful coefficients are real for hermitean operators and complex for non-hermitean ones. In the latter case, the conjugated terms are automatically included and we don't count them as independent.[2] Construction of the complete and non-redundant set of operators of a given dimension is a highly non-trivial task, especially when the existence of 3 fermion generations is taken into account. Recently, the problem of *counting* the number of independent operators built from given powers of individual fields have been fully resolved and automatized [41] while algorithmizing the explicit index contractions is still in progress [42].

For one fermion generation, there is only a single SMEFT operator of mass dimension $d = 5$, the so-called Weinberg term [43],

$$\mathcal{O}_W = \left[({\ell_L}^i)^{\mathrm{T}} \mathcal{C} {\ell_L}^j \right] \phi^k \phi^l \, \varepsilon_{ik}\, \varepsilon_{jl} \tag{1.9}$$

which breaks the lepton number by $\Delta\mathcal{L} = 2$. This operator provides an effective description of the seesaw scenarios [38, 44, 45] such as the model in Section 1.1.2.

At $d = 6$, the number of independent operators for a single fermion generation is 63, out of which 59 conserve both $\mathcal{B}$ and $\mathcal{L}$ [46]. The four remaining ones induce $\Delta\mathcal{B} = \Delta\mathcal{L} = 1$ and read [47]

$$\mathcal{O}_{duq\ell} = \varepsilon_{\alpha\beta\gamma} \left[({d_R}^\alpha)^{\mathrm{T}} \mathcal{C} {u_R}^\beta \right] \left[({q_L}^{\gamma i})^{\mathrm{T}} \mathcal{C} {\ell_L}^j \right] \varepsilon_{ij}\,, \tag{1.10a}$$

$$\mathcal{O}_{qque} = \varepsilon_{\alpha\beta\gamma} \left[({q_L}^{\alpha i})^{\mathrm{T}} \mathcal{C} {q_L}^{\beta j} \right] \left[({u_R}^\gamma)^{\mathrm{T}} \mathcal{C} e_R \right] \varepsilon_{ij}\,, \tag{1.10b}$$

$$\mathcal{O}_{qqq\ell} = \varepsilon_{\alpha\beta\gamma} \left[({q_L}^{\alpha i})^{\mathrm{T}} \mathcal{C} {q_L}^{\beta j} \right] \left[({q_L}^{\gamma k})^{\mathrm{T}} \mathcal{C} {\ell_L}^l \right] \varepsilon_{il}\, \varepsilon_{jk}\,, \tag{1.10c}$$

$$\mathcal{O}_{duue} = \varepsilon_{\alpha\beta\gamma} \left[({d_R}^\alpha)^{\mathrm{T}} \mathcal{C} {u_R}^\beta \right] \left[({u_R}^\gamma)^{\mathrm{T}} \mathcal{C} e_R \right]. \tag{1.10d}$$

[2] Our convention is in accordance with the `WCxf` standards [40]. In contrast, the non-hermitian operators and their conjugates have been counted separately in Ref. [41]; the number of operators then corresponds to the number of real parameters to be fixed.

Generally, SMEFT operators of mass dimension d follow the rule $\Delta\mathcal{B} - \Delta\mathcal{L} = 2d + 4k$ where $k \in \mathbb{Z}$ [48].

Notice that all the dimension-6 BNV operators (1.10) contain the $SU(3)_C$ Levi-Civita tensor, unlike the other 59 ones. This simple observation suggests there is an equivalence between baryon number violation and appearance of the fully antisymmetric color tensor, which can actually be proven to hold for SMEFT operators of arbitrary dimension. In Section 1.2, we will put this relation on deeper grounds, showing that similar conclusions may be drawn also for models with extended field content and even for other symmetry groups.

1.2 Relations between imposed and accidental symmetries: "$SU2U$ approach"

In this section, we will discuss some broader contexts of the notice made above – that baryon number violating SMEFT operators can be identified by the color Levi-Civita symbol. After better specifying and proving this statement, we will discuss its extended validity for more general BSM models as well as the counterparts of this observation for other symmetries.

Some parts of the discussion here are well known or trivial. Nevertheless, we believe that this section as is stands has a certain pedagogical value, bringing an intuitive insight to the appearance of some accidental symmetries. The key idea of this chapter has been published in the appendix or Ref. [25].

1.2.1 Baryon number and color

Let us start with a simple observation that the baryon numbers of all the SM fields $(\ell_L, e_R, {q_L}^\alpha, {u_R}^\alpha, {d_R}^\alpha, \phi, W_\mu, B_\mu, G^\alpha_{\mu\beta})$ can be related to their $SU(3)_C$ quantum numbers by

$$\mathcal{B} = \frac{1}{3}\Big(\big(\#\text{ upper } SU(3)_C \text{ indices}\big) - \big(\#\text{ lower } SU(3)_C \text{ indices}\big)\Big). \tag{1.11}$$

As hermitean conjugation swaps the position of these indices, antiparticles carry opposite $\mathcal{B}$ charges. Generally, the only objects carrying color indices which might appear in a Lagrangian are the dynamical fields and the invariant tensors – the Kronecker symbol δ^α_β and the Levi-Civita tensor ($\varepsilon_{\alpha\beta\gamma}$ or $\varepsilon^{\alpha\beta\gamma}$) [49]. From the perspective of Eq. (1.11), δ is a neutral *spurion* while ε is charged. The $SU(3)_C$ invariance of a given model ensures that every upper index is contracted with a lower one in the corresponding Lagrangian. This implies that **any $SU(3)_C$ invariant Lagrangian that does not contain the color Levi-Civita tensor is invariant with respect to $U(1)_\mathcal{B}$**, action of which is defined by Eq. (1.11). More generally, for any interaction term, the following relation between $\mathcal{B}$ violation and appearance of the Levi-Civita tensors holds:

$$\Delta\mathcal{B} = \big(\#\,\varepsilon_{\alpha\beta\gamma}\big) - \big(\#\,\varepsilon^{\alpha\beta\gamma}\big). \tag{1.12}$$

This observation is not very useful for the SM as it is just a slightly complicated proof of baryon number conservation. For SMEFT, it provides a rather elegant explanation of the rule of thumb in Eq. (1.12).[3] This equation also holds for any explicit BSM model provided that $\mathcal{B}$ is ascribed to the new fields in accordance with (1.11). As we will see later, prescription (1.11) provides a convenient choice[4] of baryon number in many BSM models.

Notice that our approach relates an imposed $SU(N)$ symmetry to an accidental $U(1)$ symmetry. Eq. (1.11) implies that the elements of the *center* of the non-Abelian group, $e^{2\pi i n/3}\mathbb{1} \in \mathbb{Z}_3 \subset SU(3)_C$ for $n = 0, 1, 2$, always act the same way as $e^{2\pi i n \mathcal{B}} \in U(1)_{\mathcal{B}}$. Therefore, ignoring the local nature of the non-Abelian part, both $SU(3)_C$ and $U(1)_{\mathcal{B}}$ can be understood as subgroups of $U(3)$ rather than of the larger group $SU(3)_C \times U(1)_{\mathcal{B}}$. The $U(3)$ group, with its Cartan subalgebra spanned by

$$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \qquad \mathcal{B} = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.13}$$

acts on all the fields according to their color indices by proper multiplications by a single unitary matrix $\mathcal{U}^{\alpha}_{\beta}$. For this reason, we call the above described procedure of formally extending of the $SU(N)$ symmetry group to $U(N)$ *the SU2U approach*. Its applicability range is wider than $N = 3$. In the rest of this section we give some additional remarks about *SU2U*, all of which apply or can be generalized to any $SU(N)$ gauge group.

First, the *SU2U* applied on any $SU(N)$ gauge factor in any model always generates a good symmetry of the *gauge* interactions since the Levi-Civita tensors do not naturally appear in the kinetic terms.

Secondly, note that a different basis of the Cartan subalgebra of $U(3)$ can be chosen than the one in Eq. (1.13), namely

$$r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad g = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1.14}$$

This demonstrates that the intuitive concept of *conservation of color in the strict sense*, implying never-ending colored lines in Feynman diagrams, relies not only on the $SU(3)_C$ invariance but also on the baryon number conservation – see Fig. 1.1 for an illustration.

Last but not least, the *SU2U* approach requires that only the vector indices $\alpha, \beta \in \{1, 2, 3\}$ are used even for larger $SU(3)_C$ representations (such as gluons in the SM). This is always possible since every irreducible representation of $SU(3)$ can be found in the tensor product of sufficiently large number of fundamental triplets and anti-triplets. On the other hand, such a product is not unique, which, in turn, brings a redundancy in the definition of $\mathcal{B}$. This will be commented on in the following parts.

[3] Our reasoning could not have been done in Ref. [46] since upper and lower indices are not rigorously distinguished there.

[4] If a BSM model features a global symmetry which coincides with baryon number for the SM fields, a *convenient choice* of $\mathcal{B}$ for the new fields should generate that symmetry completely.

[5] This and all subsequent Feynman diagrams have been drawn using `tikz-feynman` [51].

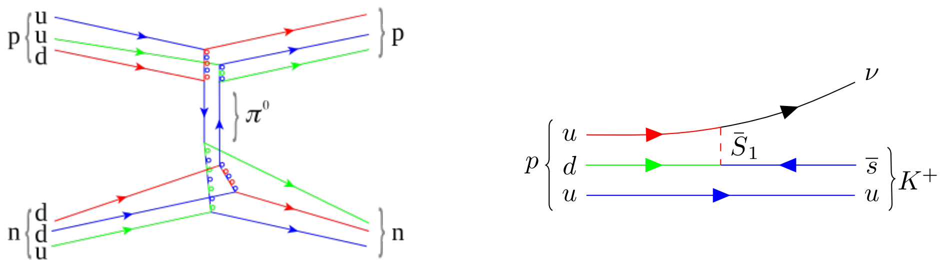

Figure 1.1: *Left* [50] – In the illustrations of SM processes, colored lines never end. *Right*[5] – This does not apply for baryon number violating models, even though $SU(3)_C$ remains a good symmetry there.

$\Psi(Y)^{3\mathcal{B}}_{\mathcal{L}}$	$\overline{e_R}\,(1)^0_{-1}$	${\ell_L}^j\,(-\frac{1}{2})^0_1$	$\overline{d}_{R\beta}\,(\frac{1}{3})^{-1}_0$	$\overline{u_{R\beta}}\,(-\frac{2}{3})^{-1}_0$	${q_L}^{j\beta}\,(\frac{1}{6})^1_0$
${q_L}^{i\alpha}\,(\frac{1}{6})^1_0$	$R^\dagger_{2\alpha i}(-\frac{7}{6})^{-1}_1$	$S_{1\alpha}\varepsilon_{ij}$ $S_{3\,\alpha ij}$ $(\frac{1}{3})^{-1}_{-1}$	$\phi^\dagger_i\delta^\beta_\alpha$ $G^{\dagger\,\beta}_{2\,\alpha i}$ $(-\frac{1}{2})^0_0$	$\phi^j\varepsilon_{ij}\delta^\beta_\alpha$ ${G_2}^{j\beta}_{\alpha}\varepsilon_{ij}$ $(\frac{1}{2})^0_0$	${S_1^\dagger}^\gamma\varepsilon_{\alpha\beta\gamma}\varepsilon_{ij}$ $\chi_{\alpha\beta}\varepsilon_{ij}$ ${S^\dagger_{3ij}}^\gamma\varepsilon_{\alpha\beta\gamma}$ $X_{\alpha\beta ij}$ $(-\frac{1}{3})^{-2}_0$
$\overline{u_{R\alpha}}\,(-\frac{2}{3})^{-1}_0$	${S_1^\dagger}^\alpha(-\frac{1}{3})^1_1$	${R_2}^{\alpha i}\,\varepsilon_{ij}\,(\frac{7}{6})^1_{-1}$	$S_{1\gamma}\,\varepsilon^{\alpha\beta\gamma}$ $\chi^{\dagger\alpha\beta}$ $(\frac{1}{3})^2_0$	$\tilde{S}_{1\gamma}\,\varepsilon^{\alpha\beta\gamma}$ $\tilde{\chi}^{\dagger\alpha\beta}$ $(\frac{4}{3})^2_0$	
$\overline{d}_{R\alpha}\,(\frac{1}{3})^{-1}_0$	${\tilde{S}_1^\dagger}{}^\alpha(-\frac{4}{3})^1_1$	${\tilde{R}_2}{}^{\alpha i}\,\varepsilon_{ij}\,(\frac{1}{6})^1_{-1}$	$\bar{S}_{1\gamma}\,\varepsilon^{\alpha\beta\gamma}$ $\bar{\chi}^{\dagger\alpha\beta}$ $(-\frac{2}{3})^2_0$		
${\ell_L}^i(-\frac{1}{2})^0_1$	$\phi^\dagger_i\,(-\frac{1}{2})^0_0$	$\varphi^+\,\varepsilon_{ij}$ Δ_{ij} $(1)^0_{-2}$			
$\overline{e_R}\,(1)^0_{-1}$	$\varphi^{--}\,(-2)^0_2$				

Table 1.2: Catalogue of all possible Yukawa interactions of SM fermions with scalar fields. Their the hypercharges are denoted explicitly while the $SU(3)_C$ and $SU(2)_L$ quantum numbers can be read off from the indices (multiple indices of the same kind are symmetric). Scalars with multiple interactions are denoted by the same symbol. The naming scheme for leptoquarks has been adopted from Ref. [52].

1.2.2 Yukawa-interacting BSM scalars

In this part we illustrate both the usefulness and caveats of the *SU2U* approach on a series of single scalar boson extensions of the SM.

Let us begin with listing all possible Yukawa interactions of the SM fermions with hypothetical scalars. The procedure is straightforward: for a given pair of left-handed (or conjugated right-handed) fermions ψ_1, ψ_2 transforming as R_1 and R_2 under G_{SM}, the boson σ must transform according to an irreducible representation R_σ contained in $R_1 \otimes R_2$. The result is cast in the fermion-pair-indexed form in Table 1.2. An extension of this table including interactions with right-handed neutrinos, a linearized version of these tables and analogous tables for vector bosons can be found in Appendix A. From Table 1.2, we pick a few examples.

Scalar leptoquark

First, consider extending the SM by the scalar $R_2 \sim (3, 2, +7/6)$. As it carries a color index, the *SU2U* approach suggests $\mathcal{B}(R_2) = +1/3$. Looking at the most general renormalizable Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}\oplus R_2} = \mathcal{L}_{\mathrm{SM}} + (D_\mu R_2)^\dagger_{i\alpha} (D^\mu R_2)^{i\alpha} - \mu^2_{R_2} \left(R_2{}^\dagger_{i\alpha} R_2{}^{i\alpha}\right) \\ - \left(\overline{q_L}_{i\alpha} Y_4\, e_R\, R_2{}^{i\alpha} + \overline{u_R}_\alpha Y_2\, \ell_L{}^j \varepsilon_{ij}\, R_2{}^{j\alpha} + \text{h.c.}\right) \\ - \kappa \left(R_2{}^\dagger_{i\alpha} R_2{}^{i\alpha}\right)^2 - \rho \left(R_2{}^\dagger_{i\alpha} R_2{}^{i\alpha}\right) \left(\phi^\dagger_j \phi^j\right) - \rho_3 \left(\phi^\dagger_i R_2{}^{\alpha i} R_2{}^\dagger_{\alpha j} \phi^j\right),\end{aligned} \tag{1.15}$$

one finds that the Yukawa interactions on the second line indeed call for $\mathcal{B}(R_2) = +1/3$ and $\mathcal{L}(R_2) = -1$, while the other terms, built of the $(R_2{}^\dagger R_2)$ quadratics, do not pose any constraints on any additional quantum numbers. Both baryon and lepton numbers remain conserved in this model. The R_2 leptoquark will play a prominent role in Chapter 4, whence the labels of the Yukawa couplings have been adopted from.

Scalar diquark

As a second example, let us consider the scalar $\chi \equiv (\bar{S}_1)^\dagger \sim (3, 1, +2/3)$ added to the Standard Model. The Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}\oplus\bar{S}_1} = \mathcal{L}_{\mathrm{SM}} + (D_\mu \chi)^\dagger_\alpha (D^\mu \chi)^\alpha - \left(d_R{}^{\alpha\mathrm{T}} \mathcal{C}\, Y_{dd}\, d_R{}^\beta \chi^\gamma \varepsilon_{\alpha\beta\gamma} + \text{h.c.}\right) \\ - m^2_\chi \left(\chi^\dagger_\alpha \chi^\alpha\right) - \kappa \left(\chi^\dagger_\alpha \chi^\alpha\right)^2 - \rho \left(\chi^\dagger_\alpha \chi^\alpha\right) \left(\phi^\dagger_j \phi^j\right).\end{aligned} \tag{1.16}$$

A naive application of the *SU2U* perspective yields $\mathcal{B}(\chi) = +1/3$ and warns that the Yukawa interaction between two quarks[6] and χ violates the baryon number. However, we can exploit the possibility to realize color triplets by antisymmetric rank-two tensors and redefine the field as

$$\chi_{\underbrace{\alpha\beta}} = \varepsilon_{\alpha\beta\gamma}\, \chi^\gamma. \tag{1.17}$$

Using the inverse relation $\chi^\gamma = \frac{1}{2} \chi_{\underbrace{\alpha\beta}}\, \varepsilon^{\alpha\beta\gamma}$, the entire Lagrangian can be rewritten without the Levi-Civita symbol as

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}\oplus\bar{S}_1} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2} (D_\mu \chi)^{\dagger\underbrace{\alpha\beta}} (D^\mu \chi)_{\underbrace{\alpha\beta}} - \left(d_R{}^{\alpha\mathrm{T}} \mathcal{C}\, Y_{dd}\, d_R{}^\beta \chi_{\underbrace{\alpha\beta}} + \text{h.c.}\right) \\ - \frac{1}{2} m^2_\chi \left(\chi^{\dagger\underbrace{\alpha\beta}} \chi_{\underbrace{\alpha\beta}}\right) - \frac{1}{4} \kappa \left(\chi^{\dagger\underbrace{\alpha\beta}} \chi_{\underbrace{\alpha\beta}}\right)^2 - \frac{1}{2} \rho \left(\chi^{\dagger\underbrace{\alpha\beta}} \chi_{\underbrace{\alpha\beta}}\right) \left(\phi^\dagger_j \phi^j\right).\end{aligned} \tag{1.18}$$

Hence, with the proper assignment of baryon number $\mathcal{B}(\chi) = -2/3$, the $U(1)_\mathcal{B}$ transformation is an exact symmetry of the model.

In this example we have explicitly shown that there is a *redundancy* hidden in the prescription (1.11). We will comment on this later in Subs. 1.2.3. For the moment, just recall that the statement above Eq. (1.12) is a one way implication: presence of the Levi-Civita tensor in the Lagrangian does not immediately imply baryon number violation – it might be just an inconvenient choice of realization of the color structures.[7]

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[6] Multiple generations necessitate to be considered in the Yukawa interaction, since the Yukawa coupling Y_{dd} is antisymmetric in the flavour space.

[7] Similar suitable-basis-specific formulations are usual in theoretical physics. For example, CP is conserved if there is a basis in which all the couplings are real; the timespace is flat if there are coordinates in which $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

Baryon number violating scalar

Finally, consider again the same scalar field as in the previous example, this time in the model containing also right-handed neutrinos. In such a case, $\chi = \bar{S}_1$ can have both diquark and leptoquark interactions:

$$-\mathcal{L}^{\bar{S}_1-\text{Yuk}}_{\text{SM}\oplus\nu_R\oplus\bar{S}_1} = {d_R}^{\alpha\text{T}} \mathcal{C}\, Y_{dd}\, {d_R}^{\beta} \chi^{\gamma} \varepsilon_{\alpha\beta\gamma} + \overline{u_{R\alpha}}\, \mathcal{C}\, Y_{u\nu}\, \overline{\nu_R}^{\text{T}} \chi^{\alpha} + \text{h.c.} \tag{1.19}$$

Clearly, the redefinition in (1.17) would not help getting rid of ε anymore since it would occur in the second term instead. This model therefore violates baryon number. If the ν_R is light enough, such a scalar field could contribute to the proton decay via the diagram in Fig. 1.1. In the FPW model studied in Chapters 2 and 4, the $\bar{S}_1$ field actually features only the second type of Yukawa couplings in Eq. (1.19) and thus violates neither lepton nor baryon number.

1.2.3 Redundancies and congruence classes

In this part, we elucidate some connection between the *SU2U* approach and a more formal treatment in the group theory [53, 49].

Every irreducible representation (irrep) R of $SU(N)$ with the Dynkin labels $(a_1, \dots, a_{N-1})$ belongs to a certain congruence class characterized by its N*-ality*:

$$c_R = \sum_{n=1}^{N-1} n a_n \pmod{N}. \tag{1.20}$$

The trivial representation has $c_1 = 0$. If $R \otimes R' = \bigoplus_m R_m$, then for each R_m

$$c_{R_m} = c_R + c_{R'} \pmod{N}. \tag{1.21}$$

This implies that the N-alities c_{φ_k} of fields φ_k constituting an $SU(N)$-invariant interaction $\prod_k \varphi_k$ must add up to a multiple of N. The same holds also for any following modification of the sets of N-alities:

$$c'_{\varphi} = c_{\varphi} + \kappa_{\varphi} N, \qquad \kappa_{\varphi} \in \mathbb{Z}. \tag{1.22}$$

If the modified N-alities c'_{φ_k} add up strictly to zero for every term in the $SU(N)$ invariant Lagrangian, these numbers form the charges of a global $U(1)$ symmetry. Nevertheless, there is no guarantee that there exist such a set of κ_{φ}'s that this indeed happens.

What has just been described is actually the *SU2U* approach in a slightly different language. The connection relies on the fact that the N-ality of an $SU(N)$ representation R realised as a tensor with upper and lower vector indices is equal to

$$c_R = \Big[(\#\ \text{upper indices}) - (\#\ \text{lower indices}) \Big] \pmod{N}. \tag{1.23}$$

Considering the $SU(3)_C$ group in particular, the (modified) trialities correspond to $3\mathfrak{B}$, while the arbitrariness in the choice of κ_{φ}'s corresponds to the possibility of notation changes like (1.17). The discussion after Eq. (1.21) corresponds to the fact that all indices must be contracted and the only nontrivial $SU(3)_C$ invariant tensor is the Levi-Civita symbol which has 3 color indices.

1.2.4 Lepton number and the weak group?

Could the *SU2U* be applied also on the weak isospin group $SU(2)_L$ to obtain the lepton number? Apparently, using the $SU(2)_L$ directly on the SM Lagrangian in the form of Eq. (1.2) fails due to the two-dimensional antisymmetric tensor in the up-quark Yukawa interaction. Nevertheless, after the following change of notation,

$$ {u_R}^{\alpha \underset{\smile}{ij}} \equiv {u_R}^{\alpha} \varepsilon^{ij} \,, \tag{1.24} $$

the whole SM Lagrangian can be rewritten, with use of ${u_R}^{\alpha} = \frac{1}{2} {u_R}^{\alpha \underset{\smile}{ij}} \varepsilon_{ij}$, to the form without explicit ε^{ij}. Indeed, the relevant terms then read

$$ \mathcal{L}_{\mathrm{SM}}^{u_R} = \frac{1}{2} \overline{u_R}_{\alpha \underset{\smile}{ij}} \left(i\partial\!\!\!/ + \frac{2}{3} g' B\!\!\!/ \right) {u_R}^{\alpha \underset{\smile}{ij}} + \frac{1}{2} \overline{u_R}_{\alpha \underset{\smile}{ij}} \, g_3 \, G\!\!\!/^{\alpha}_{\ \beta} {u_R}^{\beta \underset{\smile}{ij}} - \overline{q_L}_{i\alpha} Y_u {u_R}^{\alpha \underset{\smile}{ij}} \phi_j^{\dagger} \,. \tag{1.25} $$

Using this awkward but valid notation, the *SU2U* approach can be applied on the $SU(2)_L$ gauge group, guaranteeing the existence of a global symmetry $U(1)_{\mathcal{W}}$ with the charges of the fields given by

$$ \mathcal{W} = \frac{1}{2} \Big(\big(\#\text{ upper } SU(2)_L \text{ indices} \big) - \big(\#\text{ lower } SU(2)_L \text{ indices} \big) \Big). \tag{1.26} $$

The $U(1)_{\mathcal{W}}$ symmetry is spontaneously broken because $\mathcal{W}(\phi) = +1/2$, but the combination

$$ (\mathcal{B} + \mathcal{L}) = \mathcal{W} - Y \tag{1.27} $$

yields a good symmetry even after the electroweak symmetry breaking. Of course, the quantity $\mathcal{L}$, defined implicitly in Eq. (1.27), is nothing else but the well known lepton number.

To conclude, the *SU2U* approach *can* be in principle applied on both non-Abelian gauge factors of G_{SM} in order to rediscover the $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$ symmetry of the model. However, the price paid consists in losing its key benefit: the intuitiveness.

1.2.5 Fermion number and Lorentz symmetry

Another well-known conserved quantity of the SM is the fermion number $\mathcal{F}$, given by the following linear combination:

$$ \mathcal{F} = 3\mathcal{B} + \mathcal{L}. \tag{1.28} $$

It is trivially recognized that $\mathcal{F}$ of various fields is intimately related to their representation or another symmetry of the SM: the (proper ortochronal) Lorentz group $\mathcal{L}_0^{\uparrow}$. In the SM, the situation is quite simple: half-integer-spin particles have $\mathcal{F} = 1$, integer-spin particles have $\mathcal{F} = 0$. When generalizations of SM are considered, this rule must be also generalized: fermions carry odd-integer $\mathcal{F}$, while $\mathcal{F}$ of bosons is even. In other words, fermion number of a given field must respect the congruency class or its Lorentz representation.

Let us recall the emergence of fermion number conservation in field theories. The models in relativistic quantum field theories (QFTs) are defined by their gauge symmetry group G and by listing the irreducible representations of $G \times \mathcal{L}_0^\uparrow$. In the renormalizable theories, just scalars and left-handed Weyl fermions need to be listed while the spin-1 fields follow from the gauge structure. The left-handed spinors can be optionally denoted as right-handed spinors instead, using the relation

$$(\psi_L)^c = \mathcal{C}\, \overline{\psi_L}^{\mathrm{T}} = (\psi^c)_R \equiv \widetilde{\psi}_R. \tag{1.29}$$

Using the Dirac 4-spinor notation, the model conserves fermion number provided that its Lagrangian can be written, exploiting the freedom from Eq. (1.29), without the conjugation matrix $\mathcal{C}$. Indeed, the objects carrying upper and lower Dirac indices which might appear in the formulas are the fermion fields $\psi^M, \overline{\psi}_M$ and the covariant symbols $(\gamma^\mu)^M_N, (\gamma_5)^M_N$ and $\mathcal{C}^{MN}$ or[8] $\mathcal{C}_{MN}$. Analogously to the *SU2U* approach, one ascribes the *fermion number* to all the objects as

$$\mathcal{F} = \Big(\#\text{ upper Dirac indices}\Big) - \Big(\#\text{ lower Dirac indices}\Big). \tag{1.30}$$

Then, the corresponding group $U(1)_\mathcal{F}$ is a good symmetry of the model provided that the only possible $\mathcal{F}$-charged spurion – the conjugation matrix $\mathcal{C}$ – is absent.

If desired, the previously presented models can be used as simple examples: the SM in Eq. (1.2) and its leptoquark extension in Eq. (1.15) are free of the $\mathcal{C}$ matrix and conserve the fermion number; the $\Delta\mathcal{F} = 2$ seesaw operator in Eq. (1.9) and its explicit realization in Subs. 1.1.2 indeed contain $\mathcal{C}$ in the Lagrangian; the $\Delta\mathcal{F} = 4$ SMEFT operators of dimension 6 in (1.10) contain $\mathcal{C}$ twice while the others do not. The correct assignment of the fermion number to the diqarks in the $\mathcal{B}$- and $\mathcal{L}$-conserving models is $\mathcal{F} = 2$; formally, the corresponding absence of the conjugation matrix in Eq. (1.18) can be achieved by including the $\mathcal{C}$ matrix into the definition of the diquark itself, $\chi^\alpha_{MN} = \mathcal{C}_{MN}\chi^\alpha$.

It should be clarified that the purpose of the last two subsections is *not* to persuade the reader that u_R should be written with two antisymmetric $SU(2)_L$ indices, nor to assert that the conjugation matrix must be strictly avoided when casting fermion number conserving Lagrangians. Rather than that, we wanted to argue that the relation between baryon number and $SU(3)_C$ symmetry lies on the same grounds as the relation between the fermion number and Lorentz symmetry. (In this respect, the chapter *Tests of conservation laws* in Ref. [54] presents a different attitude).

1.2.6 Weak hypercharge rules them all

Another well known fact is that all the SM fields (see Table 1.1) satisfy, as representations of G_{SM},

$$Y \overset{\text{mod}\,1}{=\!=} \frac{1}{2}c^{SU(2)_L} - \frac{1}{3}c^{SU(3)_C} \tag{1.31}$$

where $c^{SU(2)_L}$ and $c^{SU(3)_C}$ refer to the weak isospin duality and color triality, respectively.[9] This relation holds also for any BSM field φ for which an interaction

[8] Strictly speaking, the position of Dirac indices is $\mathcal{C}^{MN}$ and $\mathcal{C}^\dagger_{MN}$. However, in the chiral basis, $\mathcal{C}^\dagger = -\mathcal{C}$.

[9] In our notation, $A = B$ (mod 1) denotes cutting off the integer part is applied on the right-hand side of the exact equation. On the other hand, $A \overset{\text{mod n}}{=\!=} B$ means that that A and B may differ by an integer multiple of n.

vertex containing a single φ-leg and several SM-field legs exists, such as the scalars discussed in Subs. 1.2.2. Furthermore, Eq. (1.31) is valid for any irrep of G_{SM} which can stem from a representation of the $SO(10)$ grand unified theory. In the broken phase, a similar well known relation between color triality and electric charge follows from Eq. (1.31),

$$Q \overset{\mathrm{mod}\,1}{=\!=} -\frac{1}{3}c^{SU(3)_C}, \tag{1.32}$$

which, in turn, implies that colorless particles (both elementary and composite) carry integer electric charge.

Equation (1.31) in some sense selects irreps of G_{SM} which, for various reasons, are being considered interesting for model building. In fact, it has the same structure as the relations discussed so far, which in the similar formalism read

$$\mathcal{B} \overset{\mathrm{mod}\,1}{=\!=} \frac{1}{3}c^{SU(3)_C}, \tag{1.33}$$

$$\mathcal{L} \overset{\mathrm{mod}\,1}{=\!=} \frac{1}{2}c^{SU(2)_L} - \frac{1}{3}c^{SU(3)_C} - Y, \tag{1.34}$$

$$\mathcal{F} \overset{\mathrm{mod}\,1}{=\!=} J. \tag{1.35}$$

However, the role of Equations (1.33) – (1.35) is different: they suggest convenient choices of global charges once the model is build. Provided that Eq. (1.31) holds, Eq. (1.34) simplifies to

$$\mathcal{L} \overset{\mathrm{mod}\,1}{=\!=} 0, \tag{1.36}$$

claiming that the lepton number of any field in the theory should be an integer. In contrast, Eq. (1.33) implies a nontrivial relationship between baryon number and weak hypercharge:

$$\mathcal{B} \overset{\mathrm{mod}\,1/2}{=\!=} -Y. \tag{1.37}$$

1.2.7 A counterexample

There are situations in which the *SU2U* approach fails. Here, a counterexample BSM model will be presented, featuring the baryon number symmetry which is not in strict connection with the triality of color. Such a situation can arise when the matter field content of the model is divided into several sectors, and only gauge and scalar-portal ($\phi^\dagger\phi\, X^\dagger X$) interactions between different sectors exist. Then, the would-be global $U(1)_{\mathcal{B}}$ symmetry suggested by the *SU2U* approach may be explicitly broken in the non-standard sector but remain conserved in the sector containing the SM fields. Such models arise, for example, when the SM is augmented by a field with quantum numbers which do *not* respect Eq. (1.31).

As a particular example, consider extending the SM by a scalar $X \sim (3, 3, 0)$. As explained above, it cannot have any Yukawa interactions with the SM fermions and only the portal coupling with the SM Higgs. Thus, the $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$ invariance remains intact if we simply ascribe $\mathcal{B}(X) = \mathcal{L}(X) = 0$. The *SU2U* approach, on the other hand, inappropriately calls for $\mathcal{B}(X^\alpha) \overset{\mathrm{mod}\,1}{=\!=} +1/3$. Nevertheless, the X field cannot carry any conserved charge because of its trilinear self-interaction

$$\mathcal{L}^{X^3} = m\, X^{\alpha i}_{\ \ j} X^{\beta j}_{\ \ k} X^{\gamma k}_{\ \ i}\, \varepsilon_{\alpha\beta\gamma}\,. \tag{1.38}$$

non-Abelian symmetry	generator of Abelian symmetry
$\mathcal{L}_0^\uparrow$	$\mathcal{F}$
$SU(2)_L$	$\mathcal{L} + \mathcal{B} + Y$
$SU(3)_C$	$\mathcal{B}$
$SU(4)_C$	$\mathcal{B} - \frac{1}{\sqrt{6}} T_C^{15}$
$SU(5)$	$\mathcal{B} - \mathcal{L} - \frac{4}{5} Y$

Table 1.3: Pairs of related imposed and accidental symmetries.

1.3 Summary and overview

In gauge field theories, global symmetries commonly appear without being imposed. The accidental $U(1)_\mathcal{B} \times U(1)_\mathcal{L}$ symmetry of the SM is explicitly violated in many of its extensions.

The charges of accidental $U(1)$ symmetries can be often related to the congruency classes of the imposed non-Abelian (space-time and gauge) symmetries. For several important examples, these relations are summarized in Table 1.3: the first three relations have been thoroughly discussed in this chapter; the case of $SU(4)_C$ will be elaborated on in Chapter 2; for $SU(5)$ in the GUT framework see Appendix B.

If the Lagrangian of an $SU(N)$ invariant theory (written with explicit indices) is free of the corresponding Levi-Civita tensor, the existence of an accidental $U(1)$ symmetry is guaranteed and its action can be inferred simply from the difference between number of upper and lower $SU(N)$ indices carried by the dynamical fields.

In SMEFT, also the opposite implication holds: baryon number violating terms can be simply identified by the color Levi-Civita symbols. Similar conclusions will be made in Section 2.3 in context of $SU(4)_C$ theories of the Pati-Salam type.

2. Quark-lepton unification

Quark-lepton unification (QLU) is a framework introduced by Pati and Salam in the article called *Lepton number as a fourth color* [26]. As this title suggests, the main idea of QLU is based on extending the QCD gauge factor $SU(3)_C$ to $SU(4)_C$ and accommodating the quarks and leptons in common representations. In the minimal setting, these read

$$F_L{}^{ia} = \begin{pmatrix} q_L{}^{\alpha i} \\ \ell_L{}^{i} \end{pmatrix}, \qquad f_R^{u\,a} = \begin{pmatrix} u_R{}^{\alpha} \\ \nu_R \end{pmatrix}, \qquad f_R^{d\,a} = \begin{pmatrix} d_R{}^{\alpha} \\ e_R \end{pmatrix}, \tag{2.1}$$

where $a \in \{1,2,3,4\}$ denotes the $SU(4)_C$ fundamental index while $\alpha \in \{1,2,3\}$.

The original Pati-Salam model combined this assumption with left-right symmetry. However, this work focuses mainly on more modest scenarios with the

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_R \tag{2.2}$$

gauge structure, as detailed in Section 2.1. In Section 2.2, the *Minimal quark-lepton symmetry model* (MQLSM) is introduced, the smallest SM extension featuring the $SU(4)_C$ unification. For more general situations but with this particular example in mind, we overview the status of baryon and lepton numbers in models of QLU in Sections 2.3 and 2.4, and argue that the title of Ref. [26] may be slightly misleading for some cases.

Afterwards, an overview of the gauge boson sector in theories based on G_{421} is given in Section 2.5, followed by a detailed analysis of the scalar sector of the MQLSM in Section 2.6 where the most general scalar potential is identified and studied [25]. In Section 2.7, the Fileviez Pérez – Wise (FPW) model is presented, an extension of the MQLSM by the inverse seesaw mechanism. Since both MQLS and FPW models have identical bosonic sectors, Sections 2.5 and 2.6 hold for both models. Section 2.8 is focused on fermionic interactions with the BSM bosons in the considered models. Finally, in Section 2.9 we count the number of free parameters and discuss the number of physical phases in the flavour mixing matrices between quarks and leptons which, to our best knowledge, has not been described in the literature yet.

Such a detailed analysis provides a solid background for Chapters 4 and 5 where phenomenological analyses of those (or similar) models will be presented.

2.1 Gauge symmetry and its breaking

Let us begin with a closer look at the structure and fate of the extended gauge symmetry cast in Eq. (2.2). The $SU(4)_C$ factor is supposed to be spontaneously broken at some high scale way above the electroweak one, which (unlike for GUTs) can be chosen arbitrarily since our framework unifies the fermions but not the gauge interactions. The smallest possible first step of the $SU(4)_C$ breaking is

$$SU(4)_C \to SU(3)_C \times U(1)_{[B\text{–}L]}\,. \tag{2.3}$$

The Abelian factor in Eq. (2.3) is generated by

$$T_C^{15} = \frac{1}{2\sqrt{6}} \begin{pmatrix} \mathbb{1}_{3\times 3} & 0 \\ 0 & -3 \end{pmatrix} \tag{2.4}$$

G_{421}	$SU(4)_C$		$U(1)_R$	$SU(2)_L$
	$T_C^{1,\dots,8}$ $g_3 = g_4$ $G_\mu^{1,\dots,8} = A_\mu^{1,\dots,8}$	$[B{-}L] = \sqrt{\frac{8}{3}} T_C^{15}$ $g_{BL} = \sqrt{\frac{3}{8}} g_4$ A_μ^{15}		
G_{3121}	$SU(3)_C$	$U(1)_{[B-L]}$		
$\downarrow$		$Y = \frac{1}{2}[B{-}L] + R$ $g' = \frac{g_{BL} g_R}{\sqrt{g_{BL}^2 + (g_R/2)^2}} = 2 g_{BL} \sin\theta'$ $B_\mu = \sin\theta' A_\mu^{15} + \cos\theta' B'_\mu$		
G_{SM}		$U(1)_Y$		
$\downarrow$		$Q = T_L^3 + Y$ $e = \frac{g g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W$ $A_\mu = \cos\theta_W B_\mu - \sin\theta_W W_\mu^3$		
G_{vac}		$U(1)_Q$		

Table 2.1: Scheme of the sequential symmetry breaking in the quark-lepton symmetry scenarios. For each step, the corresponding branching rules, matching equations and gauge bosons which remain massless are specified.

and its name becomes clear when one considers its multiple

$$[B{-}L] = \sqrt{\frac{8}{3}} T_C^{15} = \text{diag}\left(^1\!/_3, ^1\!/_3, ^1\!/_3, -1\right) \tag{2.5}$$

applied to the unified representations in Eq. (2.1). Nevertheless, we will always keep the square brackets in order to indicate that $[B{-}L]$ is an indivisible symbol.

In the next step, the intermediate symmetry

$$G_{3121} = SU(3)_C \times U(1)_{[B-L]} \times SU(2)_L \times U(1)_R \tag{2.6}$$

is further broken down to G_{SM}, following the branching rule

$$Y = R + \frac{1}{2}[B{-}L]\,. \tag{2.7}$$

More details about the spontaneous symmetry breaking can be found in Table 2.1.

Notice that G_{421} is a subgroup of the Pati-Salam group

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R, \tag{2.8}$$

Lie algebra of which is a maximal subalgebra of $so(10)$. Therefore, the BSM models considered may well be low-energy limits of the Pati-Salam-like framework or even of a grand unified theory. On the other hand, in recent years several models were built around the gauge group $SU(4)_{C_L} \times SU(n)_{C_R} \times SU(2)_L \times U(1)$ with $n = 3$ [55, 56, 57, 58, 59, 60] or $n = 4$ [55, 61, 62, 63] where the standard QCD generators arise as $T_C^A = T_{C_L}^A + T_{C_R}^A$ for $A = 1, \dots, 8$. These models can not be effectively described in our framework.

2.2 Minimal quark-lepton symmetry model

In order to give a specific example, we will shortly present the MQLSM, the simplest model featuring the $SU(4)_C$ group which may be compatible with the

G_{421}	$\rightarrow$	$G_{\rm SM}$	$[B-L]$	$\mathcal{M}\,\mathcal{F}$	$\mathcal{B}$	$\mathcal{L}$
Field content of the MQSLM						
Fermions:						
$F_{L\,(4,2,0)}$	$=$	$\begin{pmatrix} q_L \\ \ell_L \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	$1\ \ 1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$f^u_{R\,(4,1,+1/2)}$	$=$	$\begin{pmatrix} u_R \\ \nu_R \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	$1\ \ 1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$f^d_{R\,(4,1,-1/2)}$	$=$	$\begin{pmatrix} d_R \\ e_R \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	$1\ \ 1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Gauge bosons:						
$A_{\mu(15,1,0)}$	$=$	$\begin{pmatrix} G_\mu + \frac{A^{15}_\mu}{2\sqrt{6}} & U_{1\mu(3,1,+2/3)} \\ U_{1\mu}{}^\dagger & \frac{-3}{2\sqrt{6}} A^{15}_{\mu(1,1,0)} \end{pmatrix}$	$\begin{pmatrix} 0 & 4/3 \\ -4/3 & 0 \end{pmatrix}$	$0\ \ 0$	$\begin{pmatrix} 0 & 1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
$W_{\mu(1,3,0)}$			0	$0\ \ 0$	0	0
$B'_{\mu(1,1,0)}$			0	$0\ \ 0$	0	0
Scalar bosons:						
$\chi_{(4,1,+1/2)}$	$=$	$\begin{pmatrix} \bar{S}^\dagger_{1(3,1,+2/3)} \\ \chi^0_{(1,1,0)} \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	$1\ \ 0$	$\begin{pmatrix} 1/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
$H_{(1,2,+1/2)}$			0	$0\ \ 0$	0	0
$\Phi_{(15,2,+1/2)}$	$=$	$\begin{pmatrix} G_{2(8,2,+1/2)} & R_{2(3,2,+7/6)} \\ \tilde{R}^\dagger_{2(\bar{3},2,-1/6)} & 0 \end{pmatrix} + \sqrt{2}\,T^{15}_C H_{2(1,2,+1/2)}$	$\begin{pmatrix} 0 & 4/3 \\ -4/3 & 0 \end{pmatrix}$	$0\ \ 0$	$\begin{pmatrix} 0 & 1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
Fields which extend the MQLSM to the FPW model						
Fermions:						
$N_{L\,(1,1,0)}$			0	$0\ \ 1$	0	1

Table 2.2: Field content of the MQLS and FPW models. On the right hand side of the table, the corresponding quantum numbers discussed in Secs. 2.3 and 2.4 are displayed.

SM at low energies. The MQLSM has been introduced in Ref. [27] (with an optional extra scalar field $(15,1,0)$) and further elaborated on in Refs. [64, 65, 66, 61, 67, 68, 69]. It is also a starting point for construction of other similar models [28] which we will investigate later on.

The meaning of minimality of this model is three-fold: the first aspect is its gauge group G_{421}, indeed a minimal extension of $G_{\rm SM}$ containing the $SU(4)_C$ factor (barring the case of $SU(4)_C \times SU(2)_L$ with weak hypercharge identified with T^{15}_C which obviously does not lead to the SM at low energies). The other two aspects relate to the minimal fermion and scalar sectors, respectively, as summarized in Table 2.2. In particular, the fermionic sector consist only of the fields in Eq. (2.1) and thus contains no other fields but the SM fermions and right-handed neutrinos. The scalar sector consists of $\chi \sim (4,1,+1/2)$ which breaks the $SU(4)_C$ symmetry, one Higgs doublet $H \sim (1,2,+1/2)$ and an additional field $\Phi \sim (15,2,+1/2)$ which contains another Higgs doublet, H_2. We parametrize their VEVs as[1]

$$\langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_\chi\end{pmatrix}\,, \quad \langle H\rangle = \frac{\sin\beta}{\sqrt{2}}\begin{bmatrix}0\\ v_{\rm ew}\end{bmatrix}\,, \quad \langle\Phi\rangle = \frac{\cos\beta}{2\sqrt{6}}\begin{pmatrix}\mathbb{1} & 0\\ 0 & -3\end{pmatrix}\otimes\begin{bmatrix}0\\ v_{\rm ew}\end{bmatrix}. \quad (2.9)$$

Under the assumption $v_\chi \gg v_{\rm ew}$, the scalars participate in the spontaneous gauge symmetry breaking as follows:

$$G_{421} \xrightarrow{\langle\chi\rangle,\,\langle\Phi\rangle} G_{\rm SM} \xrightarrow{\langle H\rangle,\,\langle\Phi\rangle} G_{\rm vac}\,, \quad (2.10)$$

effectively skipping the intermediate stage G_{3121}. The presence of two Higgs doublets stemming from different $SU(4)_C$ representations (singlet and adjoint, no other options exist [70]) is needed in order to give masses independently to the quarks and leptons in the common quadruplets. Indeed, the fermion mass matrices are given by[2]

$$M_u = \left(Y_1\sin\beta + Y_2\frac{\cos\beta}{\sqrt{12}}\right)\frac{v_{\rm ew}}{\sqrt{2}}\,, \quad M_d = \left(Y_3\sin\beta + Y_4\frac{\cos\beta}{\sqrt{12}}\right)\frac{v_{\rm ew}}{\sqrt{2}}\,, \quad (2.11a)$$

$$M_\nu^D = \left(Y_1\sin\beta - 3Y_2\frac{\cos\beta}{\sqrt{12}}\right)\frac{v_{\rm ew}}{\sqrt{2}}\,, \quad M_e = \left(Y_3\sin\beta - 3Y_4\frac{\cos\beta}{\sqrt{12}}\right)\frac{v_{\rm ew}}{\sqrt{2}}\,, \quad (2.11b)$$

where $Y_{1,2,3,4}$ are Yukawa matrices entering the Lagrangian of the model which we cast in the full form for future purposes:

$$\begin{aligned}
\mathcal{L}_{\rm MQLSM} = &-\frac{1}{2}A^a_{\mu\nu b}A^{b\mu\nu}_a - \frac{1}{2}W^i_{\mu\nu j}W^{\mu\nu j}_i - \frac{1}{4}B'_{\mu\nu}B'^{\mu\nu} + \overline{F}_{Lia}(\not{D}F_L)^{ia} + \overline{f^u_R}_a(\not{D}f^u_R)^a \\
&+ \overline{f^d_R}_a(\not{D}f^d_R)^a + (D_\mu\chi)^\dagger_a(D^\mu\chi)^a + (D_\mu H)^\dagger_i(D^\mu H)^i + (D_\mu\Phi)^{\dagger a}_{ib}(D^\mu\Phi)^{ib}_a \\
&- \left(\overline{f^u_R}_a Y_1 H^i F_L{}^{ja}\varepsilon_{ij} + \overline{f^u_R}_a Y_2 \Phi^{ia}_b F_L{}^{jb}\varepsilon_{ij} + \overline{f^d_R}_a Y_3 H^\dagger_i F_L{}^{ia} + \overline{f^d_R}_a Y_4 \Phi^{\dagger a}_{ib} F_L{}^{ib} + \text{h.c.}\right) \\
&- \mu_H^2 H^\dagger_i H^i - \mu_\chi^2\chi^\dagger_a\chi^a - \mu_\phi^2\Phi^{\dagger a}_{ib}\Phi^{ib}_a - \lambda_1 H^\dagger_i H^i \Phi^{\dagger a}_{ib}\Phi^{ib}_a \\
&- \lambda_2\Phi^{\dagger a}_{ib}\Phi^{ib}_a\chi^\dagger_c\chi^c - \lambda_3 H^\dagger_i H^i\chi^\dagger_a\chi^a - \left(\lambda_4 H^\dagger_i\chi^\dagger_a\Phi^{\dagger ia}_b\chi^b + \text{h.c.}\right) - \lambda_5 H^\dagger_i\Phi^{ia}_b\Phi^{\dagger b}_{ja}H^j \\
&- \lambda_6\chi^\dagger_a\Phi^{ia}_b\Phi^{\dagger b}_{ic}\chi^c - \lambda_7\left(H^\dagger_i H^i\right)^2 - \lambda_8\left(\chi^\dagger_a\chi^a\right)^2 - \lambda_9\Phi^{\dagger a}_{ib}\Phi^{ib}_c\Phi^{\dagger c}_{jd}\Phi^{jd}_a - \lambda_{10}\left(\Phi^{\dagger a}_{ib}\Phi^{ib}_a\right)^2 \\
&- \left(\lambda_{11}H^\dagger_i\Phi^{ia}_b\Phi^{jb}_a H^\dagger_j + \lambda_{12}H^\dagger_i\Phi^{ia}_b\;\Phi^{jb}_c\Phi^{\dagger c}_{ja} + \lambda_{13}H^\dagger_i\Phi^{ia}_b\Phi^{\dagger b}_{jc}\Phi^{jc}_a + \text{h.c.}\right) \\
&- \lambda_{14}\chi^\dagger_a\Phi^{\dagger a}_{ib}\Phi^{ib}_c\chi^c - \lambda_{15}\Phi^{\dagger a}_{ib}\Phi^{jb}_c\Phi^{\dagger c}_{jd}\Phi^{id}_a - \lambda_{16}\Phi^{\dagger a}_{ib}\;\Phi^{jb}_a\Phi^{\dagger c}_{jd}\Phi^{id}_c \\
&- \lambda_{17}\Phi^{\dagger a}_{ib}\Phi^{\dagger b}_{ja}\Phi^{ic}_d\Phi^{jd}_c - \lambda_{18}\Phi^{\dagger a}_{ib}\Phi^{\dagger b}_{jc}\Phi^{ic}_d\Phi^{jd}_a - \lambda_{19}\Phi^{\dagger a}_{ib}\Phi^{\dagger b}_{jc}\Phi^{jc}_d\Phi^{id}_a\,.
\end{aligned} \quad (2.12)$$

The MQLSM is able to fully reproduce the SM at low energies. We postpone the description and investigation of new physics (second Higgs doublet, Z' boson, heavy scalar gluons, various kinds of leptoquarks) to later sections and chapters. In the next two sections we will provide some more general insight into the the status of baryon and lepton number in models with quark-lepton symmetry, taking the MQLSM as an important example.

2.3 Baryon number and the $\mathcal{M}$ symmetry

In this part we apply the *SU2U* approach described in Section 1.2 on the $SU(4)_C$ group. The $SU(4)_C$ gauge interactions conserve a global charge $\mathcal{M}$ which is

[1] Recall that the round brackets denote the $SU(4)_C$ space (using the 3+1 block structure) while the square brackets identify $SU(2)_L$ doublets.

[2] In Ref. [28] the numerical factors multiplying Y_2 and Y_4 are wrong due to inappropriate normalization of H_2 within the Φ field.

ascribed uniformly to all elements of a given irreducible representation as

$$\mathcal{M} = \Big(\#\text{ upper } SU(4)_C \text{ indices}\Big) - \Big(\#\text{ lower } SU(4)_C \text{ indices}\Big)\,. \tag{2.13}$$

In other words, in the formalism where the $SU(4)_C$ structure is realized by tensors over 4-dimensional vector spaces and where $T_C^{1,\ldots,15}$ act via traceless Hermitean 4×4 matrices, $\mathcal{M}$ acting on the same structures via a unit matrix generates a global symmetry of the $SU(4)_C$ gauge interactions. Moreover, $U(1)_\mathcal{M}$ is a valid accidental symmetry of the entire model (including interactions of scalars) if its complete Lagrangian does not contain any antisymmetric tensor ε_{abcd}. As one can verify by a brief look at Eq. (2.12), the MQLSM is a good example of such a model.

Eventually, $U(1)_\mathcal{M}$ gets spontaneously broken together with $U(1)_{[B\text{–}L]}$, but the combination

$$\mathcal{B} = \frac{1}{4}\left(\mathcal{M} + [B\!-\!L]\right) = \operatorname{diag}\left({}^1\!/\!_3, {}^1\!/\!_3, {}^1\!/\!_3, 0\right), \tag{2.14}$$

a natural candidate for the baryon number, remains unbroken. For the MQLSM, this statement can be easily verified by confronting Table 2.2 with Eq. (2.9). A general proof is also simple: provided that $SU(3)_C$ is an unbroken part of the $SU(4)_C$ symmetry, all VEVs are annihilated by its generators T_C^A for $A = 1,\ldots,8$. Thus, all VEVs also vanish under the action of the quadratic Casimir operator of the $SU(3)_C$ subgroup, $C_2^{SU(3)_C} = \sum_{A=1}^{8} T_C^A T_C^A$, which acts on every $SU(4)_C$ multiplet as a multiple of $\mathcal{B}$ defined above.[3] Hence, $\mathcal{B}$ also annihilates all the VEVs.

The arguments from the last two paragraphs together result in the following useful conclusion: **Absence of $\boldsymbol{\varepsilon_{abcd}}$ in the Lagrangian of an $\boldsymbol{SU(4)_C}$ gauge theory implies conservation of baryon number.** An alternative, but equivalent reasoning may be given as follows: as we know, the baryon number defined according to the *SU2U* perspective can be violated only if the Levi-Civita tensor $\varepsilon_{\alpha\beta\gamma}$ appears in the Lagrangian. When rewriting the $SU(4)_C$ invariant interactions to the broken phase by splitting the indices like $a \to (\alpha, 4)$, the 3-dimensional $\varepsilon_{\alpha\beta\gamma}$ can only emerge from the 4-dimensional antisymmetric tensor ε_{abcd}.

There are many examples of $SU(4)_C$ models featuring the $U(1)_\mathcal{M}$ symmetry and thus conserving $\mathcal{B}$, including the MQLSM and its extensions inspected in Chapters 4 and 5.

On the other hand, there are also models with different scalar sectors for which the $U(1)_\mathcal{M}$ defined in (2.13) is not a good symmetry. As the first example, consider the scalar self-interaction

$$\varepsilon^{a_1a_2a_3a_4}\varepsilon^{b_1b_2b_3b_4}\Delta^{ii'}_{a_1b_1}\Delta^{jj'}_{a_2b_2}\Delta^{kk'}_{a_3b_3}\Delta^{ll'}_{a_4b_4}\varepsilon_{ij}\varepsilon_{i'j'}\varepsilon_{kl}\varepsilon_{k'l'} \tag{2.15}$$

of the $(10,3,0)$ irrep of G_{421}, which also has the Yukawa interaction ${F_L}^{\mathrm{T}}\mathcal{C}\Delta F_L$. Within the G_{422} framework, Δ can stand for $(10,3,1)$ or also[4] $(10,1,3)$, both

[3] Notice though that the proportionality factor between the actions of $C_2^{SU(3)_C}$ and $\mathcal{B}$ may differ among various field multiplets. Moreover, as discussed in Section 1.2, our definitions of $\mathcal{M}$ and $\mathcal{B}$ above contain redundancies. Hence, the $\mathcal{M}$ and $\mathcal{B}$ generators can not be strictly related to the Casimir operators of $SU(4)_C$ and $SU(3)_C$, respectively.

[4] The indices $i, i', \ldots$ in (2.15) denote either $SU(2)_L$ or $SU(2)_R$ structure, depending on the representation considered.

emerging in the "classic" left-right symmetric model of Ref. [71]. Such scalar fields induce neutron-antineutron oscillations (i.e., $\Delta\mathcal{B} = 2$) but keep proton stable due to an exact "remnant" discrete symmetry.

As another example, let us take the interaction of Δ with an $SU(4)_C$ adjoint, $SU(2)_L$ doublet Φ (cf. also the MQLSM) via the term

$$\varepsilon^{abcd}\Delta_{aa'}\Delta_{bb'}\Phi_c^{a'}\Phi_d^{b'} \tag{2.16}$$

(with $SU(2)$ indices supressed). Baryon numbers of the substructures in Δ and Φ are fixed by their Yukawa interactions in the Pati-Salam model [70] which, together with (2.16), trigger proton decay ($\Delta\mathcal{B} = 1$). Both these examples can be also found in Section 6.5 of Mohapatra's coursebook [72].

Finally, the *Alternative Pati-Salam model* [73] has been suggested as a TeV realization of the G_{422} symmetry. However, it can be shown that $\mathcal{B}$ would be violated if there was the interaction

$$\varepsilon_{abcd}\,\chi_L^{ia}\chi_L^{jb}\chi_R^{ci'}\chi_R^{dj'}\,\varepsilon_{ij}\varepsilon_{i'j'} \tag{2.17}$$

of the scalar fields $\chi_L \sim (4,2,1)$ and $\chi_R \sim (4,1,2)$.

Notice that identifying baryon number violating models is fairly easy using the $U(1)_\mathcal{M}$ symmetry considerations, since Eq. (2.14) together with the *SU2U* approach tell us that $\mathcal{B}$-changing interaction terms can be recognized by the presence of the $SU(4)_C$ Levi-Civita tensors:

$$\Delta\mathcal{B} = \Delta\mathcal{M} = \Big(\#\,\varepsilon_{abcd}\Big) - \Big(\#\,\varepsilon^{abcd}\Big). \tag{2.18}$$

For another $SU(4)$-based model where Eq. (2.18) is applicable see Ref. [74]. In the model of Ref. [57] with an $SU(4)\times SU(3)$ gauge subgroup, an interaction term with the corresponding Levi-Civita symbols triggers explicit violation of another would-be extra accidental symmetry.

2.4 Lepton number *not* as the fourth color

As described above, the MQLSM features a global $U(1)_\mathcal{M}$ symmetry which at the end guarantees conservation of $\mathcal{B}$. One might thus conclude that the lepton number must be spontaneously broken because $[B-L]$ is so; such a deduction would be, however, wrong. It can be easily verified by looking at the MQLSM Lagrangian (2.12) that it conserves also the fermion number $\mathcal{F}$ (since there is no $\mathcal{C}$ matrix). Hence, the lepton number in MQLSM must be conserved, provided that it is *defined* by

$$\mathcal{L} = \mathcal{F} - 3\mathcal{B}\,. \tag{2.19}$$

One can easily verify at Table 2.2 that such a definition leads to correct values of $\mathcal{L}$ for all the SM fields and that $\mathcal{L}$ is respected by the vacuum [cf. (2.9)]. It should be stressed that it differs from the notion of *fourth color* which unambiguously means acting on $SU(4)_C$ tensors by

$$L^{\text{4th-col}} = \mathrm{diag}(0,0,0,1) = \frac{\mathcal{M} - 3[B-L]}{4} = \mathcal{B} - [B-L]\,, \tag{2.20}$$

$G_{422} \quad \rightarrow \quad G_{\rm SM}$	$[B\!-\!L]$	$\mathcal{M}\!=\!\mathcal{F}$	$\mathcal{B}$	$\mathcal{L}$
Field content of the Mohapatra-Marshak model [71]				
Fermions:				
$F_{L(4,2,1)} = \begin{pmatrix} q_L \\ \ell_L \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	1	$\begin{pmatrix} 1/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$F_{R(4,1,2)} = \begin{pmatrix} [u_R \;\; d_R] \\ [\nu_R \;\; e_R] \end{pmatrix}$	$\begin{pmatrix} 1/3 \\ -1 \end{pmatrix}$	1	$\begin{pmatrix} 1/3 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Scalar bosons:				
$\phi_{(1,2,2)} = \begin{bmatrix} H & H'^{\dagger} \end{bmatrix}$	0	0	0	0
$\Delta_{L(10,3,1)} = \begin{pmatrix} X_{(\bar{6},3,-1/3)} & S_{3(\bar{3},3,+1/3)} \\ S_{3(\bar{3},3,+1/3)} & \Delta_{L44(1,3,+1)} \end{pmatrix}$	$\begin{pmatrix} -2/3 & 2/3 \\ 2/3 & 2 \end{pmatrix}$	-2	$\begin{pmatrix} -2/3 & -1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ -1 & -2 \end{pmatrix}$
$\Delta_{R(10,1,3)} = \begin{pmatrix} \chi_{(\bar{6},1,T^3_R-1/3)} & S_{1(\bar{3},1,T^3_R+1/3)} \\ S_{1(\bar{3},1,T^3_R+1/3)} & \Delta_{R44(1,1,T^3_R+1)} \end{pmatrix} \otimes [T^3_R = -1, 0, +1]$	$\begin{pmatrix} -2/3 & 2/3 \\ 2/3 & 2 \end{pmatrix}$	-2	$\begin{pmatrix} -2/3 & -1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ -1 & -2 \end{pmatrix}$
Fields which extend the above model to the Pati-Salam model [70]				
Scalar bosons:				
$\Phi_{(15,2,2)} = \left[\begin{pmatrix} G_{2(8,2,+1/2)} & R_{2(3,2,+7/6)} \\ \tilde{R}^{\dagger}_{2(\bar{3},2,-1/6)} & 0 \end{pmatrix}, \begin{pmatrix} G'^{\dagger}_2 & \tilde{R}'_2 \\ R'^{\dagger}_2 & 0 \end{pmatrix} \right] + \sqrt{2} T^{15}_C \left[H_2, H'^{\dagger}_2 \right]$	$\begin{pmatrix} 0 & 4/3 \\ -4/3 & 0 \end{pmatrix}$	0	$\begin{pmatrix} 0 & 1/3 \\ -1/3 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Table 2.3: Fermions and scalars in the models of Pati-Salam type and their quantum numbers. Horizontal structures in square brackets denote $SU(2)_R$.

a quantity which is spontaneously broken indeed. It is of course just a matter of notation introduced in Eq. (2.5) which leads us to write down that, generally,

$$\mathcal{B} - \mathcal{L} \neq [B\!-\!L]\,. \tag{2.21}$$

For the MQLSM, this bizarre inequality is manifest for the elements of the scalar field χ (see Table 2.2).

Despite that, denoting the generator of $U(1)_{T^{15}_C}$ simply by $B - L$ is quite commonly used. To this end, we would like to stress that such a notation should be used with great caution as it may become a source of confusion. In Chapter 5.2, we will discuss a particular example from the literature where overlooking the inequality (2.21) lead to incorrect phenomenological conclusions. The observation that a spontaneously broken gauge symmetry which charges quarks by $+1/3$ and leptons by -1 needs not necessarily be identical to $\mathcal{B} - \mathcal{L}$ has been recently advocated also in Ref. [75].

It is perhaps worth comparing the situation with the "classic" models of quark-lepton unification with left-right symmetry based on the G_{422} group [71, 70]. Due to a different scalar sector (see Table 2.3), the $\mathcal{M}$ charge defined according to Eq. (2.13) exactly coincides with the most reasonable attempt to ascribe the fermion number. Hence, there is just a single candidate for an accidental non-local $U(1)$ symmetry, which is explicitly broken by the scalar interaction (2.15) and potentially also by (2.16). Provided $\mathcal{M} = \mathcal{F}$, Eqs. (2.14) and (2.19) imply that $[B\!-\!L] = \mathcal{B} - \mathcal{L}$ *does* hold for all the fields in a given model. Eventually, this gauge symmetry is broken spontaneously by $\langle \Delta_{R44} \rangle$ and no combination of $\mathcal{B}$ and $\mathcal{L}$ represents a precise symmetry in the broken phase.

Let us recapitulate the last two sections: quark-lepton $SU(4)_C$ unification does not necessarily induce BNV nor LNV. Violation of $\mathcal{L}$ can occur both explicitly or spontaneously, while $\mathcal{B}$ can be violated only explicitly. E.g., in the MQLSM, both $\mathcal{B}$ and $\mathcal{L}$ are conserved, while the FPW model which will be introduced later violates $\mathcal{L}$ explicitly. In the models of the Pati-Salam type, both $\mathcal{B}$ and $\mathcal{L}$ are explicitly violated, with a combination $\mathcal{B} - \mathcal{L}$ broken only spontaneously. If the explicitly violating scalar interactions in these models were absent, lepton number would be spontaneously broken but $\mathcal{B}$ would remain conserved.

2.5 Vector bosons

In this section, we describe in detail the gauge field content in the quark-lepton unification scenarios based on the G_{421} gauge group as outlined in Table 2.1. A similar study can be found, for example, in Ref. [27]; we present it here for the sake of completeness. We strive to be as general as possible, going beyond the specific scalar content of the MQLSM. To this end, we use the notation

$$\langle T_X \rangle = \sum_{\varphi} \langle \varphi \rangle^\dagger T_X \langle \varphi \rangle \,, \qquad \langle T_X T_Y \rangle = \sum_{\varphi} \langle \varphi \rangle^\dagger T_X T_Y \langle \varphi \rangle \tag{2.22}$$

where the sum runs over all scalar multiplets in the model and T_X, T_Y stand for specific gauge symmetry generators.

The gauge fields as they arise from the associated factors of G_{421} read (see also Table 2.2):

$$SU(4)_C : \qquad A_\mu = \begin{pmatrix} G_\mu + \frac{1}{2\sqrt{6}} A_\mu^{15} & U_{1\mu}/\sqrt{2} \\ U_{1\mu}^\dagger/\sqrt{2} & -\frac{3}{2\sqrt{6}} A_\mu^{15} \end{pmatrix} \tag{2.23a}$$

$$SU(2)_L : \qquad W_\mu = \frac{1}{2} \begin{bmatrix} W_\mu^3 & \sqrt{2}\, W_\mu^+ \\ \sqrt{2}\, W_\mu^- & -W_\mu^3 \end{bmatrix} \tag{2.23b}$$

$$U(1)_R : \qquad B'_\mu. \tag{2.23c}$$

Gauge leptoquark

The first step of the spontaneous symmetry breaking (SSB), i.e. $SU(4)_C \to SU(3)_C \times U(1)_{[B\text{–}L]}$, generates mass of the leptoquark field $U_{1\mu} \sim (3, 1, +2/3)$:

$$\frac{1}{2} m_{U_1}^2 = g_4^2 \langle T_C^9 T_C^9 \rangle \,. \tag{2.24}$$

Its interactions with the SM fermions are described in Section 2.8.1. The low-energy effects of this leptoquark will be studied thoroughly in Chapter 5.

Neutral gauge bosons

The renormalization group (RG) evolution of the gauge couplings between the $SU(4)_C$ breaking scale and that of $[B-L]$ breaking is more complicated than usually: because the G_{3121} group contains two $U(1)$ factors, the gauge kinetic mixing term $A^{15}_{\mu\nu}B'^{\mu\nu}$ appears during the running, bringing a new parameter into the game [76]. For simplicity, we will assume that the energy interval with two unbroken Abelian factors is so short that these effects are negligible. Then, the neutral vector boson mass matrix in the $\{A^{15}, B', W^3\}$ basis reads

$$\frac{1}{2}M^2_{ZZ'\gamma} = \begin{pmatrix} g^2_{BL}\langle[B-L]^2\rangle & g_{BL}\,g_R\langle[B-L]R\rangle & g_{BL}\,g\langle[B-L]T^3_L\rangle \\ g_R\,g_{BL}\langle R\,[B-L]\rangle & g^2_R\langle RR\rangle & g_R\,g\langle R\,T^3_L\rangle \\ g\,g_{BL}\langle T^3_L[B-L]\rangle & g\,g_R\langle T^3_L R\rangle & g^2\langle T^3_L T^3_L\rangle \end{pmatrix}. \tag{2.25}$$

Provided $Q = T^3_L + R + \frac{1}{2}[B-L]$ is unbroken, $\langle QT\rangle = 0$ for any generator T, and the matrix in (2.25) has a zero eigenvalue, corresponding to the vanishing photon mass. The other two eigenstates yield the Z boson and a new massive field, Z'.

Usually, the limit of two separate symmetry breaking stages, $G_{3121} \to G_{\rm SM} \to G_{\rm vac}$, can be considered. After the first of these steps, with the symmetries generated by $Y = R + [B-L]/2$ and T^3_L yet unbroken, Eq. (2.25) simplifies to

$$\frac{1}{2}M^2_{ZZ'\gamma} = \langle R\rangle^2\,\xi\,\xi^T \qquad \text{with} \qquad \xi = \begin{pmatrix} 2g_{BL} \\ -g_R \\ 0 \end{pmatrix}, \tag{2.26}$$

which can be diagonalized via the rotation by the angle θ' satisfying

$$\tan\theta' = \frac{g_R}{2g_{BL}}, \qquad \sin\theta' = \frac{g'}{2g_{BL}}, \qquad \cos\theta' = \frac{g'}{g_R}. \tag{2.27}$$

If the running effects between the G_{3121}-breaking scale μ_{BL} and that of G_{421}-symmetry breaking can be neglected, one has $g_{BL}(\mu_{BL}) \approx \sqrt{\frac{3}{8}}g_3(\mu_{BL})$ and $\sin\theta'$ can be given readily in terms of the SM parameters. This rotation mixes the gauge fields A^{15}_μ and B'_μ into a massive state Z'_0 with

$$\frac{1}{2}m^2_{Z'_0} = \frac{g^2_{BL}}{\cos^2\theta'}\langle[B-L]\rangle^2 \tag{2.28}$$

and a B_μ field corresponding to the weak hypercharge Y which, after the electroweak symmetry breaking (EWSB), mixes further with W^3_μ in the SM-like fashion. To sum up,

$$\begin{pmatrix} A^{15}_\mu \\ B'_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta' & \sin\theta' & 0 \\ -\sin\theta' & \cos\theta' & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Z'_{0\mu} \\ B_\mu \\ W^3_\mu \end{pmatrix} \approx \begin{pmatrix} c_{\theta'} & s_{\theta'} & 0 \\ -s_{\theta'} & c_{\theta'} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_w & s_w \\ 0 & -s_w & c_w \end{pmatrix} \begin{pmatrix} Z'_\mu \\ Z_\mu \\ A_\mu \end{pmatrix} \tag{2.29}$$

with $c_{\theta'} = \cos\theta'$, $s_w = \sin\theta_w$ etc. The coupling of the Z' boson can be obtained by rewriting the relevant terms in the covariant derivative accordingly,

$$g_{BL}[B-L]A^{15}_\mu + g_R R B'_\mu \;=\; g'YB_\mu + \frac{g_{BL}}{\cos\theta'}\left([B-L] - 2Y\sin^2\theta'\right)Z'_{0\mu}\,, \tag{2.30}$$

which is an analogue to the similar relation well known from the SM:

$$g'YB_\mu + gT^3_L W^3_\mu \;\approx\; e\,QA_\mu + \frac{g}{\cos\theta_w}\left(T^3_L - Q\sin^2\theta_w\right)Z_\mu\,. \tag{2.31}$$

The approximate equalities in Eqs. (2.29) and (2.31) become exact in the limit $\langle R\rangle/\langle T_L^3\rangle \to \infty$. With the finite ratio of the symmetry breaking scales, the true mass eigenstate Z' does not fully coincide with Z_0'; however, this small difference is currently unimportant as there have been no convincing signals of the very existence of the Z' field so far. On the other hand, a small admixture of Z_0' in the known Z boson (with $m_Z = 91\,\text{GeV}$) might slightly modify its couplings with the SM fermions as well as the SM formula for the Z-boson mass.

2.5.1 Vector boson masses in MQLSM

Now, let us study the gauge boson masses in the MQLSM in particular. The VEVs of the scalar fields given in Eq. (2.9) imply the following relations:

$$\langle T_C^9 T_C^9\rangle = \frac{v_\chi^2}{8} + \frac{v_{\text{ew}}^2\cos^2\beta}{3}, \qquad \langle [B-L]^2\rangle = \frac{1}{2}v_\chi^2\,. \tag{2.32}$$

In the limit with separated symmetry breaking stages ($v_\chi \gg v_{\text{ew}}$) one obtains

$$m_{U_1} \approx \frac{g_4 v_\chi}{2}, \qquad m_{Z'} \approx \frac{g_{BL} v_\chi}{\cos\theta'}. \tag{2.33}$$

As there is no G_{3121} intermediate stage in the MQLSM, one can write down

$$g_4 = \sqrt{\frac{8}{3}} g_{BL} = g_3(v_\chi), \qquad \sin\theta' = \sqrt{\frac{2}{3}}\frac{g'(v_\chi)}{g_3(v_\chi)}, \tag{2.34}$$

which implies

$$\frac{m_{Z'}}{m_{U_1}} \approx \sqrt{\frac{3}{2}}\cos\theta' = \sqrt{\frac{3}{2} - \frac{g'^2(v_\chi)}{g_3^2(v_\chi)}}. \tag{2.35}$$

This ratio ranges between 1.17 for $v_\chi = 2$ TeV and 1.11 for $v_\chi = 20\,000$ TeV, assuming the one-loop SM running of the gauge couplings.

2.6 Scalar potential and scalar masses

In this section, the scalar sector of the MQLSM is described in detail. Note that it remains unchanged also for the FPW model [28] which shall be introduced in the next section. Identification of the full form of the scalar potential and its analysis has been published in Refs. [25, 29].

Recall from Table 2.2 on page 25 and from Eq. (2.9) that, after the symmetry breaking from G_{421} to G_{SM} by v_χ, the scalar fields of the model can be written in the following way:

$$\chi^a = \begin{pmatrix} \bar{S}_1^{\dagger\alpha} \\ \dfrac{v_\chi + \chi_R^0 + i z_0'}{\sqrt{2}} \end{pmatrix}, \qquad H^i, \qquad \Phi_b^{ia} = \begin{pmatrix} G_{2\beta}^{\alpha i} + \delta_\beta^\alpha \frac{H_2^i}{\sqrt{12}} & R_2^{\alpha i} \\ \tilde{R}_{2\beta j}^\dagger\,\varepsilon^{ji} & -\frac{3}{\sqrt{12}} H_2{}^i \end{pmatrix}. \tag{2.36}$$

The EWSB is triggered by the VEVs of the two Higgs doublets:

$$\langle H\rangle = \frac{\sin\beta}{\sqrt{2}}\begin{bmatrix} 0 \\ v_{\text{ew}} \end{bmatrix}, \qquad \langle H_2\rangle = \frac{\cos\beta}{\sqrt{2}}\begin{bmatrix} 0 \\ v_{\text{ew}} \end{bmatrix}. \tag{2.37}$$

The axial part z_0' of the neutral component of χ becomes eaten by the Z_0' while would-be the Goldstone boson associated to the vector leptoquark U_1 is approximately (neglecting the subdominant contribution of $\langle H_2\rangle$ to the $SU(4)_C$ breaking) given by $u_1 \approx \bar{S}_1^\dagger$. The Goldstone modes emerging after the EWSB reside in the $\hat{H}$ doublet

$$\hat{H} = \begin{bmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v_{\rm ew} + H^0 + z) \end{bmatrix} \tag{2.38}$$

which is defined by the following rotation:

$$\begin{pmatrix} \hat{H}' \\ \hat{H} \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H \\ H_2 \end{pmatrix} . \tag{2.39}$$

In the new basis, the whole electroweak VEV is hidden inside $\hat{H}$ and the remaining component H^0 has exactly the SM-Higgs-like interactions with the fermions. However, H^0 is generally not and exact mass eignestate. Nevertheless, a light physical scalar h with $m_h \propto v_{\rm ew}$ must appear in the mass spectrum [77], while the rest of the physical scalars should have larger masses of the order of v_χ, in accordance with the minimal survival hypothesis [78]. In such a case, the mass eigenstate h is well approximated by the field H^0 in Eq. (2.38) and thus has all the characteristics of the SM Higgs boson [79], as its mixing with χ_R^0 as well as with H'^0 from

$$\hat{H}' = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H'^0 + iA^0) \end{bmatrix} \tag{2.40}$$

must be suppressed by $v_{\rm ew}^2/v_\chi^2$. The field H^+ is an exact mass eigenstate as there are no other fields to mix with.

In the strongly interacting sector one finds electroweak doublets of massive *scalar gluons* G_2 and of two *leptoquark* fields R_2 and $\tilde{R}_2$ (cf. Tables 1.2 and A.3):

$$G_2{}^\alpha_\beta = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(G_R^0 + iG_I^0) \end{bmatrix}^\alpha_\beta , \qquad R_2{}^\alpha = \begin{bmatrix} R_2^{+5/3} \\ R_2^{+2/3} \end{bmatrix}^\alpha , \qquad \tilde{R}_2{}^\alpha = \begin{bmatrix} \tilde{R}_2^{+2/3} \\ \tilde{R}_2^{-1/3} \end{bmatrix}^\alpha . \tag{2.41}$$

The mass of G_2 is bounded from below by the K^0-$\overline{K}^0$ mixing and $b\bar{b}t\bar{t}$ production at the LHC [29, 80, 81]. Also the leptoquark masses are limited by the Atlas [82, 83, 84, 85, 86] and CMS [87, 88, 89] searches to be way above the electroweak scale. Hence, the mixing among $R_2^{+2/3}$, $\tilde{R}_2^{+2/3}$ and $\bar{S}_1^\dagger$, which appears only after the EWSB, is parametrically suppressed and all the fields in (2.38), (2.40) and (2.41) represent exact or approximate mass eigenstates. Moreover, the isospin mass splitting within the individual doublets must be quite tiny: since a general mass formula reads $m^2_{T_L^3=\pm 1/2} = av_\chi^2 \pm bv_{\rm ew}^2$, taking the square root gives $m_{T_L^3=\pm 1/2} \approx \sqrt{a}v_\chi \pm \frac{b}{2\sqrt{a}}v_{\rm ew}^2/v_\chi$. Hence, it makes sense to simply talk about m_{G_2}, m_{R_2} and $m_{\tilde{R}_2}$.

Although the description made above can be drawn from general principles, a detailed analysis of the scalar potential is necessary in order to find if there are any relations between the masses of various $G_{\rm SM}$ representations. The scalar potential has been cast in in Ref. [28] but 9 terms terms are missing there. We have found [25] and verified by dedicated automated tools [90] that the most general renormalizable form of the scalar potential reads

$$
\begin{aligned}
V = {}& \mu_H^2 H_i^\dagger H^i + \mu_\chi^2 \chi^\dagger \chi + \mu_\Phi^2 \mathrm{Tr}(\Phi_i^\dagger \Phi^i) + \lambda_1 H_i^\dagger H^i \chi^\dagger \chi + \lambda_2 H_i^\dagger H^i \mathrm{Tr}(\Phi_j^\dagger \Phi^j) \\
&+ \lambda_3\, \chi^\dagger \chi\, \mathrm{Tr}(\Phi_i^\dagger \Phi^i) + \left(\lambda_4\, H_i^\dagger \chi^\dagger \Phi^i \chi + \text{h.c.}\right) + \lambda_5 H_i^\dagger \mathrm{Tr}(\Phi_j^\dagger \Phi^i) H^j \\
&+ \lambda_6 \chi^\dagger \Phi^i \Phi_i^\dagger \chi + \lambda_7 (H_i^\dagger H^i)^2 + \lambda_8 (\chi^\dagger \chi)^2 + \lambda_9 \mathrm{Tr}(\Phi_i^\dagger \Phi^i \Phi_j^\dagger \Phi^j) + \lambda_{10} (\mathrm{Tr}(\Phi_i^\dagger \Phi^i))^2 \\
&+ \left(\lambda_{11} H_i^\dagger\, \mathrm{Tr}(\Phi^i\, \Phi^j) H_j^\dagger + \lambda_{12} H_i^\dagger\, \mathrm{Tr}(\Phi^i\, \Phi^j\, \Phi_j^\dagger) + \lambda_{13} H_i^\dagger\, \mathrm{Tr}(\Phi^i\, \Phi_j^\dagger\, \Phi^j) + \text{h.c.}\right) \\
&+ \lambda_{14} \chi^\dagger \Phi_i^\dagger \Phi^i \chi + \lambda_{15} \mathrm{Tr}(\Phi_i^\dagger\, \Phi^j\, \Phi_j^\dagger\, \Phi^i) + \lambda_{16} \mathrm{Tr}(\Phi_i^\dagger\, \Phi^j)\, \mathrm{Tr}(\Phi_j^\dagger\, \Phi^i) \\
&+ \lambda_{17} \mathrm{Tr}(\Phi_i^\dagger\, \Phi_j^\dagger)\, \mathrm{Tr}(\Phi^i\, \Phi^j) + \lambda_{18} \mathrm{Tr}(\Phi_i^\dagger\, \Phi_j^\dagger\, \Phi^i\, \Phi^j) + \lambda_{19} \mathrm{Tr}(\Phi_i^\dagger\, \Phi_j^\dagger\, \Phi^j\, \Phi^i), \qquad (2.42)
\end{aligned}
$$

where $SU(4)_C$ structure is fully captured in the matrix notation while the $SU(2)_L$ indices are denoted explicitly. Without loss of generality, λ_4 can be chosen real since its phase can be always absorbed by a redefinition of $H^\dagger\Phi$. After interchanging $\mu_{H,\chi,\Phi}$ for v_χ, v_{ew} and β via the minimization conditions and neglecting electroweak contributions ($v_{\text{ew}} \to 0$), one finds for the non-zero masses

$$m^2_{\chi_R^0} = 2\lambda_8 v_\chi^2\,, \qquad (2.43a)$$

$$m^2_{G_2} = \left(\frac{\sqrt{3}\lambda_4}{4}\tan\beta - \frac{3}{8}\left(\lambda_6 + \lambda_{14}\right)\right) v_\chi^2\,, \qquad (2.43b)$$

$$m^2_{R_2} = \left(\frac{\sqrt{3}\lambda_4}{4}\tan\beta + \frac{\lambda_{14} - 3\lambda_6}{8}\right) v_\chi^2\,, \qquad (2.43c)$$

$$m^2_{\tilde{R}_2} = \left(\frac{\sqrt{3}\lambda_4}{4}\tan\beta + \frac{\lambda_6 - 3\lambda_{14}}{8}\right) v_\chi^2\,, \qquad (2.43d)$$

$$m^2_{\hat{H}'} = \frac{\sqrt{3}\lambda_4}{2\sin(2\beta)} v_\chi^2\,. \qquad (2.43e)$$

Notice that despite the large number of quartic couplings in Eq. (2.42), only 3 of them are important regarding the masses of the fields stemming from Φ. Eliminating those 3 parameters from the four equations (2.43b) – (2.43e) yields the following mass sum rule:

$$m^2_{G_2} + 2m^2_{\hat{H}'}\sin^2\beta = \frac{3}{2}(m^2_{R_2} + m^2_{\tilde{R}_2})\,. \qquad (2.44)$$

Apart from perturbativity in λ's, this equation represents the only theoretical constraint on the scalar masses in the model.

2.7 Inverse seesaw and the FPW model

The MQLSM, introduced in Section 2.2, contains Dirac neutrinos, masses of which are generated the same way as masses of the other fermions. To give a natural interpretation of the smallness of the neutrino masses, a variant of a seesaw mechanism is usually employed. The $SU(4)_C$ models automatically contain leptonic G_{SM}-singlets ν_R which might suggest the type-I seesaw presented in Section 1.1.2. Type-I seesaw would require the Majorana mass term generated by the VEV of an extra scalar, transforming as a 10-dimensional representation of

$SU(4)_C$ and featuring the interaction $(f_R^{u\,a} Y_\nu f_R^u)\Delta_{ab}$. Adding Δ to the MQLSM might in principle lead to a consistent model of low-energy QLU; for example, no BNV terms can be written for mass dimension $d \leq 5$. Nevertheless, the suppression of the neutrino masses by a single negative power of $\langle \Delta_{44} \rangle$ is insufficient to provide a natural scale for the neutrino masses assuming the scale of $SU(4)_C$ breaking $\lesssim 1000$ TeV.

An alternative is provided by the *inverse seesaw* mechanism, originally developed in Ref. [91]. Implementation of the inverse seesaw on top of the MQLSM has been suggested in Ref. [28] by Fileviez Pérez and Wise, and thus we call it *the FPW model*. Note that a similar effective model combining the vector leptoquark U_1 with the inverse seesaw has been studied in Ref. [92].

The extension consists in adding 3 generations of gauge-singlet fermions N_L to the MQLSM (see Table 2.2). The part of the Lagrangian relevant for fermion masses

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{FPW}}^{\mathrm{Yuk+mass}} &= \overline{f_R^u}_a Y_1 H^i F_L{}^{aj} \varepsilon_{ji} + \overline{f_R^u}_a Y_2 \Phi_b^{ia} F_L{}^{bj} \varepsilon_{ji} + \overline{f_R^d}_a Y_3 H_i^\dagger F_L{}^{ai} + \overline{f_R^d}_a Y_4 \Phi_{ib}^{\dagger a} F_L{}^{bi} \\
&\quad + \overline{f_R^u}_a Y_5 \chi^a N_L + \frac{1}{2} N_L{}^{\mathrm{T}} \mathcal{C} \mu N_L + \text{h.c.}
\end{aligned} \tag{2.45}
$$

contains, apart from the four terms which were present in the MQLSM, the fifth Yukawa interaction which produces a Dirac mass term for $\overline{\nu_R} N_L$, and also a Majorana mass term with a symmetric matrix $\mu = \mu^{\mathrm{T}}$. The fermion number $\mathcal{F}$ is explicitly broken in this model and, due to Eq. (2.19), so is the lepton number $\mathcal{L}$. The tree-level neutrino mass terms, as they follow from (2.45), read

$$
-\mathcal{L}_{\mathrm{FPW}}^{\nu\text{-mass}} = \begin{pmatrix} \nu_L{}^{\mathrm{T}} & \nu_R^{c\mathrm{T}} & N_L{}^{\mathrm{T}} \end{pmatrix} \mathcal{C} \begin{pmatrix} 0 & M_\nu^D & 0 \\ M_\nu^{D\,\mathrm{T}} & 0 & M_\nu^\chi \\ 0 & M_\nu^{\chi\,\mathrm{T}} & \mu \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \\ N_L \end{pmatrix} \tag{2.46}
$$

where $M_\nu^D \sim v_{\mathrm{ew}}$ can be found in Eq. (2.11b) and

$$
M_\nu^\chi = \frac{1}{\sqrt{2}} Y_5 v_\chi \,. \tag{2.47}
$$

The full symmetric mass matrix above (to be denoted by M_ν^{Maj}) is diagonalized when sandwitched as $U^* M_\nu^{\mathrm{Maj}} U^\dagger$. Consistency with observations requires a *technically natural* [93] assumption of smallness of the entries in μ. In such a case, the ν_L's are close to Majorana mass eigenstates with tiny masses of order

$$
m_\nu \simeq \mu \left(\frac{M_\nu^D}{Y_5 v_\chi} \right)^2 \sim \mu \frac{v_{\mathrm{ew}}^2}{v_\chi^2} \,. \tag{2.48}
$$

The remaining fields form pseudo-Dirac pairs with masses close to eigenvalues of $Y_5 v_\chi / \sqrt{2}$. For more precise expressions see, e.g., Ref. [94].

Notice that the elements of M_ν^{Maj} which are exactly zero at the tree level gain corrections from loop diagrams and might eventually become the dominant contributions to the light neutrino masses.

2.8 Flavour structure and fermion interactions

So far, the fact that the SM fermions appear in 3 generations was not explicitly emphasized and flavour aspects of the considered models were essentially ignored.

On the other hand, almost all the discussions and formulae are fully compatible with the existence of 3 fermion generations provided that the symbols such as e_R, q_L, f_R^u etc. are columns of three copies of fermion fields with identical quantum numbers and that all Y's and μ's are $3{\times}3$ matrices.[5] From this point on, flavour will be a central subject of interest.

From the defining basis in which the gauge interactions are flavour-diagonal (as follows from the kinetic term $\overline{\psi} i \not{D} \psi$), the fermion fields are transformed into the mass basis $\hat{\psi}$ by the diagonalization matrices V^ψ:

$$\hat{\psi}_{L,R} = V^\psi_{L,R}\, \psi_{L,R} \qquad \text{for} \qquad \psi = u, d, e, \nu. \tag{2.49}$$

In the case of the FPW model, Eq. (2.49) for $\psi = \nu$ neglects the mixing between different sectors (ν_L, ν_R^c and N_L) and approximates the rotation to the mass basis by a block-diagonal structure. Such an approximation is sufficient for studying physics of hadrons and charged leptons. On the other hand, non-unitarity of V_L^ν can be probed by neutrino oscillation experiments and mixing among ν_L and N_L is important for leptogenesis.

Formally, the rotation matrices follow from the singular value decomposition of the mass matrices in the defining basis,

$$M_\psi = V_R^{\psi\,\dagger} \hat{M}_\psi V_L^\psi \qquad \text{for} \qquad \psi = u, d, e, \tag{2.50}$$

where

$$\hat{M}_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad \hat{M}_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad \hat{M}_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{2.51}$$

Analogously, we define $\hat{M}_\nu^D = V_R^\nu M_\nu^D V_L^{\nu\,\dagger}$. In the MQLSM, it can be approximated by $\hat{M}_\nu^D \approx 0_{3\times 3}$. On the other hand, $\hat{M}_\nu^D$ in the FPW model is not directly related to the physical neutrino masses, which are given by the seesaw formula instead.

In accordance with a usual convention, we introduce the basis for the weak isodoublets in which the $T_L^3 = -1/2$ components are in the mass basis,

$$\hat{q}_L = V_L^d q_L = \begin{bmatrix} V_{\text{CKM}}^\dagger \hat{u}_L \\ \hat{d}_L \end{bmatrix}, \qquad \hat{\ell}_L = V_L^e \ell_L = \begin{bmatrix} V_{\text{PMNS}}\, \hat{\nu}_L \\ \hat{e}_L \end{bmatrix} = \begin{bmatrix} \mathring{\nu}_L \\ \hat{e}_L \end{bmatrix}. \tag{2.52}$$

while

$$\mathring{\nu}_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}. \tag{2.53}$$

Experimentally, so far only the combinations

$$V_{\text{CKM}} = V_L^u\, V_L^{d\,\dagger} \qquad \text{and} \qquad V_{\text{PMNS}}^\dagger = V_L^\nu\, V_L^{e\,\dagger} \tag{2.54}$$

are directly observable. As we will see in the next section, the quark-lepton unification hypothesis predicts that also the following independent unitary matrices,

$$U_L = V_L^d\, V_L^{e\,\dagger}, \qquad U_R = V_R^d\, V_R^{e\,\dagger}, \qquad U_R' = V_R^u\, V_R^{\nu\,\dagger} \tag{2.55}$$

are of physical relevance.

[5] Only the definition of effective operators in Section 1.1.3 requires some explicit generalization.

$$
\begin{tikzcd}
u_L \arrow[r, "U'_L"] \arrow[d, "V_{\mathrm{CKM}}"'] & \nu_L \\
d_L \arrow[r, "U_L"'] & e_L \arrow[u, "V_{\mathrm{PMNS}}"']
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
u_R \arrow[r, "U'_R"] & \nu_R \\
d_R \arrow[r, "U_R"'] & e_R
\end{tikzcd}
$$

Figure 2.1: Scheme of fermion mixing in the quark-lepton symmetry models.

2.8.1 Gauge interactions

Analogously to the charged-current weak interactions

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \left(\overline{\hat{u}_{L\alpha}} \gamma^\mu V_{\mathrm{CKM}} \hat{d}_L{}^\alpha + \overline{\hat{\nu}_L} \gamma^\mu V^\dagger_{\mathrm{PMNS}} \hat{e}_L \right) W^+_\mu + \text{h.c.}, \tag{2.56}$$

the interaction among fermions and the vector leptoquark is driven by the matrices defined in Eq. (2.55):

$$\mathcal{L}_{U_1} = \frac{g_4}{\sqrt{2}} \left(\overline{\hat{q}_{L\alpha i}} \gamma^\mu U_L \hat{\ell}_L{}^i + \overline{\hat{d}_{R\alpha}} \gamma^\mu U_R \hat{e}_R + \overline{\hat{u}_{R\alpha}} \gamma^\mu U'_R \hat{\nu}_R \right) U_{1\mu}^{\alpha} + \text{h.c.} \tag{2.57}$$

This can be rewritten as

$$\mathcal{L}_{U_1} = \frac{g_4}{\sqrt{2}} \left(\overline{\hat{d}} \gamma^\mu \left[\mathbb{P}_L U_L + \mathbb{P}_R U_R \right] \hat{e} + \overline{\hat{u}} \gamma^\mu \left[\mathbb{P}_L U'_L + \mathbb{P}_R U'_R \right] \hat{\nu} \right) U_{1\mu} + \text{h.c.} \tag{2.58}$$

where $\mathbb{P}_{L,R} = (1 \mp \gamma_5)/2$ are the chirality projectors and

$$U'_L = V_{\mathrm{CKM}}\, U_L\, V_{\mathrm{PMNS}} \tag{2.59}$$

is *not* another independent matrix. Notice that the gauge LQ interaction might conserve parity if $U_R = U_L$ and $U'_R = U'_L$ but there is no simple motivation for such an assumption in this framework.

The Z' interactions with the fermions in the MQLS and FPW models are diagonal and flavour-universal, with the coupling given by Eq. (2.30). On the other hand, as shall be discussed later in Section 5.2, this is not necessarily true in more complicated scenarios.

2.8.2 Yukawa interactions

Generally, the pattern of Yukawa interactions of the scalar fields from multiplets participating in the symmetry breaking is often in some way related to the fermion masses. The notorious example is the flavour-diagonal interaction of the physical Higgs field in the SM:

$$\mathcal{L}^{H^0-\text{int}}_{\text{SM}} = \sum_{\psi=u,d,e} -\frac{1}{v_{\text{ew}}}\, \overline{\hat{\psi}}\, \hat{M}_\psi\, \hat{\psi}\, H^0\,. \tag{2.60}$$

In the quark-lepton symmetry models, similar relations are obtained by inverting the set of equations in (2.11):

$$Y_1 = \sqrt{\frac{1}{8}}\, \frac{3M_u + M^D_\nu}{v_{\text{ew}} \sin\beta}\,, \qquad Y_2 = \sqrt{\frac{3}{2}}\, \frac{M_u - M^D_\nu}{v_{\text{ew}} \cos\beta}\,, \tag{2.61a}$$

$$Y_3 = \sqrt{\frac{1}{8}}\, \frac{3M_d + M_e}{v_{\text{ew}} \sin\beta}\,, \qquad Y_4 = \sqrt{\frac{3}{2}}\, \frac{M_d - M_e}{v_{\text{ew}} \cos\beta}\,. \tag{2.61b}$$

Plugging this into the first line of Eq. (2.45), rewriting the $SU(4)_C$ structure in the broken phase and switching to the mass basis according to Eqs. (2.49) and (2.50) yields

$$
\begin{aligned}
-\mathcal{L}^{\text{Yuk-int}}_{\text{MQLSM}} = \frac{\sqrt{2}}{v_{\text{ew}}}\hat{H}^i \Big(&\overline{\hat{u}_R}\hat{M}_u V_{\text{CKM}}\hat{q}_L{}^j\varepsilon_{ji} + \overline{\hat{q}_{L}}_i\hat{M}_d\hat{d}_R + \overline{\hat{\ell}_{L}}_i\hat{M}_e\hat{e}_R + \overline{\hat{\nu}_R}\hat{M}^D_\nu V_{\text{PMNS}}\hat{\ell}_L{}^j\varepsilon_{ji}\Big) \\
+ \frac{\hat{H}'^i}{\sqrt{2}v_{\text{ew}}\sin 2\beta}\Big(&\overline{\hat{u}_R}\left[(2\cos 2\beta + 1)\hat{M}_u + U'_R\hat{M}^D_\nu U'_L{}^\dagger\right]V_{\text{CKM}}\hat{q}_L{}^j\varepsilon_{ji} \\
&+ \overline{\hat{\nu}_R}\left[(2\cos 2\beta - 1)\hat{M}^D_\nu + 3U'_R{}^\dagger\hat{M}_u U'_L\right]V^\dagger_{\text{PMNS}}\hat{\ell}_L{}^j\varepsilon_{ji} \\
&+ \overline{\hat{q}_{L}}_i\left[(2\cos 2\beta + 1)\hat{M}_d + U_L\hat{M}_e U_R{}^\dagger\right]\hat{d}_R \\
&+ \overline{\hat{\ell}_{L}}_i\left[(2\cos 2\beta - 1)\hat{M}_e + 3U_L{}^\dagger\hat{M}_d U_R\right]\hat{e}_R\Big) \\
+ \frac{\sqrt{3/2}\,G_2{}^{\alpha i}_{\beta}}{v_{\text{ew}}\cos\beta}&\left(\overline{\hat{u}_{R}}_\alpha\left(\hat{M}_u - U'_R\hat{M}^D_\nu U'_L{}^\dagger\right)V_{\text{CKM}}\hat{q}_L{}^{\beta j}\varepsilon_{ji} + \overline{\hat{q}_{L}}_{\alpha i}(\hat{M}_d - U_L\hat{M}_e U_R{}^\dagger)\hat{d}_R{}^\beta\right) \\
+ \frac{\sqrt{3/2}\,R_2{}^{\alpha i}}{v_{\text{ew}}\cos\beta}&\left(\overline{\hat{u}_{R}}_\alpha\left(\hat{M}_u U'_L - U'_R\hat{M}^D_\nu\right)V^\dagger_{\text{PMNS}}\hat{\ell}_L{}^j\varepsilon_{ji} + \overline{\hat{q}_{L}}_i\left(U_L\hat{M}_e - \hat{M}_d U_R\right)\hat{e}_R\right) \\
+ \frac{\sqrt{3/2}\,\tilde{R}_2{}^{\alpha i}}{v_{\text{ew}}\cos\beta}&\left(\overline{\hat{d}_{R}}_\alpha\left(\hat{M}_d U_L - U_R\hat{M}_e\right)\varepsilon_{ij}\hat{\ell}_L{}^j + \overline{q_{L}}_{\alpha i}V^\dagger_{\text{CKM}}\left(\hat{M}_u U'_R - U'_L\hat{M}^D_\nu\right)\hat{\nu}_R\right) \\
+ \text{h.c.}&
\end{aligned}
\tag{2.62}
$$

On the first line, one can find the well known interaction of the light Higgs boson from which Eq. (2.60) can be easily obtained. The cast form of interactions of G_2, R_2 and $\tilde{R}_2$ is consistent with Ref. [66].

The FPW model contains an additional Yukawa matrix [see Eq. (2.45)] which, apart from interactions of the unphysical Goldstones u_1 and z'_0, induces also the interaction $\chi^0_R\,\overline{\nu_R}\,Y_5 N_L$. However, as N_L mostly forms the heavy neutrino mass eigenstates, this interaction is essentially irrelevant.

2.9 Parameter counting

We will close Chapter 2 by counting the independent parameters of both models considered and suggesting a suitable choice of the inputs. For convenience, we will indicate the number of parameters by a Gaussian integer with its real and imaginary part denoting the number of real parameters and complex phases, respectively. The total dimension of the parameter space can then be obtained by means of the Manhattan measure, $||a + bi|| = a + b$.

2.9.1 Bosonic sector

In the gauge sector there are 3 couplings g_4, g and g_R which must be matched onto the SM gauge couplings g_3, g and g' at the appropriate scales, using the formulas from Table 2.1.

In the scalar potential one finds 3 dimensionful parameters $\mu_{\chi,H,\Phi}$. These can be traded for $v_{\text{ew}} = 246$ GeV, v_χ, which determines the masses of the vector LQ and of the Z' boson (as well as a *natural scale* for most of the scalar spectrum), and β which in the end dictates the overall strength of Yukawa interactions of the BSM scalars.

Furthermore, there are $19 + 4i$ degrees of freedom in the scalar quartic couplings. Only one of the four complex phases can be made real by scalar field phase redefinition. Thus, generally, the model explicitly violates CP. We suggest to rephase the three scalars such that their VEVs are real, as we have tacitly assumed so far. The couplings $\lambda_4, \lambda_6, \lambda_8, \lambda_{14}$ can be interchanged with masses of four BSM scalar multiplets among those of Equations (2.43), while the remaining mass follows from Eq. (2.44). Finally, the other λ's need to be chosen so as to reproduce the Higgs boson mass $m_{H^0} = 125$ GeV.

2.9.2 Fermionic sector in the MQLSM

Naively, there are four complex Yukawa matrices in the MQLSM Lagrangian (2.12), yielding $4\,(3\times 3)\,(1+i)$ parameters. However, some of these are physically irrelevant since there is a freedom of $3\,(3+6i)$ parameters corresponding to the unitary flavour transformations of the $SU(4)_C$ multiplets F_L, f_R^u, f_R^d, which do not change the flavour diagonality of the gauge interactions and define equivalence classes among the Yukawa matrices leading to the same predictions. (For example, it is sufficient to work in the gauge interaction basis where, simultaneously, the left- and right-handed charged leptons and right-handed up-type quarks are in the mass basis.) Two of the phase transformations correspond to the action of the $U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}}$ symmetry and, thus, should not count. In total, one arrives at $(36+36i) - [(9+18i) - 2i] = 27 + 20i$ parameters.

These should reproduce the $12 + i$ parameters of the SM (charged fermion masses and the CKM matrix) and the $6 + i$ parameters corresponding to the neutrino masses and the PMNS matrix. The remaining and $3\,(3+6i)$ extra parameters reside in the mixing matrices U_L, U_R and U_R'. Note that unlike for CKM and PMNS, no phases can be removed. This is perfectly understandable: the freedom of rephasing the left-handed fermion fields has already been used to simplify V_{CKM} and V_{PMNS} and their right-handed counterparts were rephased identically in order to keep the mass terms real.

On the other hand, the PMNS matrix is irrelevant for the physical processes where the neutrino mass differences are negligible. It might be thus more convenient to formally move three phases from U_L to V_{PMNS}.

2.9.3 Fermionic sector in the FPW model

Extending the MQLSM to the FPW model brings $9(1+i)+6(1+i)$ new parameters from Y_5 and μ, $3+6i$ new redundancies from possible flavour redefinitions of N_L and breaks the $U(1)_{\mathcal{L}}$ symmetry. Thus, there is $12+8i$ new parameters compared to the MQLSM.

In the mass basis, they appear as two extra Majorana phases in the PMNS matrix, six masses of the three heavy quasi-Dirac fermions, and the rest resides in the small mixing across $\nu_L, {\nu_R}^c$ and N_L. For the best way to parametrize extended neutrino sectors cf. Ref. [95].

3. Flavour symmetries, B-meson anomalies and leptoquarks

Chapter 1 discussed $U(1)_{\mathcal{B}}$ and $U(1)_{\mathcal{L}}$ – the (classically) *exact* accidental symmetries of the SM which do not distinguish between the fermion generations. In fact, we did not even have to specify the number of families (or generations) in Chapter 1. This chapter is devoted to *approximate* symmetries of Nature, in particular, to the lepton flavour (LF) symmetries. Throughout this work, we take flavour symmetries as accidental, as they are in the SM.

This chapter provides the fundamentals necessary for the phenomenological studies presented in Chapters 4 and 5; readers well aware of the matters can safely skip the entire Chapter 3. After introducing the concept of lepton flavour symmetries in Section 3.1, we overview the basic tool of flavour physics – the Weak effective theory (WET) – in the subsequent part. In Section 3.3, we revise in detail the formulae for pseudoscalar mesons decays $P^0 \to ll'$ in WET which will be crucial in Chapter 5. Section 3.4 reviews the current status of the hints of LFU violation in the B-physics sector and their interpretation in terms of effective theories; these anomalies will be studied in Chapter 4. Finally, in Section 3.5 we present in brief some general aspects of flavour physics of leptoquarks – the fields that will play a prominent role in both Chapters 4 and 5.

3.1 Flavour symmetries

Since the concept of (lepton) flavour group is being used in several different ways, let us fix the conventions first. By the *SM flavour group* we mean

$$\mathcal{G}_{\text{flavour}} = U(3)^3_{\text{QF}} \times U(3)^2_{\text{LF}} \tag{3.1}$$

where the *quark* and *lepton flavour groups in the wider sense* read

$$U(3)^3_{\text{QF}} = U(3)_q \times U(3)_d \times U(3)_u \,, \qquad U(3)^2_{\text{LF}} = U(3)_\ell \times U(3)_e \,. \tag{3.2}$$

We will only discuss the leptonic part which acts on the three generations of leptons via

$$\ell_L \to \mathcal{U}_\ell\, \ell_L, \quad e_R \to \mathcal{U}_e\, e_R \qquad \text{with} \qquad \mathcal{U}_\ell \in U(3)_\ell, \quad \mathcal{U}_e \in U(3)_e \,. \tag{3.3}$$

$\mathcal{G}_{\text{flavour}}$ would be an exact symmetry of the SM if there were no Yukawa interactions. Especially, $U(3)^2_{\text{LF}}$ is not respected by any nonzero entry in Y_e. On the other hand, the SM predicts that $U(3)^2_{\text{LF}}$ should be a good approximate symmetry in high-energy processes where the lepton interactions with the Higgs field, and hence also their masses, can be neglected.

For most practical purposes it is sufficient to consider only the *diagonal* subgroup $U(3)_{\ell+e} \subset U(3)^2_{\text{LF}}$ obtained by $\mathcal{U}_\ell = \mathcal{U}_e$ which, at the level of Lie algebras, can be factorized as

$$U(3)_{\ell+e} \;=\; U(1)_{\mathcal{L}} \times SU(3)_{\text{LF}}, \tag{3.4}$$

where $U(1)_{\mathcal{L}}$ is the exact symmetry of the SM discussed in Chapter 1. In contrast, $SU(3)_{\mathrm{LF}}$ – another candidate to be called *lepton flavour group* – would be a valid symmetry of the SM only if $Y_e = y\,\mathbb{1}$, i.e., if all 3 charged leptons had the same mass. In our Universe, the leptonic spectrum is hierarchical ($m_e \ll m_\mu \ll m_\tau$) which implies that $SU(3)_{\mathrm{LF}}$ is an approximate symmetry of the SM essentially only in the situations when the whole $U(3)^2_{\mathrm{LF}}$ is so.

Nevertheless, there is a subgroup of $SU(3)_{\mathrm{LF}}$ which *is* a precise symmetry of the SM: the *lepton flavour group in the strict sense*,

$$U(1)^2_{\mathrm{LF}} = U(1)_{\mathcal{L}_\mu - \mathcal{L}_e} \times U(1)_{\mathcal{L}_\tau - \mathcal{L}_e}, \tag{3.5}$$

in the charged-lepton mass basis generated by diagonal traceless matrices. In combination with the $\mathcal{L}$ conservation, the $U(1)^2_{\mathrm{LF}}$ symmetry implies conservation of the individual lepton family numbers satisfying $\mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\tau = \mathcal{L}$. Notice that despite various conventions for what is called the lepton flavour group, the term *lepton flavour violation* (LFV) is being used strictly in relation with $U(1)^2_{\mathrm{LF}}$.

Experimentally, LFV has been proven by the only discovery of particle physics beyond the SM so far – the neutrino oscillations [16]. On the other hand, no *charged* lepton flavour violation has been observed yet (e.g., [96, 97, 98, 99]), in agreement with the expectations from the SM extended with Dirac neutrino masses [100].

Inspecting other parts of the anticipated approximate LF symmetry consists especially in testing the *lepton flavour universality* (LFU) which we associate with the permutation group $(S_3)_{\mathrm{LFU}} \subset U(3)_{\ell+e}$. The LFU symmetry implies that replacements $e \leftrightarrow \mu$ and $e \leftrightarrow \tau$ should not change the amplitude for a considered process. LFU can be tested, e.g., in the leptonic W and Z boson decays or the (chirality-unsuppressed) semileptonic decays of heavy mesons. The sources of lepton flavour universality violation (LFUV) in the SM are strictly related to the lepton mass differences and are theoretically quite well understood.

Generally, since none of $U(1)^2_{\mathrm{LF}}$ or $(S_3)_{\mathrm{LFU}}$ is a subgroup of the other, *LFV does not necessarily imply LFUV nor vice versa.* On the other hand, various BSM fields (such as leptoquarks) would often induce violation of both.

Finally, we note that New Physics might violate the $U(1)^2_{\mathrm{LF}}$ or $(S_3)_{\mathrm{LFU}}$ symmetries only partially to $U(1) \times \mathbb{Z}_n$ [101] or $S_2 = \mathbb{Z}_2$, respectively.

3.2 Flavoured effective theories

In this section we return to the concept of effective field theory (EFT) and fix some additional conventions regarding the flavour structures.

Flavoured SMEFT

We have already introduced SMEFT – a way to universally describe New Physics (NP) effects in the majority of BSM models – in Section 1.1.3. At this moment, it should be perhaps emphasized that we have committed an oversimplification there as we have neglected the existence of multiple fermion generations. To this end, each line in Eqs. (1.9) and (1.10) actually represents several distinct and independent operators. We adhere to the so-called Warsaw basis within the `wcxf` standards [40] in which the fermion generations are labeled by numbers and the weak doublets are in the mass basis of their $T_L^3 = -1/2$ component, i.e., of down-type quarks and charged leptons. For example, one particular fully specified baryon number violating operator corresponding to (1.10a) reads

$$\mathcal{O}_{duq\ell\_1231} = \varepsilon_{\alpha\beta\gamma}\left[(d_R^\alpha)^{\mathrm{T}}\mathcal{C}\, c_R^\beta\right]\left[(q_{L(b)}{}^{\gamma i})^{\mathrm{T}}\mathcal{C}\ell_{L(e)}{}^{j}\right]\varepsilon_{ij}\,. \tag{3.6}$$

Needless to say, adding family indices greatly increases the dimension of the Wilson coefficient space: there is 84 linearly independent dimension-6 effective operators built from single generation SM fields, but 3045 such operators in the flavoured SMEFT [41].

Weak effective theory

Currently, the LHC experiments have probed the electroweak scale quite thoroughly, finding an impressive agreement with the Standard Model. This suggests that a possible New Physics, shall there be any, should be well described by SMEFT indeed. Limits on various SMEFT coefficients can be drawn, for example, from the electroweak precision measurements. On the other hand, SMEFT is not the most appropriate tool when it comes to phenomenology in low-energy precision physics (such as hadron decays). The reason is two-fold: Firstly, the $SU(2)_L$ invariance is lost at lower energies and it is thus more convenient to have all the fermions in the mass basis. Secondly, there are important renormalization effects which need to be taken into account; the running of the Wilson coefficients via renormalization group equations (RGE) proceeds differently above and below the electroweak scale.

Thus, instead of SMEFT, flavour physics at hadronic scales $\mu \sim$ GeV is best described using the *Weak effective theory* (WET) [102]: an $SU(3)_C \times U(1)_Q$ gauge invariant low energy model in which the higher-dimensional interaction terms encode the effects of both NP and SM fields with masses $m \gg \mu$. In particular, for the B physics, the 5-quark WET is employed. It is defined by the Langrangian

$$\mathcal{L}_{\mathrm{WET}(5)}(\nu_{Ll}, e, d, u, s, \mu, c, \tau, b, A_\mu, G_\mu) = \mathcal{L}_{QCD+QED} + \mathcal{L}_{\mathrm{eff}}\,, \tag{3.7}$$

the non-renormalizable part of which reads

$$\mathcal{L}_{\mathrm{eff}} = -\mathcal{H}_{\mathrm{eff}} = \sum_{A}^{\mathcal{O}_A=\mathcal{O}_A^\dagger} C_A\mathcal{O}_A + \sum_{A}^{\mathcal{O}_A\neq\mathcal{O}_A^\dagger}\left(C_A\mathcal{O}_A + C_A^*\mathcal{O}_A^\dagger\right). \tag{3.8}$$

The Wilson coefficients can be further written as $C_A = C_A^{\mathrm{SM}} + C_A^{\mathrm{NP}}$ with $C_A^{\mathrm{SM}}\mathcal{O}_A$ covering the interactions of W, Z, H^0 and t.

Generally, a phenomenological inspection of a BSM model may proceed as follows:

1. If the scenario is not given in terms of SMEFT but of new explicit dynamical degrees of freedom (such as new fields in the case of QFT), these need to be matched onto effective operators in SMEFT.

2. The SMEFT Wilson coefficients are ran down to the EW scale $\mu_{\mathrm{ew}} \sim m_Z$.

3. Effective SMEFT operators as well as heavy SM field dynamics is matched onto WET at the μ_{ew} scale. [103]

4. The Wilson coefficients (WCs) the of WET are ran down by RGE to the scale of the considered process.

5. Predictions are made using a general expression for a given process in the WET.

The advantage of such an approach is that the steps 2–5 are independent on the particular NP scenario, and thence need to be studied only once and can be automatized. To this end, we particularly mention the `wilson` package [104], which can accomplish the steps 2–4.

The flavoured WET(5) basis consists of more than a thousand independent operators of $d = 5$ or 6. Nevertheless, usually only a small subset significantly contributes to a particular process.

Semileptonic FCNC operators

NP is being probed in studying the *rare decays* of mesons – processes mediated by flavor changing neutral currents (FCNC) which are suppressed in the SM by loop and CKM factors as well as by the GIM mechanism. We will put a lot of focus on the FCNC meson decays with leptons in the final state. These are mainly driven by the two-quark two-lepton operators listed below.

$$\mathcal{O}_{9\text{-}qq'll'} = \mathcal{N}\left(\overline{q'_L}\gamma_\mu q_L\right)\left(\overline{l'}\gamma^\mu l\right) \qquad \mathcal{O}'_{9\text{-}qq'll'} = \mathcal{N}\left(\overline{q'_R}\gamma_\mu q_R\right)\left(\overline{l'}\gamma^\mu l\right) \tag{3.9a}$$

$$\mathcal{O}_{10\text{-}qq'll'} = \mathcal{N}\left(\overline{q'_L}\gamma_\mu q_L\right)\left(\overline{l'}\gamma^\mu \gamma_5 l\right) \qquad \mathcal{O}'_{10\text{-}qq'll'} = \mathcal{N}\left(\overline{q'_R}\gamma_\mu q_R\right)\left(\overline{l'}\gamma^\mu \gamma_5 l\right) \tag{3.9b}$$

$$\mathcal{O}_{S\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'_L} q_R\right)\left(\overline{l'}\, l\right) \qquad \mathcal{O}'_{S\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'_R} q_L\right)\left(\overline{l'}\, l\right) \tag{3.9c}$$

$$\mathcal{O}_{P\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'_L} q_R\right)\left(\overline{l'}\gamma_5 l\right) \qquad \mathcal{O}'_{P\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'_R} q_L\right)\left(\overline{l'}\gamma_5 l\right) \tag{3.9d}$$

$$\mathcal{O}_{T\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'}\sigma_{\mu\nu} q\right)\left(\overline{l'}\sigma^{\mu\nu} l\right) \qquad \mathcal{O}_{T5\text{-}qq'll'} = \mathcal{N}\zeta\left(\overline{q'}\sigma_{\mu\nu} q\right)\left(\overline{l'}\sigma^{\mu\nu}\gamma_5 l\right) \tag{3.9e}$$

Within this context, the quark-flavour changing electromagnetic dipole operators are usually also relevant:

$$\mathcal{O}_{7\text{-}qq'} = \mathcal{N}\zeta_7\left(\overline{q'_R}\sigma_{\mu\nu} q_R\right)F^{\mu\nu}, \qquad \mathcal{O}'_{7\text{-}qq'} = \mathcal{N}\zeta_7\left(\overline{q'_L}\sigma_{\mu\nu} q_L\right)F^{\mu\nu}. \tag{3.10}$$

The normalization factors $\mathcal{N}$, $\mathcal{N}\zeta$ and $\mathcal{N}\zeta_7$ differ among various conventions (and, within one convention, among quark flavours). The weak interactions only contribute to vector-type operators $C_{9,10}^{\text{SM}}$ with $l = l'$ and to the EM-dipole.

Dimension-6 SMEFT operators may match onto $C_{9,10,S,P}^{(\prime)\,\text{NP}}$ with arbitrary flavour structures, with only the following constraints:

$$C_S = -C_P \qquad \text{and} \qquad C'_S = +C'_P. \tag{3.11}$$

The semileptonic operators $\mathcal{O}_T$ and $\mathcal{O}_{T5}$ cannot arise from a G_{SM}-invariant dimension-6 operator [103] nor do they stem from the heavy sector of the SM; for this reason, they are often being ignored from the beginning. Nevertheless, they may arise from $d = 8$ SMEFT operators [105].

Lepton flavour symmetries are violated in both parts of the WET Lagrangian (3.7): $\mathcal{L}_{\text{QED+QCD}}$ breaks LFU by the lepton mass terms, while $\mathcal{L}_{\text{eff}}$ may generally break the whole lepton flavour group by the coefficients C_A^{NP}. Considering the four-fermion operators (3.9) in particular, any non-zero $C^{\text{NP}}_{X\_qq'll'}$ with $l \neq l'$ breaks $U(1)^2_{\text{LF}}$, while $(S_3)_{\text{LFU}}$ symmetry requires $C^{\text{NP}}_{X\_qq'll'} = C^{\text{NP}}_{X\_qq'\pi_l\pi_{l'}}$ for all permutations π of $\{e, \mu, \tau\}$. The weak interactions, encoded in C^{SM}'s, are symmetric with respect to the entire $SU(3)_{\text{LF}}$ group.

3.3 Leptonic meson decays

Purely leptonic decays of pseudoscalar mesons $P^0 \to l_1^+ l_2^-$ constitute the first important class of rare decays discussed above. Due to same-handedness of the lepton pairs in effective weak four-fermion interactions (i.e., $C^{(\prime)\,\text{SM}}_{P,S} = 0$) within the SM, angular momentum conservation implies chirality suppression. To this end, the LF symmetry in C^{SM}'s together with the well understood low-energy LFUV from Δm_l naively predicts relations like

$$\frac{\text{BR}(P^0 \to e^+e^-)}{\text{BR}(P^0 \to \mu^+\mu^-)} = \frac{m_e^2(m_P - m_e)^2}{m_\mu^2(m_P - m_\mu)^2}. \tag{3.12}$$

Unfortunately, the extreme suppression of the ee channel ($B^0_{d,s} \to ee$ are unobserved, with the limits many orders of magnitude below the SM prediction [106]) together with theoretical uncertainties in the $K_L^0 \to ll$ channel from long-distance corrections [107] make such an easy test inapplicable. Nevertheless, measurements or searches for these decays provide strong limits on C^{NP}'s anyway, once theory and experiment are compared individually for the individual branching ratios.

In this section, we revise the formulae for branching fractions for the decays $P^0 \to l_1^+ l_2^-$ with $P^0 = K_L^0, K_S^0, B^0, B_s$ in the WET built around the two-quark-two-lepton operators (3.9), including the lepton flavour violating cases. These formulas are not difficult to derive and can be found in the correct form, e.g., in Ref. [108], using a different operator basis than that in Eqs. (3.9). On the other hand, several different variants of the expressions can be found over the literature which are not applicable to all the considered decays since they:

- are flawed by subtle mistakes [109, 110, 111][1] and thus only allow for order-of-magnitude estimates but are insufficient for a careful analysis,
- only hold for the weak eigenstate mesons (not for $K^0_{L,S}$) [52, 111], but sometimes are applied on also on the decays of K_L^0 [113, 114, 115, 116],
- do not handle the LFV case $l_1 \neq l_2$ [52, 109, 110, 112].

Formulae for $\text{BR}(P^0 \to l_1^+ l_2^-)$ will be essential in Chapter 5 and so we re-derive them here in detail. We have found and fixed a mistake in the public code `flavio` [117, 118] (ver. `2.2.0` [119]) concerning the predictions of $\text{BR}(K^0_{L,S} \to e^\pm \mu^\mp)$; the update should be included in some of the upcoming official versions.

[1] Problems in [109, 110] have been explained in Ref. [112]. Issues with the expressions in [111] will be described here on page 47.

3.3.1 Amplitudes for weak eigenstates

Let us start the derivation by parametrizing the hadronic matrix elements for the pseudoscalar meson $P = \overline{q'}q$ as follows:

$$\langle 0|\, \overline{q'}\gamma^\mu q\, |P(k)\rangle = 0, \qquad \langle 0|\, \overline{q'}\gamma^\mu\gamma^5 q\, |P(k)\rangle = i f_P k^\mu, \tag{3.13a}$$
$$\langle 0|\, \overline{q'}q\, |P(k)\rangle = 0, \qquad \langle 0|\, \overline{q'}\gamma^5 q\, |P(k)\rangle = -i f_P \overline{m}_P, \tag{3.13b}$$

where f_P is the relevant meson decay constant and

$$\overline{m}_P = \frac{m_P^2}{m_q + m_{q'}}\,. \tag{3.14}$$

The vanishing matrix elements of the scalar and vector quark currents imply that only the WC combinations [see (3.9)]

$$C_{\Delta X\_qq'll'} = C_{X\_qq'll'} - C'_{X\_qq'll'} \tag{3.15}$$

matter. As $\overline{\psi}_1(\gamma_\mu)\gamma_5\psi_2 \xrightarrow{C} +\overline{\psi_2}(\gamma_\mu)\gamma_5\psi_1$ and pseudoscalar mesons are C-even, there are no extra phases or signs for the antiparticles in the relations (3.13).

Provided $q \neq q'$, the operators in Eqns. (3.9) are all non-hermitean. Since

$$\left(\overline{\psi}\gamma_\mu\chi\right)^\dagger = +\,\overline{\chi}\gamma_\mu\psi, \qquad \left(\overline{\psi}\gamma_\mu\gamma_5\chi\right)^\dagger = +\,\overline{\chi}\gamma_\mu\gamma_5\psi, \tag{3.16a}$$
$$\left(\overline{\psi}\chi\right)^\dagger = +\,\overline{\chi}\psi, \qquad \left(\overline{\psi}\gamma_5\chi\right)^\dagger = -\,\overline{\chi}\gamma_5\psi, \tag{3.16b}$$

the operator $\mathcal{O}_{\Delta S\_qq'll'} \equiv \mathcal{O}_{S\_qq'll'} - \mathcal{O}'_{S\_qq'll'} = \mathcal{N}\zeta(\overline{q}'\gamma_5 q)(\overline{l}'l)$ yields an extra minus sign under hermitean conjugation while the other relevant operators, $\mathcal{O}_{\Delta 9,\Delta 10,\Delta P}$ in obvious notation, do not. Consequently, the hermiticity of the effective Hamiltonian implies the following relations among the corresponding Wilson coefficients:

$$\mathcal{N}^* {C_{\Delta 9\_qq'll'}}^* = +\mathcal{N} C_{\Delta 9\_q'ql'l}\,, \qquad \mathcal{N}^* {C_{\Delta 10\_qq'll'}}^* = +\mathcal{N} C_{\Delta 10\_q'ql'l}\,, \tag{3.17a}$$
$$\mathcal{N}^* {C_{\Delta S\_qq'll'}}^* = -\mathcal{N} C_{\Delta S\_q'ql'l}\,, \qquad \mathcal{N}^* {C_{\Delta P\_qq'll'}}^* = +\mathcal{N} C_{\Delta P\_q'ql'l}\,. \tag{3.17b}$$

In the `flavio` basis [117, 120], the Wilson coefficients of the $\_sdll'$, $\_bsll'$ and $\_bdll'$ types need to be defined while those with the $\_dsll'$, $\_sbll'$ and $\_dbll'$ flavour structures can be obtained from Eqs. (3.17). For example, the part of effective Hamiltonian relevant for leptonic decays of B_s and $\bar{B}_s$ reads

$$\begin{aligned} -\mathcal{H}_{\text{eff}}^{bsll'} = \sum_{l,l'} \Big[&-\frac{\mathcal{N}}{2} C_{\Delta 9\_bsll'} \left(\overline{s}\gamma_\mu\gamma_5 b\right)\left(\overline{l'}\gamma^\mu l\right) - \frac{\mathcal{N}^*}{2} C^*_{\Delta 9\_bsll'} \left(\overline{b}\gamma_\mu\gamma_5 s\right)\left(\overline{l}\gamma^\mu l'\right) \\ &- \frac{\mathcal{N}}{2} C_{\Delta 10\_bsll'} \left(\overline{s}\gamma_\mu\gamma_5 b\right)\left(\overline{l'}\gamma^\mu\gamma_5 l\right) - \frac{\mathcal{N}^*}{2} C^*_{\Delta 10\_bsll'} \left(\overline{b}\gamma_\mu\gamma_5 s\right)\left(\overline{l}\gamma^\mu\gamma_5 l'\right) \\ &+ \frac{\zeta\mathcal{N}}{2} C_{\Delta S\_bsll'} \left(\overline{s}\gamma_5 b\right)\left(\overline{l'}l\right) - \frac{\zeta\mathcal{N}^*}{2} C^*_{\Delta S\_bsll'} \left(\overline{b}\gamma_5 s\right)\left(\overline{l}l'\right) \\ &+ \frac{\zeta\mathcal{N}}{2} C_{\Delta P\_bsll'} \left(\overline{s}\gamma_5 b\right)\left(\overline{l'}\gamma_5 l\right) + \frac{\zeta\mathcal{N}^*}{2} C^*_{\Delta P\_bsll'} \left(\overline{s}\gamma_5 s\right)\left(\overline{l}\gamma_5 l'\right) \Big]. \end{aligned} \tag{3.18}$$

The covariant S-matrix element for the decay of the weak eigenstate $\bar{P} \to l_1^+ l_2^-$, where $\bar{P}(q\bar{q}') = \bar{K}^0(s\bar{d}), \bar{B}^0(b\bar{d})$ or $\bar{B}_s(b\bar{s})$, takes the form

$$\mathcal{M}_{\bar{P}\to l_1^+ l_2^-} = -\frac{\mathcal{N}}{2} f_P\, \overline{u}(p_2)\, [m_P \mathcal{S}_{\text{-}qq'l_1l_2} + m_P \mathcal{P}_{\text{-}qq'l_1l_2}\gamma_5]\, v(p_1) \tag{3.19}$$

with

$$m_P\, \mathcal{S}_{\text{-}qq'l_1l_2} = (m_2 - m_1)\, C_{\Delta 9\text{-}qq'l_1l_2} + \overline{m}_P \zeta C_{\Delta S\text{-}qq'l_1l_2}\,, \tag{3.20a}$$

$$m_P \mathcal{P}_{\text{-}qq'l_1l_2} = (m_2 + m_1)\, C_{\Delta 10\text{-}qq'l_1l_2} + \overline{m}_P \zeta C_{\Delta P\text{-}qq'l_1l_2}. \tag{3.20b}$$

The prefactor m_P multiplying $\mathcal{S}$ and $\mathcal{P}$ is a mere convention (used in `flavio`, in Refs. [52, 112] but not in [111]). Notice that the first term in Eq. (3.20a) changes sign under $l_1 \leftrightarrow l_2$ and thus $\mathcal{S}_{\text{-}qq'l_1l_2} \neq \mathcal{S}_{\text{-}qq'l_2l_1}$ in the case of lepton flavour universal WC's [121]. This is not a paradox: LFUV is brought to the game by the fact that $m_1 \neq m_2$.

For the decays of the conjugated states $P(q'\bar{q}) = K^0(d\bar{s}), B^0(d\bar{b})$ and $B_s(s\bar{b})$, the matrix elements read

$$\mathcal{M}_{P\to l_1^+ l_2^-} = -\frac{\mathcal{N}^*}{2} f_P\, m_P\, \overline{u}(p_2)\Big[-\mathcal{S}^*_{\text{-}qq'l_2l_1} + \mathcal{P}^*_{\text{-}qq'l_2l_1}\gamma_5\Big] v(p_1)\,. \tag{3.21}$$

It is interesting to trace back the origin of the minus sign in front of $\mathcal{S}^*_{\text{-}qq'l_2l_1}$: the first term in the definition (3.20a) changes sign due to the factor $(m_1 - m_2)$ while in the second term the opposite sign arises from Eq. (3.18).

3.3.2 Instantaneous B-meson decays

The B^0 and B_s mesons usually decay before their first oscillation. To a good approximation, the oscillations can be neglected and one can effectively consider the decay of the weak eigenstates. For a more rigorous approach in the LF conserving case, see Ref. [122].

From Eq. (3.19) or (3.21), one can derive, using the standard trace techniques,

$$\begin{aligned} \mathrm{BR}(B_s \to l_1^- l_2^+) = \mathrm{BR}(\bar{B}_s \to l_1^+ l_2^-) = \tau_{B_s} \frac{|\mathcal{N}|^2}{32\pi} \frac{\sqrt{\lambda(m_{B_s}^2, m_1^2, m_2^2)}}{m_{B_s}} f_{B_s}^2 \\ \times \Big[\Big(m_{B_s}^2 - (m_1 + m_2)^2\Big)\, |\mathcal{S}_{\text{-}bsl_1l_2}|^2 + \Big(m_{B_s}^2 - (m_1 - m_2)^2\Big)\, |\mathcal{P}_{\text{-}bsl_1l_2}|^2\Big] \end{aligned} \tag{3.22}$$

with $\lambda(a^2, b^2, c^2) = [a^2 - (b-c)^2][a^2 - (b+c)^2]$. The prediction is CP-symmetric regardless of possible complex phases of the Wilson coefficients. On the other hand, if $l_1 \neq l_2$, the other pair of CP conjugated processes, $B_s \to l_1^+ l_2^-$ and $\bar{B}_s \to l_1^+ l_2^-$, is in fact absolutely independent on those in Eq. (3.22) as it is driven by different effective operators, i.e., by those multiplied by $C_{\Delta X\text{-}bsl_1l_2}$. This subtlety has been overlooked in Ref. [111] which inappropriately employs the Wilson coefficients $C_{X\text{-}bsl_2l_1}$ in predicting $B_s \to l_1^- l_2^+$ and indicates that only the WC proportional to $m_2 - m_1$ in Eq. (3.20a) should be important regarding the differences between $\mathrm{BR}(B_s \to l_1^- l_2^+)$ and $\mathrm{BR}(B_s \to l_1^+ l_2^-)$. Apart from this issue and a small typo concerning the (pseudo)scalar operator normalization coefficient ζ, Eq. (3.22) agrees with the expressions in [111].

Needless to say, incidental swapping $l_1 \leftrightarrow l_2$ in Eq. (3.22) becomes irrelevant once only the sum of both decay channels is considered. Neglecting the lighter lepton mass, this sum is driven by

$$\mathrm{BR}(B_s \to l_1^\pm l_2^\mp) \propto |\mathcal{S}_{\text{-}bsl_1l_2}|^2 + |\mathcal{S}_{\text{-}bsl_2l_1}|^2 + |\mathcal{P}_{\text{-}bsl_1l_2}|^2 + |\mathcal{P}_{\text{-}bsl_2l_1}|^2\,. \tag{3.23}$$

3.3.3 Decays of neutral kaons

Experimentally studied neutral kaons are far from the weak basis and thus the equations in the previous part need to be modified. Neglecting the indirect CP violation, the relations between weak and mass kaon eigenstates read

$$|K_L^0\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \qquad |K_S^0\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}. \tag{3.24}$$

Accordingly, one obtains

$$\mathcal{M}_{K^0_{L,S}\to l_1^+ l_2^-} = -\frac{1}{2} f_{K^0}\, \overline{u}(p_1) \left[\mathcal{S}^{L,S}_{l_1 l_2} + \mathcal{P}^{L,S}_{l_1 l_2}\gamma_5\right] v(p_2), \tag{3.25}$$

where

$$\mathcal{S}^{L,S}_{l_1 l_2} = \frac{-\mathcal{N}^*\mathcal{S}^*_{\_sdl_2l_1} \pm \mathcal{N}\mathcal{S}_{\_sdl_1l_2}}{\sqrt{2}}, \qquad \mathcal{P}^{L,S}_{l_1 l_2} = \frac{\mathcal{N}^*\mathcal{P}^*_{\_sdl_2l_1} \pm \mathcal{N}\mathcal{P}_{\_sdl_1l_2}}{\sqrt{2}} \tag{3.26}$$

with $\mathcal{S}_{\_sdl_il_j}, \mathcal{P}_{\_sdl_il_j}$ defined in (3.20). Recall that the relevant effective operators are conventionally defined in the form $(\bar{d}\,\Gamma s)(\bar{l}_2\Gamma' l_1)$, while those with the $(\bar{s}d)$ quark flavours are obtained by hermitean conjugation. The BR can be readily obtained in analogy with Eq. (3.22):

$$\begin{aligned}&\mathrm{BR}(K^0_{L,S}\to l_1^+ l_2^-) = \\ &= \frac{\tau_{K^0_{L,S}}}{32\pi}\frac{\sqrt{\lambda}}{m^2_{K^0}} m^3_{K^0} f^2_{K^0}\left[\left(1-\frac{(m_1+m_2)^2}{m^2_{K^0}}\right)\left|\mathcal{S}^{L,S}_{l_1l_2}\right|^2 + \left(1-\frac{(m_1-m_2)^2}{m^2_{K^0}}\right)\left|\mathcal{P}^{L,S}_{l_1l_2}\right|^2\right]\end{aligned} \tag{3.27}$$

This formula is compatible with the expressions in Ref. [108].

LFV decays

Notice that, as long as the indirect CPV in K^0 is neglected, we have $\left|\mathcal{S}^{L,S}_{e\mu}\right| = \left|\mathcal{S}^{L,S}_{\mu e}\right|$ and $\left|\mathcal{P}^{L,S}_{e\mu}\right| = \left|\mathcal{P}^{L,S}_{\mu e}\right|$, which leads to

$$\mathrm{BR}(K^0_{L,S}\to e^+\mu^-) = \mathrm{BR}(K^0_{L,S}\to e^-\mu^+). \tag{3.28}$$

Thus, the prediction for the sum of the two final states is, in the case of K^0_L, proportional to

$$\begin{aligned}\mathrm{BR}(K^0_L\to e^\pm\mu^\mp) \propto{}& \frac{2}{|\mathcal{N}|^2}\left(\left|\frac{-\mathcal{N}^*\mathcal{S}^*_{\_sde\mu}+\mathcal{N}\mathcal{S}_{\_sd\mu e}}{\sqrt{2}}\right|^2 + \left|\frac{\mathcal{N}^*\mathcal{P}^*_{\_sde\mu}+\mathcal{N}\mathcal{P}_{\_sd\mu e}}{\sqrt{2}}\right|^2\right) \\ ={}& |\mathcal{S}_{\_sde\mu}|^2 + |\mathcal{S}_{\_sd\mu e}|^2 + |\mathcal{P}_{\_sde\mu}|^2 + |\mathcal{P}_{\_sd\mu e}|^2 \\ &- 2\,\mathrm{Re}\left[\frac{\mathcal{N}^2}{|\mathcal{N}|^2}\left(\mathcal{S}_{\_sd\mu e}\mathcal{S}_{\_sde\mu} + \mathcal{P}_{\_sd\mu e}\mathcal{P}_{\_sde\mu}\right)\right].\end{aligned} \tag{3.29}$$

Notably enough, this structure differs from that in Eq. (3.23) by the last term. Hence, ignoring the kaon oscillations is not justified even when summing over the lepton pair charges. For K^0_S, the last double-term in (3.29) simply changes sign. Obviously, $K^0_L\to e\mu$ and $K^0_S\to e\mu$ are driven by different directions in the space of Wilson coefficients. The former process is experimentally well constrained [98] while the latter has never been searched for [54]. From the EFT point of view, a search for the $K^0_S\to e\mu$ decay would be of interest.

LF conserving decays and long distance contributions

For $l_1 = l_2 = l$, Eq. (3.27) simplifies to

$$\mathrm{BR}(K^0_{L,S} \to l^+ l^-) = \frac{\tau_{K^0_{L,S}}\, \beta_l\, m^3_{K^0} f^2_{K_0}}{32\pi} \left[\beta_l^2 \left|\mathcal{S}^{L,S}_{ll}\right|^2 + \left|\mathcal{P}^{L,S}_{ll}\right|^2\right] \tag{3.30}$$

with $\beta_l = \sqrt{\lambda(m^2_{K^0}, m^2_l, m^2_l)}/m^2_{K^0} = \sqrt{1 - 4m^2_l/m^2_{K^0}}$, and the expressions in (3.26) reduce to

$$\mathcal{S}^L_{ll} = i\sqrt{2}\,\mathrm{Im}\left(\mathcal{N}\mathcal{S}_{\_sdll}\right), \qquad \mathcal{P}^L_{ll} = \sqrt{2}\mathrm{Re}\left(\mathcal{N}\mathcal{P}_{\_sdll}\right), \tag{3.31a}$$

$$\mathcal{S}^S_{ll} = -\sqrt{2}\,\mathrm{Re}\left(\mathcal{N}\mathcal{S}_{\_sdll}\right), \qquad \mathcal{P}^S_{ll} = -i\sqrt{2}\,\mathrm{Im}\left(\mathcal{N}\mathcal{P}_{\_sdll}\right). \tag{3.31b}$$

These results are fully compatible with the expressions in Ref. [112].

For the LF conserving decays, the SM brings a long-distance (LD) contribution to $\mathcal{P}^L$ and $\mathcal{S}^S$, arising from the $\gamma^*\gamma^*$ intermediate state, which can not be encoded in the four-fermion operators in (3.9). For $K^0_L \to ll$, the LD contributions are comparable in magnitude to the short-distance ones from Eq. (3.31a) and from the dominant theoretical uncertainty to the prediction of the BR of this process. For more details, see, e.g., Refs. [107, 112].

3.4 B-meson anomalies in semileptonic decays

The second class of FCNC decays we will focus on are the pseudoscalar meson decays $P \to P'^{(*)} l_1^+ l_2^-$ which, apart from leptons, contain a pseudoscalar (P') or vector (P'^*) meson in the final state. As three-body decays, these processes provide a rich variety of kinematical observables, in contrast with the purely leptonic decays where the BR is the only observable available.

The semileptonic meson decays $P \to P' l_1^+ l_2^-$ are driven by the combinations $C_X + C'_X$ of Wilson coefficients [123], in contrast with $P^0 \to l_1^+ l_2^-$ which are triggered by $C_{\Delta X}$ defined in Eq. (3.15). Another important difference with respect to the purely leptonic decays is that there is no helicity suppression in $P \to P'^{(*)} l l'$ for the vector-type effective operators ($C_{9,10}$) which are generated in the EW interactions. Hence, the semileptonic decays proceed at much higher rates than $P^0 \to l^+ l^-$ and SM predicts that they should be approximately lepton flavour universal. For the full expressions for the differential decay widths in the WET, see Ref. [111].

In the recent years, an enormous wave of interest arose around the hints of non-SM sources of LFUV in the ratios

$$\mathrm{R}_{K^{(*)}} = \frac{\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)}{\mathrm{BR}(B \to K^{(*)} e^+ e^-)}. \tag{3.32}$$

The measured individual BR's are of the order of 10^{-6} [124, 125]. For the one-loop SM contributions to these processes, see Fig. 3.1.

R_K and R_{K^*} are usually measured separately in several bins of the lepton pair invariant mass squared q^2, with the resonance intervals around $m^2_{J/\psi(1S)}$ and $m^2_{\psi(2S)}$ omitted from the analysis. In the lowest q^2 region, dominated by the photon penguin with a pole at $q^2 \to 0$, the lepton masses become important and

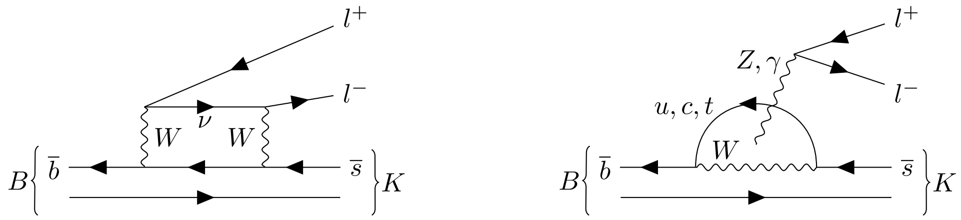

Figure 3.1: Box and penguin diagrams contributing to $\mathrm{R}_{K^{(*)}}$ in the Standard Model.

are expected to spoil the LFU. For $q^2 > 1\,\mathrm{GeV}^2$, the four-fermion WET operators dominate. The hadronic form-factor and electromagnetic uncertainties mostly cancel and the SM predicts $\mathrm{R}^{\mathrm{SM}}_{K^{(*)}} = 1$ to a better than 1% accuracy [126] (see also references in [22]).

Experimentally, R_K is easier to measure for the charged B mesons while R_{K^*} is easier for the neutral ones, due to the charged hadrons in the final state ($K^{*0} \to K^+\pi^-$). Both R_K and R_{K^*} were first measured by Belle ($\mathrm{R}_K = 1.03 \pm 0.19 \pm 0.06$, $\mathrm{R}_{K^*} = 0.83 \pm 0.17 \pm 0.08$, where the first uncertainty is statistical and the latter is systematic) [125] and BaBar [127] without finding a tension with the SM. Nevertheless, the statistical uncertainties were quite large. The hint of LFUV in R_K was reported by LHCb in 2014 [21], which found a 2.6-σ deviation from unity in $\mathrm{R}_K = 0.745^{+0.090}_{-0.074} \pm 0.036$, for the interval $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$. This has been later [128] updated to

$$\mathrm{R}_K = 0.846^{+0.060+0.016}_{-0.054-0.014} \tag{3.33}$$

for $1.1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$, with the significance of the discrepancy similar to the previous result.[2] The updated Belle measurement [130] is compatible with both SM and the LHCb value.

The R_{K^*} measurement at LHCb has been performed on neutral B mesons with the result [22]

$$\mathrm{R}_{K^*} = 0.69^{+0.11}_{-0.07} \pm 0.05 \tag{3.34}$$

in the $\langle 1.1, 6\rangle\,\mathrm{GeV}^2$ interval. The updated analysis by Belle [131] for the same q^2 range reported $\mathrm{R}_{K^*} = 0.96^{+0.45}_{-0.29} \pm 0.11$.

Apart from the ratios $\mathrm{R}_{K^{(*)}}$, some deficit has been found in the widths of $B \to K^{(*)}\mu^+\mu^-$ decays themselves [132] as well as in $\mathrm{BR}(B_s \to \phi\mu^+\mu^-)$ [133]; predictions of individual BR's, however, suffer from larger uncertainties, especially from the hadronic form factors. Furthermore, certain discrepancies have been reported in the so-called P_5' observable in an angular analysis of the $B \to K^*\mu^+\mu^-$ decay [134]. Normalized to the total branching ratio, the hadronic uncertainties are reduced but still quite important. Although neither of these additional anomalies would probably raise much of an attention alone, all of them, together with the theoretically clean observables $\mathrm{R}_{K^{(*)}}$, can be interpreted as a negative interference between NP and SM contributions to the $b \to s\mu\mu$ amplitudes.

Last but not least, an apparently detached subset of anomalies in B decays

[2] Very recently, another update was announced by LHCb: $\mathrm{R}_K = 0.846^{+0.042+0.013}_{-0.039-0.012}$ [129]. The reduced uncertainties of this measurement are not considered in this thesis.

consists of the ratios of the charged current decays

$$\mathrm{R}_{D^{(*)}} = \frac{\mathrm{BR}(\overline{B} \to D^{(*)}\tau\overline{\nu})}{\mathrm{BR}(\overline{B} \to D^{(*)}l\overline{\nu})}, \qquad l = \mu \text{ or } e. \tag{3.35}$$

Here, the m_τ forms a large but well understood source of LFUV within the SM, leading to the prediction $\mathrm{R}_D^{\mathrm{SM}} = 0.299 \pm 0.003$ and $\mathrm{R}_{D^*}^{\mathrm{SM}} = 0.258 \pm 0.005$ (see [135] and the references therein). These observables have been measured by BaBar [23], Belle [136, 137, 138] and LHCb [139, 140]. The current experimental world averages [135] are $\mathrm{R}_D = 0.340 \pm 0.027 \pm 0.013$ and $\mathrm{R}_{D^*} = 0.295 \pm 0.011 \pm 0.008$. Taking into account the correlations, the measured values of $\mathrm{R}_{D^{(*)}}$ are $3\,\sigma$ above the SM predictions [135]. The experimentally established LFU in the analogous μ/e ratio indicates that there might be a NP in the $b \to c\tau\bar{\nu}$ channel. Notice that anticipated BSM contributions should be quite large in order to significantly influence the unsuppressed charged-current decays (e.g., $\mathrm{BR}(\overline{B}^+ \to D^{*0}\tau^+\overline{\nu}) \approx 2\,\%$ [141]).

These so-called B anomalies have been analysed within the WET framework by many studies, with the results slightly varying in details, depending on the statistical methods applied and on the experimental data available at the time they were performed [142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153]. For a recent global analysis, we especially refer to [154], where the following best simplified scenarios accounting for the neutral current anomalies have been found:

1. Scenario with $C^{\mathrm{NP}}_{9\_bs\mu\mu} = -C^{\mathrm{NP}}_{10\_bs\mu\mu}$ can fit essentially all the aforementioned FCNC observables. This WC combination corresponds to the chiral operator $(\bar{b}_L\gamma_\mu s_L)(\mu_L\gamma^\mu\mu_L)$ which is also exciting regarding the explicit NP models.

2. Scenario with $C^{\mathrm{NP}}_{9\_bs\mu\mu}$.

3. Scenario with $C^{\mathrm{NP}}_{10\_bs\mu\mu}$.

4. Scenario with NP in the electron sector, namely $C^{\mathrm{NP}}_{9\_bsee} = +C^{\mathrm{NP}}_{10\_bsee}$, which induces the operator $(\bar{b}_L\gamma_\mu s_L)(\bar{e}_R\gamma^\mu e_R)$. Obviously, such a setup can not address the additional anomalies in the $bs\mu\mu$ sector; nevertheless, R_K and R_{K^*} can be described so successfully that the resulting maximal likelihood outclasses that of other simplified scenarios.

As mentioned before, it seems reasonable to assume that NP resides at some high energy scale and, therefore it should well described by SMEFT. As mentioned in Section 3.2, this puts no constraints on 2-down-type-quark-2-lepton WC's except from those in Eq. (3.11). On the other hand, operators containing left-handed quarks or leptons necessarily enter with their weak-isospin counterparts which leads to correlations with many other processes, including $\mathrm{R}_{D^{(*)}}$. Ref. [154] contains the global analysis also within the SMEFT framework.

3.5 Flavour physics of leptoquarks in a nutshell

Leptoquarks (LQs) are hypothetical bosons featuring an interaction vertex with a lepton and a quark. This defining feature implies that they must be color

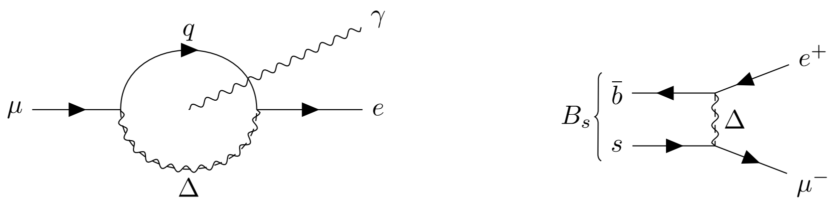

Figure 3.2: Leptoquark mediation of lepton flavour violating processes.

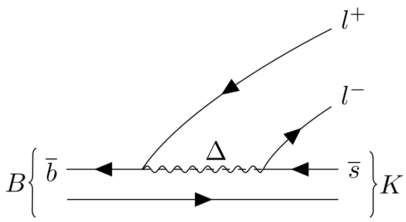

Figure 3.3: Leptoquark contribution to the semileptonic meson decays may be flavour non-universal.

triplets or antitriplets. On the other hand, their spin may be either 0 or 1 and the $SU(2)_L \times U(1)_Y$ representation is not uniquely fixed – see Tables 1.2, A.1 and A.3 or Ref. [52] for all possibilities. In this section, we assume that the LQ has no baryon number violating interaction (i.e., no diquark-type coupling).

Leptoquarks and flavour symmetries

For simplicity, we will take into account only the $SU(3)_C \times U(1)_Q$ gauge symmetry and ignore the Lorentz structures in the following few paragraphs. As an example consider a leptoquark (denoted by Δ) with the electric charge $Q = 2/3$, interacting with the charged leptons via

$$\bar{\hat{d}}_\alpha \, Y_\Delta \, \hat{e} \, \Delta^\alpha + \text{h.c.} = \begin{pmatrix} \bar{d} & \bar{s} & \bar{b} \end{pmatrix}_\alpha \begin{pmatrix} h_{de} & h_{d\mu} & h_{d\tau} \\ h_{se} & h_{s\mu} & h_{s\tau} \\ h_{be} & h_{b\mu} & h_{b\tau} \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \Delta^\alpha + \text{h.c.} \tag{3.36}$$

Such an interaction generally breaks the flavour symmetries but, in special cases, some of the important flavour subgroups may remain intact:

- If only a single column of the interaction matrix Y_Δ is nonzero, then the LQ can be ascribed the corresponding flavour number $\mathcal{L}_l$ introduced in Section 3.1 and there is no lepton flavour violation. Conversely, whenever there are at least *two non-zero columns* in Y_Δ, the $U(1)^2_{\text{LF}}$ symmetry is violated and the leptoquark mediates LFV processes. An example can be found in Fig. 3.2.

- On the other hand, respecting the $(S_3)_{\text{LFU}}$ symmetry requires that all three columns of the interaction matrix are equal. Thus, the leptoquark brings new sources of lepton flavour universality violation whenever (at least) *two columns of* Y_Δ *differ*. See Fig. 3.3 for the hypothetical LQ contributions to the processes discussed in Section 3.4.

To sum up, leptoquarks always induce LFV or LFUV as their interactions never respect the whole flavour group. (In the trivial case $Y_\Delta = 0_{3\times 3}$, the considered boson does not deserve to be called a leptoquark.) Analogous conclusions for the *quark* flavour and *rows* of Y_Δ can be drawn easily.

As the direct searches for various kinds of LQs have put the mass limits up to more than a TeV [82, 83, 84, 85, 86, 87, 88, 89], effects of LQs in precision phenomenology should be well described by the effective theories. Keeping the Lorentz structure suppressed, the Wilson coefficients of the $(\overline{q'}q)(\overline{l'}l)$ effective interactions are given by $C_{\bar{q}'ql'l} \propto h^*_{ql'}h_{q'l}/m^2_\Delta$. One can arrange the coefficients to a matrix C with multiindices (lq') and $(l'q)$ and reverse the logic: the relevant effective interaction can stem from a single LQ if and only if C has rank one [155].

For some of the many general-purpose studies of leptoquarks in flavour physics, we refer to [52, 156].

Leptoquarks and B-meson anomalies

Clearly, the LQs look like natural candidates to account for the B anomalies described in Section 3.4. Leptoquark effects depend on their $SU(2)_L \times U(1)_Y$ quantum numbers which constrain the charges and chiralities of quarks and leptons they can couple to, which, in turn, determine the Lorentz structure of the resulting semileptonic effective operators [cf. (3.9)]. For a catalogue of LQs which could contribute to the $b \to sl^+l^-$ amplitudes (a subset of the list in Table A.3) and their brief characterzation, see Ref. [123].

Numerous studies have been already published, analyzing the capabilities of various types of LQs to accommodate various subsets of the B anomalies, and confronting these scenarios with other experimental constraints. We refer to some of them in two categories:

- *Simplified models* – SM extensions consisting solely in adding a single LQ to the theory. Scalar LQs have been discussed, e.g., in [143, 121, 157, 158, 150, 159, 160]. In the case of vector leptoquarks, the gauge nature of these fields was not imposed: [161, 162, 121, 163, 149, 164, 165, 166]. Within such a simplified approach, the vector $U_1 \sim (3, 1, +\frac{2}{3})$ has been identified as a particularly feasible candidate (e.g., [167, 154]) as it can induce at the tree level the Wilson coefficients $C_9 = -C_{10}$, which points towards the effective scenario no. 1 mentioned on page 51; furthermore, it generates operators contributing to $b \to c\tau\bar{\nu}$.

- *Full dedicated models.* Strictly speaking, there is nothing incomplete on the SM enriched by a scalar LQ. Nevertheless, we include to this category only the renormalizable models containing multiple BSM fields which were build in order to resolve the tension in the B decays, for example: [168, 169]. A large subset of such models assumes an extended gauge structure; e.g. [170, 56, 57, 58, 60, 62, 171, 172, 173, 174].

4. Quark-lepton unification confronted with LFUV in B decays

In Chapter 2, two scenarios of quark-lepton unification have been thoroughly introduced: the MQLSM and the FPW model. Chapter 3 presented the B-meson anomalies. The current chapter is devoted to exploring if the neutral current B anomalies could be compatible with the models from Chapter 2, at least to some extent. Most of the material presented has been published in Refs. [25, 29, 30]. On top of that, also the deviations in $\mathrm{R}_{D^{(*)}}$ shall be shortly discussed at the end of the chapter.

The considered models are by no means *simplified models* discussed at the end of Section 3.5, neither have they been build in order to address the B anomalies. The motivation for the two models is different – they are the most modest BSM realizations of the quark-lepton $SU(4)_C$ symmetry with Dirac or Majorana neutrinos. It is therefore not expectable that the models would alleviate all the tension between the SM and experiment in B decays. On the other hand, as they contain several leptoquarks, the relevant phenomenology can be quite rich.

Refs. [25, 29] have multiple co-authors. The author of this thesis contributed to finding the complete scalar potential, identification of the promising parts of the parameter space of the FPW model, recognizing some of the key limiting observables and debugging, while the loop calculations of specific processes, their computer implementation and scanning have been performed by the others.

4.1 BSM fields: the menu

As detailed in Chapter 2, the MQLS and FPW models contain 3 distinct physical leptoquark states: the vector leptoquark (VLQ) $U_1 \sim (3, 1, +2/3)$ and two scalar doublets $\tilde{R}_2 \sim (3, 2, +1/6)$ and $R_2 \sim (3, 2, +7/6)$. Furthermore, there is a Z' boson around, another potential candidate to account for the B anomalies. However, interactions of all these fields are subject to constraints stemming from the extended gauge symmetry. Notice that the $\bar{S}_1$ field, which also has the quantum numbers of a LQ, dominates the Goldstone modes and disappears from the physical spectrum; moreover, it can not couple to the charged leptons.

As already mentioned in Section 3.5, the U_1 leptoquark has been identified as a great candidate to accommodate the anomalies in several publications [154]. However, the specific interaction patterns suggested do not obey the requirement that the VLQ interactions are unitary in the flavour space (cf. Section 2.8.1). In the quark-lepton symmetry models, U_1 is necessarily coupled to both left-handed and right-handed quark-lepton vector-type currents [see Eq. (2.57)]. The induced effective vertices thus contain also the structures $(\overline{q_L}\gamma^\mu l'_L)(\overline{l_R}\gamma_\mu q'_R)$ which after the Fierz transformation lead to the scalar-type operators $(\overline{q_L}q'_R)(\overline{l_R}l'_L)$, triggering the strongly constrained decays $P^0 \to l_1^+ l_2^-$. The mass limit on the gauge leptoquark from these processes ranges from about 86 TeV [175] to thousands of TeV [61], depending on the mixing matrices U_L and U_R defined in Eq. (2.55).

With such a heavy mass, the impact of the gauge LQ on the semileptonic decays is inevitably too small compared to the observed deviations. There are several ways to circumvent the requirement of unitarity of $U_{L,R}$ (extra fermions, more complicated gauge structure, compositeness), all of which go beyond the field of our current interest – ”economical” models of quark-lepton unification. The gauge leptoquark U_1 will be elaborated on more in Chapter 5.

Also the Z' boson has been suggested as a possible source of the LFUV in the B decays (see, e.g., [149] and references therein). However, prospects of Z' in the MQLSM and FPW model are desperate in this respect. Firstly, $m_{Z'}$ is correlated with the mass of U_1 [see Eq. (2.35)] and hence too high. Secondly, the Z' interactions are flavour diagonal and, hence, do not contribute to the quark flavour changing processes. Thirdly, the Z' couplings respect the LFU.

Hence, in the rest of this chapter we focus on the *scalar* leptoquarks in the model. We assume that the $SU(4)_C$ breaking scale is at the several-thousand-TeV ballpark, making the gauge LQ, the Z' boson and most of the scalar spectrum safely decoupled from the low-energy phenomenology, but that some of the scalar LQs are accidentally much lighter.

As explained on page 34, the only constraints on scalar masses are given by Eq. (2.44), which implies that either of the two scalar LQs may be much lighter than most of the BSM scalar spectrum but not both of them. This, among other things, precludes an interesting scenario in which the 2/3-charged components of R_2 and $\tilde{R}_2$ undergo large mixing, which has been studied in Ref. [176] in the context of anomalous muon magnetic moment.

As both R_2 and $\tilde{R}_2$ are $SU(2)_L$ doublets, they couple via Yukawa interactions to one SM fermion doublet and an $SU(2)_L$ singlet. Since the chiralities of SM fermions with $\mathcal{F} = +1$ are strictly related to their $SU(2)_L$ quantum numbers (whence the label of the group), the resulting effective operators necessarily have the '$2L2R$' chiral structure.[1] On the other hand, the SM contributions to quark-flavour changing neutral currents (see Fig. 3.1) are purely left-handed in the quark sector and accidentally *almost left-handed* in the lepton sector. In other words, the WCs from $C_9(\bar{s}_L\gamma^\mu b_L)(\bar{l}\gamma_\mu l) + C_{10}(\bar{s}_L\gamma^\mu b_L)(\bar{l}\gamma_\mu\gamma_5 l)$ induced by the electroweak physics satisfy [149]

$$C_9^{\mathrm{SM}} \approx -C_{10}^{\mathrm{SM}}\,. \tag{4.1}$$

Due to this chirality mismatch between SM and LQ-doublet mediated amplitudes, there is just a very restricted room for negative interference in the $b \to s\mu\mu$ channel, desired for accommodation of the additional discrepancies discussed in Section 3.4.

Thus, one may only hope for attributing the anomalous values of R_K and R_{K^*} to the scalar LQ contributing mainly to the $B \to K^{(*)}e^+e^-$ channel.[2] In such a case, the $\tilde{R}_2$ leptoquark leads to opposite signs of $\mathrm{R}_K - 1$ and $\mathrm{R}_{K^*} - 1$, at odds with observation. On the other hand, effects of R_2 may quite well account for both R_K and R_{K^*} [149].

[1]This can not be inferred just from the scalar nature of the LQs. Consider the interaction $({q_L}^{i\alpha}{\ell_L}^j)S_{3ij\alpha}$ of the $S_3 \sim (\bar{3}, 3, 1/3)$ leptoquark as a counterexample.

[2]Note that the angular observables in $B \to K^*ee$ have been measured [177] with larger experimental uncertainties than those for $B \to K^*\mu\mu$ [134]. See also Ref. [178].

Furthermore, R_2 has been suggested as a possible origin of the $\mathrm{R}_{D^{(*)}}$ anomalies [179, 180, 181]; we will comment on the charged-current anomalies at the end of this chapter.

Hence, in what follows we assume for simplicity that the R_2 leptoquark is the only BSM field with mass in the TeV ballpark while the rest of the BSM spectrum is much heavier. We remind the reader that such a scenario is compatible the scalar potential of the models considered.

4.2 Semileptonic decays with the R_2 leptoquark

Recall from Eq. (1.15) that the Yukawa interactions of the chosen LQ generally take the form

$$\mathcal{L}^{R_2-\text{Yuk}} = {R_2}^i \left(\overline{\hat{u}_R} \hat{Y}_2 {\hat{\ell}_L}^j \varepsilon_{ji} + \overline{\hat{q}_{Li}} \hat{Y}_4 \hat{e}_R \right) + \text{h.c.} \tag{4.2}$$

where the circumflex and upper indices of $Y_{2,4}$ indicate that these Yukawa matrices introduced in Eq. (2.45) are now in the mass basis of the denoted fermion types. At the tree level, this interaction gives rise to the following 4-fermion SMEFT operators (with generation indices suppressed) [182]:

$$Y_2 \times Y_2^* : \quad (\overline{\ell_{Li}}\, u_R)\,(\overline{u_R}\, {\ell_L}^i) = -\frac{1}{2}(\overline{u_R}\gamma^\mu u_R)(\overline{\ell_L}\gamma_\mu \ell_L) = -\frac{1}{2}\mathcal{O}_{\ell u} \tag{4.3a}$$

$$\begin{aligned} Y_2 \times Y_4^* : \quad (\overline{\ell_{Li}}\, u_R)\,\varepsilon^{ij}\,(\overline{q_{Lj}} e_R) &= -\frac{1}{2}(\overline{\ell_{Li}}\, e_R)\,\varepsilon^{ij}\,(\overline{q_{Lj}} u_R) - \frac{1}{8}(\overline{\ell_{Li}}\sigma^{\mu\nu} e_R)\,\varepsilon^{ij}\,(\overline{q_{Lj}}\sigma_{\mu\nu} u_R) \\ &= -\frac{1}{2}\mathcal{O}^{(1)}_{\ell equ} - \frac{1}{8}\mathcal{O}^{(3)}_{\ell equ} \end{aligned} \tag{4.3b}$$

$$Y_4 \times Y_4^* : \quad (\overline{q_{Li}}\, e_R)\,(\overline{e_R}\, {q_L}^i) = -\frac{1}{2}(\overline{q_{Li}}\gamma^\mu {q_L}^i)(\overline{e_R}\gamma_\mu e_R) = -\frac{1}{2}\mathcal{O}_{qe} \tag{4.3c}$$

For the the $b \to sll$ processes, only the $\mathcal{O}_{qe}$ operator is relevant since the others involve also up-type quarks. Below the electroweak scale, the $T^3_L = -1/2$ part of the sum over i in Eq. (4.3c) is matched to the WET operator $(\overline{q'_L}\gamma^\mu q_L)(\overline{l'_R}\gamma_\mu l_R)$ which in the standard basis (3.9) corresponds to the direction in the space of Wilson coefficients defined by

$$C^{\text{NP}}_{9\_qq'll'} = +C^{\text{NP}}_{10\_qq'll'}. \tag{4.4}$$

This relation together with Eq. (4.1) implies the non-interference of the SM and NP amplitudes for the $P \to P'll'$ decays. Hence,

$$\mathrm{R}_K \approx \frac{|C^{\text{SM}}_{9\_bsll}|^2 + |C^{\text{NP}}_{9\_bsee}|^2}{|C^{\text{SM}}_{9\_bsll}|^2 + |C^{\text{NP}}_{9\_bs\mu\mu}|^2} \,. \tag{4.5}$$

In the simplest scenarios, one can turn on only the be and se coupling that are needed for inducing $C^{\text{NP}}_{9,10\_bsee}$. Thus, the ideal pattern of the Yukawa matrices for addressing $\mathrm{R}_{K^{(*)}}$ by the interaction in Fig. 3.3 is

$$\hat{Y}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{Y}_4 = \begin{pmatrix} 0 & 0 & 0 \\ y_{se} & 0 & 0 \\ y_{be} & 0 & 0 \end{pmatrix}. \tag{4.6}$$

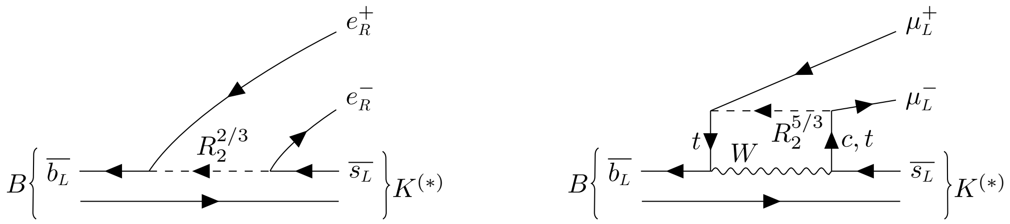

Figure 4.1: *Left* – Tree-level contribution of the R_2 leptoquark to $\mathrm{R}_{K^{(*)}}$. *Right* – Loop contribution of R_2 to the $b \to s\mu\mu$ processes, not applicable as a dominant contribution to the B anomalies in the considered models of quark-lepton unification.

With the couplings of perturbative size, the mass of the R_2 leptoquark should not exceed several units of TeV. The corresponding WET scenario, though it cannot address the additional $b \to s\mu\mu$ anomalies, ranked the 2nd position in the likelihood comparison of simplified effective models in Ref. [149] and the 4th position in the more recent study [154] (see also page 51).

In the models under consideration, Eq. (2.62) implies that the Yukawa matrices $\hat{Y}_2$ and $\hat{Y}_4$ are subject to the following constraints:

$$\hat{Y}_2 = \frac{\sqrt{3/2}}{v_{\rm ew}\cos\beta}\left(U_R' \hat{M}_\nu^D V_{\rm PMNS}^\dagger - \hat{M}_u V_{\rm CKM} U_L\right), \tag{4.7a}$$

$$\hat{Y}_4 = \frac{\sqrt{3/2}}{v_{\rm ew}\cos\beta}\left(\hat{M}_d U_R - U_L \hat{M}_e\right). \tag{4.7b}$$

Hence, the idealized case of Eq. (4.6) cannot be automatically adopted. Due to the different roles of $\hat{M}_\nu^D$ in the MQLSM and the FPW model, also the possible patterns of $\hat{Y}_2$ differ between these two models.

4.3 MQLSM model

In the MQLSM, $\hat{M}_\nu^D$ is a diagonal light-neutrino mass matrix, and as such is completely negligible in Eq. (4.7a). Hence, the largest Yukawa couplings of R_2 unavoidably reside in the 3rd row of $\hat{Y}_2$, elements of which are enhanced by the top quark mass [65]. Expanding the $SU(2)_L$ and flavour structures from Eq. (4.2), these interactions take the form

$$\mathcal{L}_{\rm MQLSM}^{\text{Yuk-large}} = \sqrt{\frac{3}{2}}\frac{m_t}{v_{\rm ew}\cos\beta}\,\overline{t_R}\left(\left(V_{te}e_L + V_{t\mu}\mu_L + V_{t\tau}\tau_L\right)R_2^{5/3} - \nu_{Lt}R_2^{2/3}\right) + \text{h.c.} \tag{4.8}$$

where V_{tl} (with $l = e, \mu, \tau$) are elements of the third row of the unitary matrix $V_{\rm CKM}U_L$ and $\nu_{Lt} = \sum_l V_{tl}\nu_{Ll}$. Due to $V_{tb} \approx 1$, the relevant mixing factors are approximately given by $V_{tl} \approx u_{bl}^L$ where u_{ql}^L denote the entries of the U_L matrix alone.

Ref. [164] suggested that the R_2 leptoquark could accommodate the neutral-current B anomalies via the loop diagram in Fig. 4.1 which induces the contributions to the 'preferred' Wilson coefficients $C_{9\_bs\mu\mu}^{\rm NP} = -C_{10\_bs\mu\mu}^{\rm NP}$. Naively, this is achievable in the MQLSM simply by setting $u_{b\mu}^L = 1$; in such a case, the Yukawa coupling for $\overline{t_R}\mu_L R_2^{5/3}$ is much larger than all the other entries in $\hat{Y}_2$ and $\hat{Y}_4$. However, as explained in Refs. [164, 159], avoiding other experimental constraints requires an even stronger interaction of $R_2^{5/3}$ with $\overline{c_R}\mu_L$; this is not consistent with the MQLSM in which Eq. (4.7a) implies that couplings of $R_2^{5/3}$ to $\overline{q_R}l_L$ are proportional to the corresponding quark masses. Hence, in what follows, we focus solely on the scenarios advocated in Section 4.2 where the LQ is supposed to address $\mathrm{R}_{K^{(*)}}$ at the tree level via its couplings to the electrons.

The two Yukawa coupling y_{de}, y_{se} from Eq. (4.6) are suppressed by m_b/m_t and m_s/m_t in comparison to the largest coupling in Eq. (4.8). Setting them strong enough to significantly influence $\mathrm{R}_{K^{(*)}}$ by choosing $\cos\beta$ small thus implies nonperturbative values of some of the vertices in Eq. (4.8).

Moreover, there are very strong bound on the lepton flavour violating muon decays such as $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [96]. These constraints in context of the MQLSM have been studied in Ref. [68]. One-loop R_2-mediated amplitudes to this decay (see Fig. 3.2) are m_q/m_μ enhanced when both $\hat{Y}_2$ and $\hat{Y}_4$ are involved [164, 25]. It can be shown that the bounds on LFV are incompatible with R_K significantly below 1. We will comment on this shortly during the next section which is devoted to the FPW model.

To conclude, the MQLSM is inconsistent with the $\mathrm{R}_{K^{(*)}}$ anomaly.

4.4 FPW model

Naively, the nature of neutrino masses has little to do with the $b \to sll$ transitions. Nevertheless, extending the MQLSM by inverse seesaw *is* relevant for our study as it brings new free parameters to the game. In particular, in the FPW model we have essentially absolute freedom in choosing the Dirac mass matrix $\hat{M}_\nu^D$. In turn, as follows from Eq. (4.7a), the $\hat{Y}_2$ matrix can be chosen arbitrarily without fixing the other relevant free parameters, U_L and U_R. We will adopt the idealized case of Eq. (4.6),

$$Y_2 = 0_{3\times3} \tag{4.9}$$

which holds in any flavour basis. This assumption does not mean that we restrict our analysis to the case where all the elements of $\hat{Y}_2$ are strictly zero. In fact, some of the elements may even lead to important signals of new physics. However, we do not have the ambition to list all possible additional NP signals. Instead, we want to focus on those predictions that cannot be avoided when accommodating the $\mathrm{R}_{K^{(*)}}$ anomaly. Such an attitude focuses on the testability of the whole model without restricting to ad-hoc chosen parts of the parameter space.

We should concede that by adopting Eq. (4.9) we leave out the potentially viable scenarios where Y_2 contributes to a negative interference among amplitudes of processes which are experimentally well constrained. In particular, consider the decays $\mu \to e\gamma$ and $\mu \to eee$. In the FPW model, these are mediated by several kinds of loops:

1. Due to existence of heavy neutrino states in the FPW model, there are interesting W-loop contributions. Unlike for the type-I seesaw, the current scenario allows for their masses well below $v_\chi \sim 10^3$ TeV and these contributions are not necessarily desperately small.

2. There are quark-R_2 loops, governed by (schematically) $Y_2 \times Y_2^*$, $Y_4 \times Y_4^*$ and especially by $Y_4 \times Y_2^*$ which contributes most significantly to the dipole

operators $(\overline{\mu_{L,R}}\sigma^{\mu\nu}e_{R,L})F_{\mu\nu}$ at the m_μ scale due to a relative m_q/m_μ enhancement [164] – see also the lepton chiralities in the $\mathcal{O}^{(3)}_{\ell equ}$ SMEFT operator in (4.3b).

In Ref. [25], a simplistic study of the amplitude interference in the $\mu \to e\gamma$ channel in the case of non-zero Y_2 has been performed. In what follows, we shall assume that the elements of $\hat{Y}_2$ are so small that their effects are irrelevant, and that the extra neutrinos are heavy enough so that their contributions to low energy processes are negligible.

Expanding the $SU(2)_L$ structure of the second Yukawa interaction in Eq. (4.2) yields

$$\mathcal{L}^{Y_4-\mathrm{int}}_{R_2} = \overline{\hat{d}_L}\,\hat{Y}_4\,\hat{e}_R\,R_2^{+2/3} + \overline{\hat{u}_L}\,V_{\mathrm{CKM}}\hat{Y}_4\,\hat{e}_R\,R_2^{+5/3} + \mathrm{h.c.} \tag{4.10}$$

Denoting the elements of the $\hat{Y}^{de}_4$ matrix by

$$\hat{Y}_4 = \begin{pmatrix} y_{de} & y_{d\mu} & y_{d\tau} \\ y_{se} & y_{s\mu} & y_{s\tau} \\ y_{be} & y_{b\mu} & y_{b\tau} \end{pmatrix}, \tag{4.11}$$

the matrix equation in (4.7b) can be rewritten into individual components: the coupling between $R_2^{2/3}$, a d-type quark q and a charged lepton l in the FPW (as well as in MQLS) model is given by [65]

$$y_{ql} = \frac{\sqrt{3/2}}{v_{\mathrm{ew}}\cos\beta}\left(u^R_{ql}m_q - u^L_{ql}m_l\right). \tag{4.12}$$

where u^R_{ql} and u^L_{ql} denote the elements of the yet unfixed mixing matrices U_R and U_L, respectively.

4.4.1 Constraining the parameter space

In what follows, the ideal Yukawa pattern identified in Eq. (4.6) will be confronted with the constraint (4.12) stemming from the extended gauge symmetry.

We approximate Eq. (4.12) by neglecting m_e and m_d everywhere, and further by neglecting the second generation masses when compared to the third generation ones.[3] The resulting structure reads

$$\hat{Y}_4 = \frac{\sqrt{3/2}}{v_{\mathrm{ew}}\cos\beta}\begin{bmatrix} 0 & -u^L_{12}m_\mu & -u^L_{13}m_\tau \\ u^R_{21}m_s & u^R_{22}m_s - u^L_{22}m_\mu & -u^L_{23}m_\tau \\ u^R_{31}m_b & u^L_{32}m_b & u^R_{33}m_b - u^L_{33}m_\tau \end{bmatrix}. \tag{4.13}$$

Generally, such a rich interaction structure leads to LFV processes (cf. Section 3.5). In our situation, the most important experimental constraints include $K^0_L \to e^\pm\mu^\mp$ which is triggered by $y_{d\mu}y^*_{se}$, the decays $\mu \to e\gamma$ and $\mu \to ee^+e^-$ (driven by various combinations of $y_{qe}y^*_{q'\mu}$), $B^+ \to K^+\mu^+e^-$ and $B^+ \to K^+\mu^-e^+$. For $O(1)$ elements of $u^{L,R}_{ql}$ and the LQ light enough to significantly affect $\mathrm{R}_{K^{(*)}}$, these limits would be violated by several orders of magnitude [25]. For example,

[3] The latter approximation will be justified a few paragraphs below.

$\mathrm{BR}(B^+ \to K^+\mu^-e^+)_{\rm exp} < 7\times 10^{-9}$ [183] while the deviation in R_K is much larger: $\mathrm{BR}(B^+ \to K^+e^+e^-)_{\rm exp} - \mathrm{BR}(B^+ \to K^+\mu^+\mu^-)_{\rm exp} = 1.1\times 10^{-7}$ [54]. The LFV processes involving also τ leptons are less stringently constrained.

Suppressing all the $\mathcal{L}_\mu$-violating processes while keeping y_{se} and y_{be} non-negligible requires that[4]

$$y_{d\mu} = y_{s\mu} = y_{b\mu} = 0 \tag{4.14}$$

approximately holds. As explained in Section 3.5, the leptoquark would not mediate *any* muon-family-number violating process in such a situation. The most general unitary parametrization of the matrices $U_{L,R}$ conforming this requirement takes the form

$$U_R = \begin{pmatrix} e^{i\delta_8}\cos\gamma\sin\alpha & e^{i(-\delta_1+\delta_7+\delta_8)}\cos\alpha & e^{i(\delta_3+\delta_8-\delta_2)}\sin\alpha\sin\gamma \\ -e^{i\delta_1}\cos\alpha\cos\gamma & e^{i\delta_7}\sin\alpha & -e^{i(\delta_1-\delta_2+\delta_3)}\cos\alpha\sin\gamma \\ -e^{i\delta_2}\sin\gamma & 0 & e^{i\delta_3}\cos\gamma \end{pmatrix}, \tag{4.15a}$$

$$U_L = \begin{pmatrix} e^{i\delta_9}\cos\phi & 0 & -e^{i\delta_4}\sin\phi \\ -e^{i(\delta_5+\delta_9-\delta_4)}\cos\alpha'\sin\phi & e^{i\delta_7}\sin\alpha' & -e^{i\delta_5}\cos\phi\cos\alpha' \\ e^{i(\delta_6+\delta_9-\delta_4)}\sin\phi\sin\alpha' & e^{i(\delta_6+\delta_7-\delta_5)}\cos\alpha' & e^{i\delta_6}\cos\phi\sin\alpha' \end{pmatrix}, \tag{4.15b}$$

where α and α' are related via $m_s\sin\alpha = m_b\sin\alpha'$. It leads to the following structure of the relevant Yukawa matrix:

$$\hat{Y}_4 = \frac{\sqrt{3/2}}{v_{\rm ew}\cos\beta}\begin{pmatrix} 0 & 0 & e^{i\delta_4}m_\tau\sin\phi \\ -e^{i\delta_1}m_s\cos\alpha\cos\gamma & 0 & e^{i\delta_5}m_\tau\cos\phi\cos\alpha' \\ -e^{i\delta_2}m_b\sin\gamma & 0 & e^{i\delta_3}m_b\cos\gamma - e^{i\delta_6}m_\tau\cos\phi\sin\alpha' \end{pmatrix}. \tag{4.16}$$

A comment about the accuracy of this form might be worth here: the mixing matrices in (4.15) solve Eq. (4.14) exactly for the approximate form (4.13) of the Yukawa matrix. For the exact form of $\hat{Y}_4$, it is easy to find the corresponding $U_{L,R}$ matrices numerically and check that they are close to those in (4.15). We have used the exact solutions to Eq. (4.14) in the numerical study [29] but in this text we stick to the approximation made above for the sake of clarity.

Notice that having successfully gotten rid of the muon number violating couplings, we cannot prevent completely the $\mathcal{L}_\tau - \mathcal{L}_e$ violation at the same time.

As far as LFV is concerned, there is no clear choice for the remaining free parameters. On the other hand, there is a unique way to maximize $|y_{se}y_{be}|$ which triggers the effective interaction contributing to $\mathrm{R}_{K^{(*)}}$:

$$\alpha \simeq 0, \qquad \gamma \simeq \frac{\pi}{4}. \tag{4.17}$$

Such a setting leads to the following pattern of the Yukawa matrix:

$$\hat{Y}_4 \simeq \frac{\sqrt{3/2}}{v_{\rm ew}\cos\beta}\begin{pmatrix} 0 & 0 & m_\tau e^{i\delta_4}\sin\phi \\ m_s e^{i\delta_1}/\sqrt{2} & 0 & m_\tau e^{i\delta_5}\cos\phi \\ m_b e^{i\delta_2}/\sqrt{2} & 0 & -m_b e^{i\delta_3}/\sqrt{2} \end{pmatrix}. \tag{4.18}$$

[4] For a detailed study of correlation between the B anomalies and μ-e violation using a different kind of leptoquark, see Ref. [166].

In what follows, we will mostly investigate this special case (4.18) in which suppress the LFV processes can be suppressed globally, simply by pushing down the overall effective operator prefactor $1/(m_{R_2}^2 \cos^2\beta)$ (with m_{R_2} stemming from the propagator) as much as possible.

Adopting Eq. (4.18), achieving the current experimental value of R_K cast in Eq. (3.33) requires

$$m_{R_2}\cos\beta \simeq 20\,\mathrm{GeV}. \tag{4.19}$$

Hence, $\cos\beta \ll 1$ must be set in order to evade the bounds from direct leptoquark searches. On the other hand, perturbativity of the Yukawa couplings, corresponding roughly to $\cos\beta \gtrsim m_b/v_{\mathrm{ew}}$, forbids extremely small values of $\cos\beta$.

Let us shortly compare this situation with the MQLSM where the $\hat{Y}_2$ matrix is essentially fixed by the choice of U_L [see. Eq. (4.7a)]. Adopting the form in Eq. (4.15b) is necessary to ensure that R_2 does not couple to μ_R via $\hat{Y}_4$; however, it also implies that the couplings of R_2 to $\overline{c_R}\mu_L$ or $\overline{t_R}\mu_L$ via $\hat{Y}_2$ are non-negligible. Furthermore, perturbativity of entries in [see Eq. (4.8)] requites $\cos\beta$ to be $O(1)$. Thus, the MQLSM can not significantly address the $\mathrm{R}_{K^{(*)}}$ anomaly.

4.4.2 Predictions

In this part we present the predictions within the FPW model based on the Yukawa matrices given in Eqs. (4.9) and (4.18). The relevant free parameters are the LQ mass m_{R_2}, the Higgs doublet mixing parameter $\cos\beta$ and the angle ϕ and phases δ_i entering the $\hat{Y}_4$ matrix. The detailed numerical computation of predictions that presented below was performed in Ref. [29] using and extending the computer package `FlavorKit` [184, 185] based on `SPheno` [186, 187, 188] and `SARAH` [189, 190]. Rather than going into details of the calculation of individual processes, we provide some simplified arguments here.

To get some insight also into the dominant interactions of the $Q = +5/3$ component of the R_2 doublet [see Eq. (4.10)], consider the case $\delta_i = 0$ with m_s neglected and approximate of the CKM matrix by the Cabibbo rotation:

$$V_{\mathrm{CKM}}\hat{Y}_4 \sim \begin{pmatrix} c_{\theta_c} & s_{\theta_c} & 0 \\ -s_{\theta_c} & c_{\theta_c} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & m_\tau s_\phi \\ 0 & 0 & m_\tau c_\phi \\ \frac{m_b}{\sqrt{2}} & 0 & -\frac{m_b}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & m_\tau \sin(\phi+\theta_c) \\ 0 & 0 & m_\tau \cos(\phi+\theta_c) \\ m_b/\sqrt{2} & 0 & -m_b/\sqrt{2} \end{pmatrix}. \tag{4.20}$$

In what follows we shall carefully distinguish between (i) *unavoidable* effects of this scenario, (ii) predictions which vary significantly within the considered part of the parameter space and (iii) *additional* possible signals which may arise due to small deviations from the considered Yukawa patterns. Interestingly enough, the first category turns out to be quite rich.

Low energy phenomenology

As we have already mentioned, the tree-level interactions of $R_2^{+2/3}$ lead to the $(\overline{q_L'}\gamma^\mu q_L)(\overline{l_R'}\gamma_\mu l_R)$ four-fermion operators with down-type quarks and charged leptons ($C_9^{\mathrm{NP}} = +C_{10}^{\mathrm{NP}}$), with the flavour structure given by products of two pairs

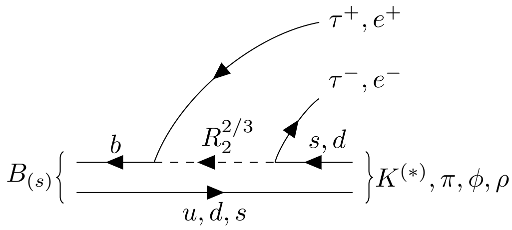

Figure 4.2: Leptoquark-mediated B-meson decays appearing in our scenario.

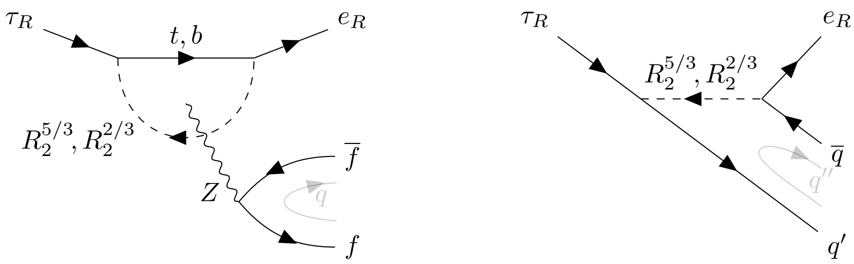

Figure 4.3: Dominant penguin and tree diagrams which contribute to various LFV τ decays. Notice that the latter depends strongly on the remaining parameters in Eq. (4.18) while the magnitude of the former is essentially constant.

of the large Yukawa elements $y_{be}, y_{b\tau}, y_{s\tau}$ and/or $y_{d\tau}$, and the smaller but important coupling y_{se} [see Eq. (4.18)]. The $R_2^{+5/3}$ partner induces analogous operators with the up-type quarks involved. Notice that all the large elements of $\hat{Y}_4$ and $V_{\mathrm{CKM}}\hat{Y}_4$ correspond to the LQ couplings to a third generation fermion. Furthermore, important loop-induced interactions contribute to other Lorentz and flavour structures.

First, let us focus on the B-physics where we expect important contributions from R_2 to the semileptonic decays. Recall that the considered part of the parameter space has been chosen such that $|y_{be}y_{se}|$ is as large as possible in order to address the neutral-current B anomalies $\mathrm{R}_{K^{(*)}}$ and that Eq. (4.19) determines the condition for accommodating the central value of R_K, regardless of the choice of the remaining free parameters ϕ and δ_i. Due to the relative smallness of y_{se}, parametrically larger amplitudes are predicted for the semileptonic decays containing at least one τ lepton in the final state. Nevertheless, due to the notorious difficulties with the tauon identification, these predictions of the considered model are well under the experimental sensitivity [191, 192, 54].

On the other hand, we have found that more important bounds stem from the decays of the τ lepton itself. Recall that the non-vanishing first and third columns of $\hat{Y}_4$ and $V_{\mathrm{CKM}}\hat{Y}_4$ in Eqs. (4.16) and (4.20) facilitate LFV decays $\tau \to eX$. As depicted in Figs. 3.2 and 4.3, the leptoquarks contribute to $\tau \to e\gamma$ and $\tau \to eee$ at the loop level and to the decays containing mesons already at the classical level. However, due to the magnitudes of the relevant couplings, the penguins are often even more important than trees in the current situation and their interference must be properly taken into account.

Among the $\tau^- \to e^- X$ processes including $X = \gamma, e^+e^-, \mu^+\mu^-, \pi^0, K_S^0, \phi$, K^+K^-, π^+K^-, $K^+\pi^-$, the most stringent bound turns out to arise from $\mathrm{BR}(\tau \to e\pi^+\pi^-)$ including – but not consisting solely of – the ρ-resonance intermediate state. The process $\tau \to e\pi^+\pi^-$ is mediated by the Z- and γ-penguins (which are

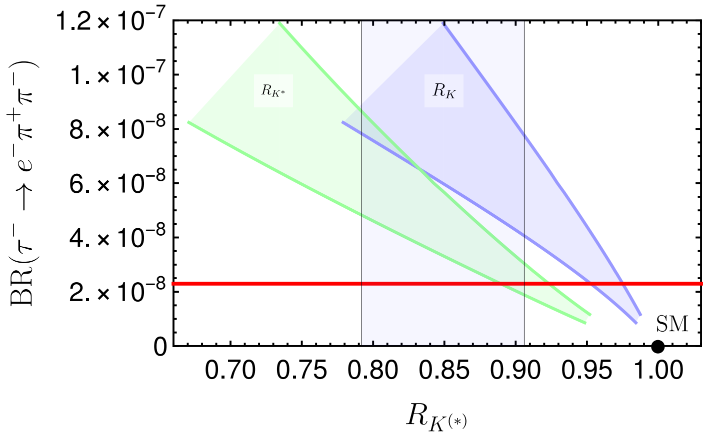

Figure 4.4: From Ref. [29]. Correlation between $\mathrm{R}_{K^{(*)}}$ and branching fraction of $\tau \to e\pi^+\pi^-$ in the region corresponding to the Yukawa pattern approximately given in Eq. (4.18). The red line denotes the current experimental limit on the LFV decay from Belle [10] while the vertical belt is the 1-σ region of the 2019 LHCb measurement [128].

essentially independent on ϕ and δ_i's) and, furthermore, by an $R_2^{5/3}$-mediated tree graph. The latter is proportional to the small parameter y_{ue} whose dependence on the ϕ and δ_i goes beyond the approximations made in Eq. (4.20).

Figure 4.4 shows the correlation between R_K and $\mathrm{BR}(\tau \to e\pi\pi)$ resulting from the scan over m_{R_2} and unfixed angles and phases in (4.18). The only remaining parameter $\cos\beta$ enters the low-energy calculations in a product with m_{R_2} and was fixed to the value 0.02. The picture shows a clear tension between the currently measured value of R_K and the limit on $\mathrm{BR}(\tau \to e\pi\pi)$. Note that there is no simple way of overcoming this problem by departing from the Yukawa pattern in Eq. (4.18). Thus, if the current value of R_K is confirmed by more statistics in the future, the FPW model will be ruled out.

In what follows, we consider the limit case which is just on the edge of invalidation by the $\tau \to e\pi^+\pi^-$ search, which allows for

$$m_{R_2}\cos\beta = 30\,\mathrm{GeV}. \tag{4.21}$$

In such a case, a smaller but still potentially significant departure from LFU is possible (with $\mathrm{R}_K = 0.95$). For such a scenario, predictions for many other LFV τ decay channels have been given in Ref. [29]. Most of them, including $\mathrm{BR}(\tau \to e\gamma) = (3\sim 4)\times 10^{-9}$, are going to be probed by the Belle II experiment in the near future [193].

There are also other possible additional signals which might arise from a small departure from Eqs. (4.9) and (4.18), especially the processes changing both $\mathcal{L}_\mu$ and $\mathcal{L}_e$. On the other hand, the LFV decays $\tau \to \mu X$ should not be observed in the near future. In Ref. [29], we have also mentioned possible low-energy signals of the color-octet G_2 which could be another relatively light BSM scalar around.

Collider searches

Let us briefly comment on direct LQ searches. The Yukawa interactions in Eq. (4.18), regardless on the values of the free parameters in there, imply the following relations between the dominant LQ decay channels:

$$\mathrm{BR}(R_2^{+2/3} \to e^+ j_b) \simeq \mathrm{BR}(R_2^{+2/3} \to \tau^+ j_b) \simeq \frac{m_b^2}{2m_\tau^2} \mathrm{BR}(R_2^{+2/3} \to \tau^+ j)\,, \tag{4.22a}$$

$$\mathrm{BR}(R_2^{+5/3} \to e^+ t) \simeq \mathrm{BR}(R_2^{+5/3} \to \tau^+ t) \simeq \frac{m_b^2}{2m_\tau^2} \mathrm{BR}(R_2^{+5/3} \to \tau^+ j)\,, \tag{4.22b}$$

where j denotes a light-quark jet and j_b is a b-jet. The other branching fractions are negligible. Since $m_b^2/2m_\tau^2 \simeq 1.2$ at the 1.5 TeV scale, each of the ratios displayed in (4.22) amounts roughly to 1/3.

Leptoquarks could be produced in pp collisions in pairs via gluon-gluon fusion. Notice that while the low energy precision observables constrain the product $m_{R_2} \cos\beta$, the LQ on-shell pair production is driven by the strong force and depends on m_{R_2} solely. Experimental limits on LQs with several decay modes are generally weaker than those on LQs decaying dominantly into a single final state. Thus, the mass limit found in Ref. [29] reached only up to 900 GeV. Very recently, new searches for scalar LQs have been performed by ATLAS: in Ref. [85], new limit $m_{\mathrm{LQ}} < 1250$ GeV is given for $\mathrm{BR}(R_2^{+2/3} \to eb) \approx 0.31$. The limits from the searches considering the $t\tau$ [86] or te [84] final state are slightly lower. The current limits on the LQ single production are generally weaker.

4.5 Conclusions and discussion

In this chapter, we have studied the capability of two simple quark-lepton symmetry models introduced in Chapter 2 of describing the hints of NP in the neutral current B-meson decays. We have found that the MQLSM is unable to alleviate the tension among theory and experiment in these processes.

For the FPW model, we have found a setup with a light R_2 leptoquark doublet which can *partially* accommodate the deviations in R_K and R_{K^*} but not the additional $b \to s\mu\mu$ anomalies. In such a case, the LFV decays $\tau \to e\pi^+\pi^-$ and $\tau \to e\gamma$ are bound to be observed at BELLE II. As the LQ mass remains unfixed in that setup, disproving the scenario by negative collider LQ searches would be quite hard.

In looking for viable scenarios accommodating $\mathrm{R}_{K^{(*)}}$, we have explored essentially all the parameter space of the FPW model. The only cases which might have escaped our attention are the contrived schemes with a negative interference among various BSM amplitudes.

While we have focused on the neutral-current B anomalies, the R_2 leptoquark has also been suggested as a candidate to account for the charged-current ones [179, 180, 181]. A natural question is whether R_2 could significantly influence both $\mathrm{R}_{K^{(*)}}$ and $\mathrm{R}_{D^{(*)}}$ simultaneously. In this respect, the minimal set of couplings required has been advocated recently in Ref. [160]; it consists in adding two more entries to the Yukawa matrices compared to Eq. (4.6):

$$\hat{Y}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \tilde{y}_{c\tau} \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{Y}_4 = \begin{pmatrix} 0 & 0 & 0 \\ y_{se} & 0 & 0 \\ y_{be} & 0 & y_{b\tau} \end{pmatrix}. \tag{4.23}$$

Notice that this form is quite close to the pattern in Eqs. (4.9) and (4.18) studied in context of the FPW model. Especially, note that a large the non-zero value of $y_{b\tau}$ in (4.18) has been enforced by the extended gauge symmetry and by the requirement of complying the limits on muon number violation. Furthermore, since $\hat{Y}_2$ can be chosen arbitrarily in the FPW model, $\tilde{y}_{c\tau}$ can be simply set to a desired value. On the other hand, as $\hat{Y}_4$ in Eq. (4.18) contains also sizable elements $y_{s\tau}$ and/or $y_{d\tau}$, the predictions and constraints in the FPW model might differ considerably from those in the simplified scenario of Ref. [160]. Addressing both $\mathrm{R}_{K^{(*)}}$ and $\mathrm{R}_{D^{(*)}}$ within the FPW model of quark-lepton unification might be a topic of a future work.

5. Gauge leptoquark in $SU(4)_C$

This chapter presents a yet unpublished study [31] of the low-energy phenomenology of the gauge leptoquark (GLQ) in the models the $SU(4)_C$ gauge symmetry, such as the MQLSM or the FPW model which we have elaborated on in Chapters 2 and 4.

One can verify in Table A.1 that a general interaction of the vector leptoquark (VLQ) field $U_1 \sim (3, 1, +2/3)$ with the SM fermions consistent with the gauge symmetries of the SM can be written as

$$\mathcal{L}_{U_1} = \frac{g_4}{\sqrt{2}} \left(\overline{\hat{d}} \gamma^\mu \left[\mathbb{P}_L U_L + \mathbb{P}_R U_R \right] \hat{e} + \overline{\hat{u}_L} \gamma^\mu V_{\mathrm{CKM}} U_L \mathring{\nu}_L \right) U_{1\mu} + \text{h.c.} \tag{5.1}$$

where U_L and U_R are complex matrices in the flavour space.

As we have already noted in Section 3.5, vector leptoquarks have raised a lot of attention recently, being identified as excellent candidates to account for the neutral-current and potentially also charged-current B-meson anomalies. Nevertheless, the suggested flavour and chirality patterns are not consistent with the gauge nature of the LQ. As explained in Chapter 2, quark-lepton unification (QLU) à la Pati-Salam determines the gauge coupling at the scale of $SU(4)_C$ breaking and restricts the interaction patterns to unitary matrices, i.e.

$$g_4(m_{U_1}) = g_3(m_{U_1}) \qquad \text{and} \qquad U_L, U_R \in U(3). \tag{5.2}$$

In contrast, the current benchmark VLQ setup for acommoddation of the B anomalies is roughly [154]

$$\frac{g_4 U_L}{m_{U_1}} = \frac{1}{2\,\mathrm{TeV}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.05\xi & 0.6 \\ 0 & 0.05/\xi & 0.7 \end{pmatrix}, \qquad \frac{g_4 U_R}{m_{U_1}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{5.3}$$

where ξ is a positive $O(1)$ number. Clearly, such a form is in clash with the conditions (5.2). For this reason, the B anomalies could not be attributed to the GLQ in the two models of QLU in Chapter 4.

Despite its inability to account for the discrepancies in the B-meson decays, we still find the gauge leptoquark in the QLU framework worth a detailed and dedicated study as it is a common feature of many specific models. In Section 5.1, assuming that the B anomalies will eventually disappear or are caused by other BSM fields, we shall analyse the possible near-future first signals[1] of the U_1 field with interactions satisfying (5.2). Many phenomenological studies of the GLQ in quark-lepton unification are available. The novelty of the one presented here consists in simultaneously focusing on two points:

1. Essentially all possible forms of the unitary matrices U_L and U_R are considered, keeping in mind that there is *no prior measure* on the parameter space. In particular, the setups which might be labeled as *fine-tuned scenarios* or *small parts of the parameter space* are not dismissed. To our best knowledge, such a study has only been performed in Ref. [175].

[1] The meaning of *first signal* will be specified later.

2. An attempt is made to consider all observables which might become the first signal of the gauge LQ. To this end, we employ the packages `flavio` [118, 117] and `smelli` [194, 195] which make it possible to easily consider much more processes and measurements than Ref. [175].

Furthermore, in Section 5.2, modest generalizations of the scenario studied in the previous part shall be investigated, consisting in adding 1 or 2 generations of extra leptons to the model. Like in Section 5.1, we will make the key steps towards finding the catalogue of all possible first signals in each of the models considered. Finally, we will discuss the number of extra leptons necessary to significantly alleviate the discrepancies between theory and experiment in the B-meson decays.

In both sections, the other potentially important BSM fields are neglected, such as the scalars studied in Chapter 4 for the case of FPW and MQLS models. Notice, however, that scenarios where the VLQ effects are stronger than those of scalars are quite eligible; for example, consider the scalar sector of the MQLSM (cf. Chapter 2) with $\beta \approx \pi/4$ and rather large λ's. After all, in the electroweak physics, the low-energy effects of the gauge bosons $W^{\pm}$ and Z^0 are much stronger than the effects of the fluctuations of the Higgs field around its ground-state configuration. Neglecting the rest of the BSM spectrum implies that the only relevant free parameters are the LQ mass m_{U_1} and the interaction matrices U_L and U_R.

5.1 Gauge LQ in quark-lepton unification

This section is devoted to the low-energy signals of the gauge LQ in the models with SM quarks and leptons in common representations of the $SU(4)_C$ gauge group, such as the MQLS [27] and the FPW models [28] introduced in Chapter 2 or the Pati-Salam model [26, 70]. Interactions of such a leptoquark satisfy the conditions in (5.2).

As follows from the discussion in Section 3.5, the unitarity of the $U_{L,R}$ matrices implies that the gauge LQ interactions always violate the lepton flavour symmetries. In particular, the condition of column normalization implies that none of the columns can be empty, the $U(1)^2_{\mathrm{LF}}$ symmetry is broken and the LQ inevitably mediates LFV processes. Complementarily, the column orthogonality condition implies violation of $(S_3)_{\mathrm{LFU}}$.

Concerning the Lorentz structure of the GLQ interactions, integrating out the interaction in Eq. (5.1) at the tree level gives rise to the following SMEFT operators (supressing the flavour indices):

$$\mathcal{O}_{ed} = (\overline{e_R}\gamma_\mu e_R)(\overline{d_R}\gamma^\mu d_R)\,, \tag{5.4a}$$

$$\mathcal{O}_{\ell edq} = (\overline{\ell_L} e_R)(\overline{d_R} q_L)\,, \tag{5.4b}$$

$$\mathcal{O}^{(1)}_{\ell q} + \mathcal{O}^{(3)}_{\ell q} = (\overline{\ell_L}\gamma_\mu \ell_L)(\overline{q_L}\gamma^\mu q_L) + (\overline{\ell_L}\gamma_\mu \sigma^I \ell_L)(\overline{q_L}\gamma^\mu \sigma^I q_L)\,, \tag{5.4c}$$

where σ^I represents the Pauli matrices in the $SU(2)_L$ space. With explicit flavour structure, the corresponding Wilson coefficients are given by [182]

$$C_{ed\,\_\bar{l}l\bar{q}q} = -1\frac{g_4^2}{2m_{U_1}^2} u_{q\bar{l}}^{R*}\, u_{\bar{q}l}^{R}\,, \tag{5.5a}$$

$$C_{\ell edq\,\_\bar{l}l\bar{q}q} = +2\frac{g_4^2}{2m_{U_1}^2} u_{q\bar{l}}^{R*}\, u_{\bar{q}l}^{L}\,, \tag{5.5b}$$

$$C^{(1)}_{\ell q\,\_\bar{l}l\bar{q}q} = C^{(3)}_{\ell q\,\_\bar{l}l\bar{q}q} = -\frac{1}{2}\frac{g_4^2}{2m_{U_1}^2} u_{q\bar{l}}^{L*}\, u_{\bar{q}l}^{L}\,, \tag{5.5c}$$

where $u_{ql}^{L,R}$ denotes elements of $U_{L,R}$, respectively, and $q, \bar{q}, l, \bar{l}$ are four independent flavour indices. Recall that we use the basis in which the $T_L^3 = -\frac{1}{2}$ components of fermion doublets are the mass eigenstates.

The tree level matching of the SMEFT coefficients (5.5) to those of the Weak effective theory such as those in (3.9) is straightforward. In particular, notice that that the $\mathcal{O}_{\ell q}^{(1)}$ operator, triggered by products of two elements of U_L, gives rise to the WC's $C_9^{\rm NP} = -C_{10}^{\rm NP}$ which are well suited for addressing the $b \to s\mu\mu$ anomalies [cf. page 51]. However, way more stringent bounds are set on the *scalar-type* operators $\mathcal{O}_{\ell edq}$, which under the EW scale perform as the neutral-current interactions $\mathcal{O}_S - \mathcal{O}_P = \left(\overline{q_R'}\, q_L\right)\left(\overline{l_L'} l_R\right)$ as well as to the charged current operators like $(\overline{q_R}\, u_L)\,(\overline{\nu_L}\, l_R)$, where q denotes a d-type quark. As discussed in detail in Section 3.3, these interactions trigger the purely leptonic meson decays without any chirality suppression. Such processes are measured with an extreme absolute precision [196, 197, 198, 199, 200, 201] or severely limited [98, 202, 203, 204, 205, 106, 206] and hence form important constraints on the couplings and mass of the gauge LQ.

Many studies of VLQ effects have been issued during the last few decades, adopting special patterns of U_L and U_R. Some of them do not take into account the $SU(4)_C$ constraints; for example, Ref. [207] assumed only chiral coupling to the first generation; also [113] considered mostly the *chiral leptoquark*, i.e., the setup in which one of the interaction matrices vanishes. Ref. [108] simply assumed order-one elements in the relevant entries of $U_{L,R}$.

Among the literature which took the constraints (5.2) seriously, Refs. [208, 209] from the early 1980's considered $U_L = U_R = \mathbb{1}$ and found a mass limit $m_{U_1} >$ 310 TeV stemming from the experimental bound on the LFV decay $K_L^0 \to e\mu$. Since then, this limit has been risen up by more precise experimental limits on $\mathrm{BR}(K_L^0 \to e\mu)$ [210, 98] to the incredibly high value of $m_{U_1} > 2000$ TeV [61] for the case with "trivial" quark-lepton mixing.

However, the GLQ can have different phenomenology with different forms of $U_{L,R}$. Valencia and Willenbrock [211] in 1994 considered special cases where $U_L = U_R$ are the permutation matrices, i.e. where each lepton is coupled to a single quark, and studied various two-body meson and tau decays. They found that apart from $K_L^0 \to e\mu$, the gauge LQ mass was for some mixing patterns limited from below to 250 TeV by the test of the SM prediction of the LFU violation

$$\mathrm{R}_{e/\mu}(\pi^+ \to l^+\nu) \;=\; \Gamma(\pi^+ \to e^+\nu)\,/\,\Gamma(\pi^+ \to \mu^+\nu), \tag{5.6}$$

by an analogous observable $\mathrm{R}_{e/\mu}(K^+ \to l^+\nu)$ or by $\mathrm{BR}(B^+ \to e^+\nu)$ which lead to a much weaker bound $m_{U_1} > 13$ TeV.

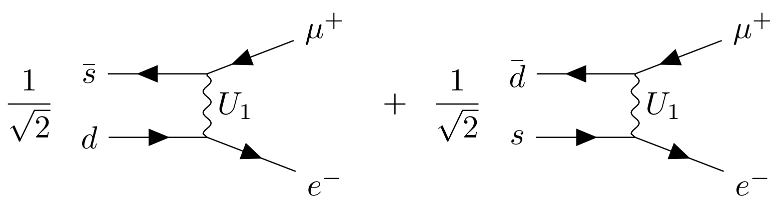

Figure 5.1: Tree-level amplitude for a LFV decay of K_L^0.

Kuznetsov and Mikheev in 1994 [212] considered various (semi)leptonic K and π decays and the $\mu \to e$ conversion on nuclei, and cast inequalities employing the GLQ mass and elements of quark-lepton mixing matrices, virtually taking the full freedom in the quark-lepton mixing into account, but still tacitly assuming $U_L = U_R$. Apart from $\text{BR}(K_L^0 \to e\mu)$ and $\text{R}_{e/\mu}(K^+ \to l^+\nu)$, important bounds have been found to stem also from BR's of $K_L^0 \to l^+l^-$, $K \to \pi\mu e$ and from coherent $\mu \to e$ conversion on Titanium nuclei. Needless to say, both analyses [211] and [212] are outdated nowadays due to the new experimental data.

Ref. [61] considered $K_L^0 \to e\mu$ and $B^0 \to e\tau$ for general $U_{L,R}$ but did not confront the obtained limits with other observations. In Ref. [213], which is the 2012 update of [212], also the recent B factory results on B and τ decays have been included and the general case $U_L \neq U_R$ has been considered. A specific form of U_L and U_R has been found for which the stated GLQ mass limit was as low as 38 TeV. However, as pointed out in Ref. [175], this finding is invalid because the authors forgot to include the predictions for the $\mu^- e^+$ final state when studying $\text{BR}(B^0 \to \mu^\pm e^\mp)$ and $\text{BR}(B_s \to \mu^\pm e^\mp)$.

The correct treatment has been applied in the recent study by Smirnov [175] who considered all kinematically allowed decays $P^0 \to l_1^+ l_2^-$ for $P^0 = K_L^0, B^0, B_s$ and took fully into account the freedom in the fermion mixing by performing a scan. The global lower limit stemming from these processes was found to be

$$m_{U_1} > 86 \text{ TeV}, \tag{5.7}$$

the corresponding forms of U_L and U_R were given as well as detailed predictions for all the considered BR's in that scenario. We have completely recalculated Ref. [175] and verified that the computations in there are correct. Some aspects of the calculations can be found in Section 5.1.1.

Despite that, there is still some work left to be done. Firstly, other important processes like $P^+ \to l^+\nu$, semileptonic meson decays or $\mu \to e$ conversion on nuclei should be taken into account. Secondly, even if considering more bounds did not change the answer (5.7), there are other interesting aspects to be studied. There is no theoretical reason to assume that the quark-lepton mixing matrices really follow the pattern which minimizes the VLQ mass limit, neither is such a setup more interesting from the point of view of the high-energy frontier: 86 TeV is still too heavy to be observed on any of the currently discussed 21$^{\text{th}}$ century colliders.

Hence, rather than giving detailed predictions for the particular point in the parameter space corresponding to the global LQ mass limit (like in [175]), we are interested in listing *all possible first signals* of the GLQ. To this end, we construct a *catalogue of all observables which currently determine the VLQ mass limit for some form of the mixing matrices $U_{L,R}$*. These observables are excellent candidates for future NP signals since even a small improvement in precision will investigate a new part of the parameter space.

In Section 5.1.1, the simplified approach used Ref. [175] is presented, since it has been used as a starting point for our analysis. In Section 5.1.2 some concepts are explained which can be well understood on that level of rigor. Section 5.1.3 presents the advancement in the analysis gained by our approach which is based on the public general-purpose tools `flavio` and `smelli`. Postopning the technical details about the scanning of the parameter space to Appendix C, we present the results in Section 5.1.4.

5.1.1 Smirnov's approach

In this part, let us overview the strategy adopted from Ref. [175] which has been applied as the first stage of our analysis. Especially, we want to point at the following aspects of the investigation:

1. The effects of the U_1 leptoquark are taken into account at the tree level.

2. Four-loop QCD running of the induced effective operators is taken into account [214]. For simplicity, the effective operators are defined at the scale 100 TeV regardless of the considered LQ mass.

3. SM contributions to the considered processes are completely neglected in the calculation. To highlight this approximation, the predictions for branching ratios are labelled as BR_V. The measured BR's of the observed decays ($K_L^0 \to ee$, $K_L^0 \to \mu\mu$, $B_s^0 \to \mu\mu$) are taken as limits on BR_V. Such a rough approximation is meaningful due to large relative theoretical uncertainties for the SM amplitudes.

4. Ref. [175] has taken into account the branching ratios of $P \to l^\pm l'^\mp$ decays where $P = K_L^0, B^0, B_s^0$ and ll' corresponds to various kinematically allowed combinations of leptons and antileptons. In this work, also the leptonic decays of K_S^0 are considered and updated limits on $B_{d,s}^0 \to e\mu$ [206] are taken into account.

5. No processes with neutrinos are analyzed; therefore, the study holds for both situations with light or heavy right-handed neutrinos.

6. The masses of electrons and muons in the final state are neglected, as well as the indirect CP violation in the neutral kaon mass eigenstates.

7. The VLQ mass limits for given $U_{L,R}$ are determined as the maximum of individual limits obtained from the considered observables. The decay responsible for the strongest limit is considered to be the candidate for the *first future signal* of the LQ for the investigated form of the quark-lepton mixing matrices.

The branching ratio for a process with light leptons only is calculated by the following formula:

$$\mathrm{BR}_V(P \to l^+ l'^-) = \frac{m_P \pi \alpha_s^2 f_P^2 \overline{m}_P^2 (R_P^V)^2}{2 m_{U_1}^4 \Gamma_P^{\mathrm{tot}}} \, \beta_{P,ll'}^2 \, . \tag{5.8}$$

The formfactors are $f_K = 155.72\,\text{MeV}$, $f_{B^0} = 190.9\,\text{MeV}$, $f_{B_s^0} = 227.2\,\text{MeV}$ and the gluonic corrections to the pseudoscalar quark currents amount to $R_K^V = 3.47$ and $R_B^V = 2.1$ [214]. The lepton-flavour-dependent factor is a sum over two different helicity combinations

$$\beta^2_{P,ll'} = \frac{|a_{LR}(P,l,l')|^2 + |a_{RL}(P,l,l')|^2}{2} \tag{5.9}$$

where for weak eigenstates

$$a_{LR}(P,l,l') = u^L_{\bar{q}l}\, u^{R\,*}_{ql'}, \qquad\qquad a_{RL}(P,l,l') = u^R_{\bar{q}l}\, u^{L\,*}_{ql'}, \tag{5.10}$$

with $\bar{q}$ and q standing for (the index of) the valence antiquark and quark of P, respectively. For CP eigenstates,

$$a(K^0_{L,S},l,l') = \frac{a(K^0,l,l') \pm a(\bar{K}^0,l,l')}{\sqrt{2}}, \tag{5.11}$$

where $+$ and $-$ relate to K^0_L and K^0_S, respectively, and a stands for either a_{LR} or a_{RL}.

For processes with a single τ-lepton in the final state, the expression for BR_V in Eq. (5.8) must be multiplied by a phase space factor $(1-m_\tau^2/m_P^2)^2$. Along with that, the substitution $u^{L,R}_{r\tau} \to \left[u^{L,R}_{r\tau} - u^{R,L}_{r\tau} m_\tau/(2\overline{m}_P R_P^V)\right]$ for $r = q, \bar{q}$ is applied in Eq. (5.10). For two τ-leptons in the final state, see Ref. [40]. The formulae above can be obtained by matching the SMEFT Wilson coefficients (5.5) to those of the WET, plugging them into the general formulae from Section 3.3 and neglecting m_e and m_μ.

To review the CP properties of the predictions, notice that $a_{RL}(P,l,l') = a_{LR}(\bar{P},l',l)^*$ always holds for the weak eigenstates P. This implies $\beta^2_{P,ll'} = \beta^2_{\bar{P},l'l}$ and therefore the predictions are CP symmetric, i.e. $\text{BR}_V(P \to l^+l'^-) = \text{BR}_V(\bar{P} \to l'^+l^-)$, even if the entries in $U_{L,R}$ are complex. This corresponds to the fact that we consider a single Feynman graph for each amplitude and there is thus no room for interference. Accordingly, $\text{BR}_V(P \to l^\pm l'^\mp) = \text{BR}_V(\bar{P} \to l^\pm l'^\mp)$ and $K^0_L = \overline{K^0_L}$ implies $\text{BR}_V(K^0_L \to l^+l'^-) = \text{BR}_V(K^0_L \to l^-l'^+)$.

5.1.2 Dominance of K^0_L limits, subdeterminants and maximal non-unitarity of the leptoquark interactions

As pointed out in [175], the experimental limits on BR's of $K^0_L \to e^\pm\mu^\mp$ and $K^0_L \to e^+e^-$ and the measured value of $\text{BR}(K^0_L \to \mu^+\mu^-)$ put far more stringent constraints on m_{U_1} than those from the decays of the $B^0_{d,s}$ mesons. Naively, one could say that the K^0_L decays determine the limits on m_{U_1} in the "vast majority of the parameter space". For an illustration see Fig. 5.2.

Nevertheless, we are interested especially in the "special" cases where the strongest constraints on m_{U_1} are set by other processes than $K^0_L \to ll'$. In order to efficiently scan over this interesting part of the parameter space, it is both convenient and sufficient to investigate the hypersurface in the parameter space where the VLQ does not give *any* contribution to $K^0_L \to ll'$, i.e.,

$$\beta^2_{K^0_L,11} = \beta^2_{K^0_L,12} = \beta^2_{K^0_L,21} = \beta^2_{K^0_L,22} = 0. \tag{5.12}$$

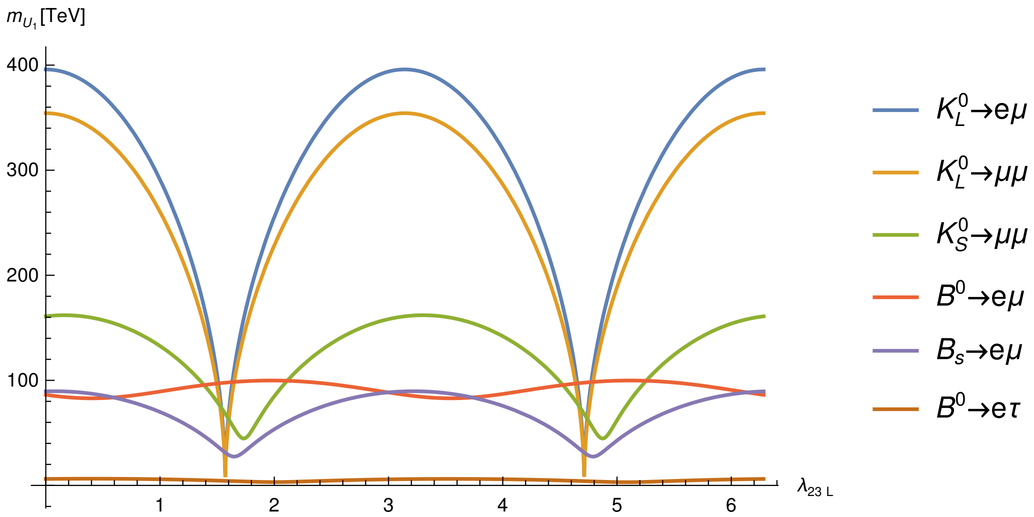

Figure 5.2: Illustration of lower limits on the gauge LQ mass stemming from several observables, calculated along a one-dimensional subset of the mixing parameter space, defined by Table C.1 in Appendix C. In this slice, the most important bounds are given by $\mathrm{BR}(K_L^0 \to e^\pm \mu^\mp)$ and $\mathrm{BR}(B^0 \to e^\pm \mu^\mp)$. The mass limits are obtained in using the approach described in Section 5.1.1.

In Ref. [175], these equations have been solved for a particular parametrization of $U_{L,R}$. Here, we shall investigate them without an explicit parametrization in order to get some more insight which will be useful also in the next section.

Since $a_{LR}(K_L^0, l, l') = a_{RL}(K_L^0, l', l)^*$, Eqs. (5.12) are equivalent to

$$a_{LR}(K_L^0, l, l') = 0 \qquad \text{for} \qquad (ll') = (ee), (e\mu), (\mu e), (\mu\mu). \tag{5.13}$$

Interestingly enough, these four equations can be written as

$$\begin{pmatrix} u_{22}^R & u_{12}^R \\ u_{21}^R & u_{11}^R \end{pmatrix}^* \begin{pmatrix} u_{11}^L & u_{12}^L \\ u_{21}^L & u_{22}^L \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{5.14}$$

For U_R fixed, (5.14) represents two pairs of linear homogeneous equations for elements of U_L. The trivial solution $u_{11}^L = u_{12}^L = u_{21}^L = u_{22}^L = 0$ is in conflict with the unitarity of the 3×3 matrix U_L. Existence of nontrivial solutions requires that the determinant of the coefficient matrix must be zero. Apparently, this determinant is identical to the 3, 3-minor (subdeterminant) of U_R. Analogously, existence of nontrivial solutions for elements of U_R requires that the second matrix in (5.14) is singular. Hence, preventing the U_1 leptoquark from mediating $K_L^0 \to l^+ l'^-$ decays necessitates that the determinants of the upper left 2×2 submatrices of *both* U_R and U_L vanish.

Furthermore, absolute values of determinants of two complementary submatrices of any unitary matrix are the same. Therefore, the subdeterminants of our interest are equal in magnitude to the 3, 3−elements of U_R and U_L, respectively. Hence, we conclude that a necessary condition for avoiding the $K_L^0 \to l^+ l'^-$ decays via VLQ in the quark-lepton symmetry models reads

$$u_{b\tau}^R = u_{b\tau}^L = 0, \tag{5.15}$$

with an implication mentioned also in [175]: $\mathrm{BR}_V(B_{d,s}^0 \to \tau\tau) = 0$.

Notice that the "solutions" to the B anomalies which leave out the unitarity constrains on $U_{L,R}$, such as the one in Eq. (5.3), usually assume that the $b\tau$ elements are the largest ones in order to address also $\mathrm{R}_{D^{(*)}}$. As we can see, such a setting in a certain sense violates the unitarity of the mixing matrix in maximal possible amount.

For solutions of Eq. (5.14) in a specific unitary parametrization of $U_{L,R}$ and technical details about scanning over the resulting parameter subspace, see Appendix C.

5.1.3 A more robust approach

In paralell with the approach introduced in Section 5.1.1, we have also performed a similar analysis using the family of general-purpose open-source tools `wilson` [104, 215], `flavio` [118, 117] and `smelli` [194, 195]. We present the features of this approach as a list which can be compared with that on page 71.

1. Like in the previous approach, we match the VLQ interactions onto SMEFT at the tree level. We have implemented a python function taking $U_{L,R}$ and m_{U_1} as input arguments and returning a dictionary of SMEFT Wilson coefficients given by Eq. (5.5) in the format compatible with the `wcxf` standard [40, 120], which is used by the packages mentioned above.

2. The RGE running of the SMEFT effective operators from the scale $\mu = m_{U_1}$ to the EW scale, tree-level matching onto WET and further evolution to the meson mass energy scales is handled automatically by the `wilson` package. The full numerical solution to the one-loop SMEFT RGEs (the `'integrate'` option) is performed since we have exemplified that the `'leadinglog'` approximation leads to $O(1)$ relative differences in certain predictions. Analytical solution to the one-loop QCD and QED running is applied under the electroweak scale in `wilson`. For more details see [104] and references therein.

3. The SM contributions to the amplitudes of the calculated processes are automatically taken into account by `flavio`. As a result of this (and of the RGE running), the predictions do not scale uniformly as $m_{U_1}^{-4}$, which was a simplifying feature of the previous approach [see Eq. (5.8)].

4. The global likelihood tool `smelli` is employed. This package uses `flavio` for predictions and confronts them with the measurements, including the correlations. By default, version `2.2.0` of `smelli` takes into account hundreds of observables, most of which are, however, irrelevant for our scenarios. On the other hand, the very interesting processes $\mathrm{BR}(B^0_{d,s} \to e^+e^-)$ as well as $\mu \to e$ conversion on nuclei were not included. To this end, we have modified the package to regard also these observables. As already mentioned on page 45, we have found and fixed an important bug in the calculation of $K^0_{L,S} \to e^\pm\mu^\mp$ decays in `flavio v2.2.0` [119]. The complete list of considered observables can be found in Table C.3 on page 99.

5. As tacitly assumed during the entire Chapter 5, no light right-handed neutrinos are taken into account. Since the ν_R fields are mandatory in quark-lepton unification, the analysis, strictly speaking, holds only for the case with heavy ν_R

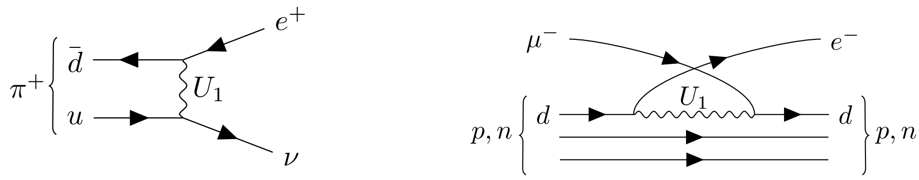

Figure 5.3: Various important processes mediated by the gauge leptoquark.

(e.g., for the FPW model). In the case of light Dirac neutrinos (e.g., MQLSM), new channels like $B^- \to e_L^- \overline{\nu_R}$ open, and the constraints on the gauge LQ are generally slightly stronger. Extending the packages `flavio` and `wilson` in order to take into account also the effective operators containing ν_R would be far beyond the scope of this work.

6. Light lepton masses are taken into account in `flavio` for all observables, but indirect CPV in neutral kaons remains neglected in the $K^0_{L,S} \to ll'$ decays.

7. For U_L and U_R fixed, we determine the *first signal* by finding a LQ mass for which some of the following conditions are fulfilled:

   (a) In the list of individual observables (neglecting all correlations) with a pull $\Delta\chi^2/2 > 2.5$ between the current prediction and experiment there is a *single observable* which is absent in the analogous list of observables with a pull $\Delta\chi^2/2 > 2.0$ comparing experiment with SM. This observable is then taken as a potential *future first signal*.

   (b) The global log-likelihood of the situation worsens by $3.5 \sim 5$ units compared to the log-likelihood of the SM. In such a case, the possible *future first signal* is identified with the observable whose contribution to the likelihood changed most significantly compared to the SM.

   (c) The global log-likelihood of the situation improves at least by 4 units compared to the SM. In such a case, the observable (or set of observables) contributing most to the improvement is considered as a possible *current signal of the LQ*.

   In all cases, the contributions of individual observable measurements to the global likelihood are obtained via the `obstable` method provided by the `smelli` package [194].

   Admittedly, these criteria seem to be chosen slightly arbitrarily. Nevertheless, we have exemplified that they lead to the same results as the more intuitive criterion in paragraph 7 of Section 5.1.1 whenever the other differences among the two methods (paragraphs 2 – 6) are irrelevant.

5.1.4 Results

We have performed a scan over the space of pairs of unitary matrices U_L and U_R. For each investigated point in this space, the limiting LQ mass has been found, along with the predictions for various observables based on that mass. For technical details about the scan see Appendix C.

Observable	Experiment		SM prediction	
$\mathrm{BR}(K_L^0 \to e^\pm\mu^\mp)$	$< 4.7 \times 10^{-12}$	[98]	0	
$\mathrm{BR}(K_L^0 \to e^+e^-)$	$8.7^{+5.7}_{-4.1} \times 10^{-12}$	[198]	$(9.0 \pm 0.5) \times 10^{-12}$	[216, 217]
$\mathrm{BR}(K_L^0 \to \mu^+\mu^-)$	$(6.84 \pm 0.11) \times 10^{-9}$	[54]	$(7.4 \pm 1.3) \times 10^{-9}$	
$\mathrm{BR}(K_S^0 \to \mu^+\mu^-)$	$< 2.1 \times 10^{-10}$	[205]	$(5.2 \pm 1.5) \times 10^{-12}$	[218]
$\mathrm{BR}(B^0 \to e^\pm\mu^\mp)$	$< 1.0 \times 10^{-9}$	[204]	0	
$\mathrm{BR}(B_s \to e^\pm\mu^\mp)$	$< 5.4 \times 10^{-9}$	[204]	0	
$\mathrm{BR}(B^0 \to \mu^+\mu^-)$	$1.1^{+1.4}_{-1.3} \times 10^{-10}$	[54]	$(1.1 \pm 0.1) \times 10^{-10}$	
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	$(3.0 \pm 0.4) \times 10^{-9}$	[54]	$(3.7 \pm 0.2) \times 10^{-9}$	
$\mathrm{CR}(\mu \to e, \mathrm{Au})$	$< 7 \times 10^{-13}$	[219]	0	
$\mathrm{R}_{e/\mu}(\pi^+ \to l^+\nu)$	$1.2327(23) \times 10^{-4}$	[54]	$1.2352(1) \times 10^{-4}$	[220]
$\mathrm{R}_{e/\mu}(K^+ \to l^+\nu)$	$2.488(9) \times 10^{-5}$	[54]	$2.476(2) \times 10^{-5}$	

Table 5.1: Complete list of observables which currently constrain the gauge LQ mass for some form of unitary quark-lepton mixing matrices. The experimental limits are given at 90% C.L. The SM predictions have been calculated in `flavio` unless cited.

As stated earlier in Eq. (5.7), the *simplified approach* of Ref. [175] described in Section 5.1.1 leads to the global lower leptoquark mass limit of 86 TeV. When taking into account more observables in the *more robust approach* presented in Section 5.1.3, the experimental limit on the $\mu \to e$ coherent conversion on nuclei, $\mathrm{CR}(\mu \to e, \mathrm{Au})$, turns out to be violated by 3 orders of magnitude by the predictions based on the benchmark point of Ref. [175] which is supposed to saturate the global limit on m_{U_1}. Nevertheless, we have found another form of U_L and U_R which allows for a similarly light gauge LQ even when all the constraints included in `smelli` are considered. Thus, the global lower mass limit is indeed given by Eq. (5.7).

Table 5.1 presents the catalogue of observables which currently give the largest constraint on the GLQ mass for some configuration of U_L and U_R. (These observables correspond to the *future first signals* as defined above.) For each of these processes, an example of the corresponding $U_{L,R}$ is given in Appendix C.2. We would like to stress that this list is *complete*. To fully appreciate the result, notice that even a very small improvement in precision of any experimental limit listed in Table 5.1 will probe a so-far allowed part of the parameter space of the model, and could potentially detect a NP signal. (The only exception is the observed decay $K_L^0 \to \mu\mu$ for which the SM uncertainties dominate.) Conversely, under a very idealized assumption that the experimental sensitivity will grow uniformly for all the observables considered, neither of the other processes is predicted to be the first seen signal of the GLQ. More realistically, the precision of the measurement of (or search for) any other observable needs to be improved by a larger step in order to put a new constraint on the model parameters or to have a theoretical chance of observing a signal of the gauge LQ. How large these steps must be is shown for several important examples in Table 5.2.

Concerning searches for LFV, Table 5.1 contains limits on $K_L^0, B_{d,s}^0 \to e\mu$ and on the $\mu \to e$ coherent conversion on nuclei; further searches for these processes are therefore of great interest.

The remaining observables in Table 5.1 all employ the leptonic decays of pseudoscalar mesons which are chirality suppressed in the SM and can be understood

Observable	Exp. limit		Model prediction	SM	
$\mathrm{BR}(K_S^0 \to e^+e^-)$	$< 9 \times 10^{-9}$	[203]	$\leq 2 \times 10^{-9}$	2×10^{-14}	[203]
$\mathrm{BR}(K_S^0 \to e^\pm\mu^\mp)$	N/A	[54]	$\leq 3 \times 10^{-10}$	0	
$\mathrm{BR}(B^0 \to e^+e^-)$	$< 2.5 \times 10^{-9}$	[106]	$\leq 1.1 \times 10^{-10}$	3×10^{-15}	[221]
$\mathrm{BR}(B_s \to e^+e^-)$	$< 9.4 \times 10^{-9}$	[106]	$\leq 3 \times 10^{-9}$	9×10^{-14}	[221]
$\mathrm{BR}(B^0 \to e^\pm\tau^\mp)$	$< 2.8 \times 10^{-5}$	[202]	$\leq 6 \times 10^{-9}$	0	
$\mathrm{BR}(B_s \to e^\pm\tau^\mp)$	N/A	[54]	$\leq 2.5 \times 10^{-9}$	0	
$\mathrm{BR}(B^0 \to \mu^\pm\tau^\mp)$	$< 1.2 \times 10^{-5}$	[206]	$\leq 5 \times 10^{-9}$	0	
$\mathrm{BR}(B_s \to \mu^\pm\tau^\mp)$	$< 3.4 \times 10^{-5}$	[206]	$\leq 2.3 \times 10^{-9}$	0	
$\mathrm{BR}(B^0 \to \tau^+\tau^-)$	$< 1.6 \times 10^{-3}$	[222]	2×10^{-8}	2×10^{-8}	[221]
$\mathrm{BR}(B_s \to \tau^+\tau^-)$	$< 5.2 \times 10^{-3}$	[222]	8×10^{-7}	8×10^{-7}	[221]

Table 5.2: Examples of processes which are not listed in Table 5.1. The third column shows predictions stemming from the situations (defined by U_L, U_R and m_{U_1}) which are fully compatible with all the current experiments, obtained by the numerical scan. We also list the SM predictions for comparison.

as *tests of LFUV in the SM.*

Firstly, significant deviations could arise in the ratios of charged current decays $R_{e/\mu}(P^+ \to l\nu)$ with $P = \pi, K$ when the LQ couples mostly to the electrons. Although the decay widths involved cannot be measured with an absolute precision similar to the rare decays above, the deviations from the SM can be significant due to the interference among the NP and SM amplitudes (and subdominantly also due to including the other neutrino flavours).

Secondly, limits on m_{U_1} stem also from the observed BRs of $K_L^0 \to ee, \mu\mu$ and $B_{d,s}^0 \to \mu\mu$. Concerning $K_L^0 \to \mu\mu$, the experimental precision is better than the theoretical error estimates in the SM stemming from long distance contributions, as briefly discussed in Section 3.3.

Finally, a very interesting limit on the GLQ mass for some patterns of quark-lepton mixing is set by the recent LHCb search for $K_S^0 \to \mu\mu$; the anticipated discovery of this decay after the upcoming LHC runs thus provides an exciting opportunity for the Pati-Salam-type leptoquark.

In Table 5.2, the $P^0 \to ll'$ decays that currently do not pose the largest bound on m_{U_1} are listed, together with the predictions from the parameters fully compatible with all the current experimental searches. All parameter values studied during the scanning are included.

As τ leptons are generally experimentally hard to handle, all process involving τ's belong to this category. In fact, $3 \sim 4$ orders of magnitude improvements in limits on $B_{d,s}^0 \to l\tau$ would be necessary in order compete with the other constraints, which is far under the prospected sensitivity of Belle II [193] and hardly achievable even at LHCb at the high-luminosity phase. Furthermore, as discussed in Section 5.1.2, due to the unitarity of $U_{L,R}$, the LQ amplitudes mediating of $B_{d,s}^0 \to \tau\tau$ are severely limited by the probes of $K_L^0 \to ll'$ and our predictions for the former essentially coincide with the SM. Hence, the prospected sensitivity of Belle II about 10^{-6} for $\mathrm{BR}(B^0 \to \tau\tau)$ [223] is not sufficient.

On the other hand, the experimental sensitivities to K_S^0, B^0 and B_s decays to e^+e^- require less than 1 order of magnitude improvement in order to probe the currently unexplored parts of the parameter space. Note that $\mathrm{BR}_V(B_{d,s}^0 \to$

$e^+e^-) = \mathrm{BR}_V(B^0_{d,s} \to \mu^+\mu^-)$ is predicted for any parameter point for which $\mathrm{BR}_V(K^0_L \to ll') = 0$ [175]; currently, the muonic channel is measured more accurately. However, when further searches for NP in the $P^0 \to \mu^+\mu^-$ decays will become limited by the SM uncertainties, new searches for $B^0_{d,s}, K^0_S \to ee$ will become essential.

No experimental limits on the decay $K^0_S \to e\mu$ are available [54]. Based on the current limits on $K^0_S \to ee$ [203] and $K^0_S \to \mu\mu$ [205], the required experimental sensitivity around 10^{-10} for $K^0_S \to e\mu$ should be reachable by KLOE II or LHCb.

5.2 $SU(4)_C$ models with extra leptons

The second part of Chapter 5 is devoted to more complicated models featuring the vector LQ. Although they could be considered as aesthetically less appealing, such models have been studied thoroughly in the recent years, mainly due to the attempts to accommodate the B-meson anomalies, as mentioned at the beginning of this chapter. Generally, several tricks to circumvent the theoretical requirement of unitarity of $U_{L,R}$ have been suggested in the literature. They can be divided into three categories, according to the paradigm abandoned:

1. Adding extra generations of fermions while maintaining the gauge symmetry group G_{421} or G_{422} [170, 172].

2. Assuming more complicated gauge structure. Especially, the models based on the $G_{4N21} = SU(4)_{C_L} \times SU(N)_{C_R} \times SU(2)_L \times U(1)_R$ gauge symmetry become popular; here $N = 3$ or 4 and the QCD generators are given by $T^A_C = T^A_{C_L} + T^A_{C_R}$ for $A = 1, \dots, 8$.

   In the basic setting of *chiral quark-lepton symmetry* [62], where left-handed fermions are charged by $SU(4)_{C_L}$ while the right-handed ones transform non-trivially under $SU(N)_{C_R}$, the U_1^μ field interacting with the left-handed quark-lepton currents is a *chiral leptoquark* – it has no or suppressed couplings to the right-handed currents, avoiding the scalar-type effective operators $\mathcal{O}_{\ell edq}$ which are responsible for all the most stringent limits in Table 5.1.

   In more general cases with $N = 3$, some quark and lepton fields are unified within the $SU(4)$ factor while others live in separate irreps of $SU(3)$ [59]. Usually, more than 3 generations of fermions are considered in total [224, 56, 57, 58, 60].

3. Assuming that the vector LQ is not a gauge field but a composite resonance formed by some more fundamental strongly interacting fields [162, 225, 226].

This work is focusing solely on the first option. Since the SM leptons do not entirely stem from the same $SU(4)_C$ representations as the quarks, we shall not use the term *quark-lepton unification* for these theories but rather call them *extended $SU(4)_C$ models*.

5.2.1 Specification of the models

Like in the previously considered $SU(4)_C \times SU(2)_L \times U(1)_R$ scenarios, the models contain 3 generations of each of the following chiral fermion $SU(4)_C$ quadruplets:

$$F_{L(4,2,0)} = \begin{pmatrix} q_L \\ {}^4\ell_L \end{pmatrix} , \qquad f^u_{R\,(4,1,+1/2)} = \begin{pmatrix} u_R \\ \nu_R \end{pmatrix} , \qquad f^d_{R\,(4,1,-1/2)} = \begin{pmatrix} d_R \\ {}^4e_R \end{pmatrix} , \tag{5.16}$$

Notice that we have slightly updated the notation by adding an "isotopic index" to the leptons living inside the quadruplets. On top of that, k_L generations of $SU(2)_L$-doublet vector-like fermions

$${}^1\ell_{L(1,2,+1/2)} + {}^1\ell_{R(1,2,+1/2)} \tag{5.17}$$

and k_R generations of weak-singlet vector-like fermions

$${}^1e_{L(1,1,-1)} + {}^1e_{R(1,1,-1)} , \tag{5.18}$$

are assumed. These new fields are intact to the gauge LQ interactions. After the $G_{421} \to G_{\rm SM}$ symmetry breaking, they can mix with the leptons from the quadruplets. We assume that the 3 lightest eigenstates correspond to e, μ, τ, while the $k_L + k_R$ remaining ones are are too heavy to be observed. As the weak hypercharges of the 3 known leptons are quite precisely measured, they must be composed solely from the fields ${}^1e_R, {}^4e_R, {}^1\ell_L$ and ${}^4\ell_L$. For all practical purposes, it is sufficient to assume the following mixing pattern in the charged lepton sector:

$$\begin{pmatrix} \hat{e}_R \\ E_R \\ {}^1\ell_R^{\,-} \end{pmatrix} = \begin{pmatrix} \multicolumn{2}{c}{V^e_R} & 0_{3\times k_L} \\ & & 0_{k_R\times k_L} \\ 0_{k_L\times 3} & 0_{k_L\times k_R} & \mathbb{1}_{k_L\times k_L} \end{pmatrix} \begin{pmatrix} {}^4e_R \\ {}^1e_R \\ {}^1\ell_R^{\,-} \end{pmatrix} , \tag{5.19}$$

$$\begin{pmatrix} \hat{e}_L \\ E_L \\ {}^1e_L \end{pmatrix} = \begin{pmatrix} \multicolumn{2}{c}{V^e_L} & 0_{3\times k_R} \\ & & 0_{k_L\times k_R} \\ 0_{k_R\times 3} & 0_{k_R\times k_L} & \mathbb{1}_{k_R\times k_R} \end{pmatrix} \begin{pmatrix} {}^4\ell_L^{\,-} \\ {}^1\ell_L^{\,-} \\ {}^1e_L \end{pmatrix} , \tag{5.20}$$

where, generally, ℓ^- denotes the electrically charged component an ℓ doublet[2], $\hat{e} = \hat{e}_L + \hat{e}_R$ is the triplet of light leptons while E_R and ${}^1\ell_R^{\,-}$ with their chiral counterparts E_L and ${}^1e_L$ form the heavy mass eigenstates. The form of the mixing in the heavy lepton sector is irrelevant for our considerations. Finally, V^e_L and V^e_R are arbitrary unitary matrices of dimension $3 + k_L$ and $3 + k_R$, respectively. Including the "non-standard" fields ${}^1\ell_R$ and ${}^1e_L$ in the model ensures the ABJ anomaly cancellation and enables one to write down arbitrarily large Dirac mass terms for the vector-like pairs.

The $Q = 0$ components of ${}^4\ell_L$ and ${}^1\ell_L$ naturally follow their charged $SU(2)_L$ partners during the mixing at the first stage of SSB: those belonging to E_L become equally heavy while the companions of $\hat{e}_L$ become the light neutrinos, eventually gaining mass after the EWSB.

Finally, let us have a look at the gauge LQ interactions. Like in Section 5.1, we assume that ν_R are heavy due to the inverse seesaw which implies that the

[2] Notice that ${}^1e_L \neq {}^1\ell_L^{\,-}$ and ${}^1e_R \neq {}^1\ell_R^{\,-}$.

U_1 leptoquark interactions from $\overline{f_R^u} i \not{D} f_R^u$ are unimportant for the low-energy phenomenology. Interactions with the other fermions can be rewritten as follows:

$$
\begin{aligned}
\mathcal{L} &= \frac{g_4}{\sqrt{2}} \left(\overline{q_L}\, \gamma^\mu\, {}^4\ell_L + \overline{d_R}\, \gamma^\mu\, {}^4 e_R \right) U_{1\mu} + \text{h.c.} \\
&= \frac{g_4}{\sqrt{2}} \Bigg[\begin{pmatrix} \overline{\hat{q}_L} & 0 \end{pmatrix} \gamma^\mu \begin{pmatrix} V_L^d & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} V_L^e \end{pmatrix}^\dagger \begin{pmatrix} \hat{\ell}_L \\ L_L \end{pmatrix} \\
&\qquad + \begin{pmatrix} \overline{\hat{d}_R} & 0 \end{pmatrix} \gamma^\mu \begin{pmatrix} V_R^d & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} V_R^e \end{pmatrix}^\dagger \begin{pmatrix} \hat{e}_R \\ E_R \end{pmatrix} \Bigg] U_{1\mu} + \text{h.c.} \\
&= \frac{g_4}{\sqrt{2}} \left[\begin{pmatrix} \overline{\hat{q}_L} & 0 \end{pmatrix} \gamma^\mu \begin{pmatrix} U_L \end{pmatrix} \begin{pmatrix} \hat{\ell}_L \\ L_L \end{pmatrix} + \begin{pmatrix} \overline{\hat{d}_R} & 0 \end{pmatrix} \gamma^\mu \begin{pmatrix} U_R \end{pmatrix} \begin{pmatrix} \hat{e}_R \\ E_R \end{pmatrix} \right] U_{1\mu} + \text{h.c.}
\end{aligned}
\tag{5.21}
$$

The L_L field on the last line is the heavy $SU(2)_L$ doublet containing E_L as a component. Apparently, the novelty of such extended $SU(4)_C$ models consists in the fact that the unitary matrices $U_{L,R}$, defined by the last line of Eq. (5.21), are of dimension $3 + k_{L,R}$. Using the block-form notation,

$$
U_L = \begin{pmatrix} (U_L)_{ff} & (U_L)_{fF} \\ (U_L)_{Ff} & (U_L)_{FF} \end{pmatrix}, \qquad U_R = \begin{pmatrix} (U_R)_{ff} & (U_R)_{fF} \\ (U_R)_{Ff} & (U_R)_{FF} \end{pmatrix}, \tag{5.22}
$$

only the 3×3 submatrices $(U_{L,R})_{ff}$ are relevant for the interactions among the SM fermions. The larger the numbers $k_{L,R}$ of extra lepton generations, the more parametric freedom in $(U_{L,R})_{ff}$ is available. With $k_L = k_R = 3$, the arbitrariness in U_L, U_R and m_{U_1} already allows to choose *any* form of $\frac{g_4}{m_{U_1}}(U_{L,R})_{ff}$, which is all that is relevant for the low-energy phenomenology at the leading order [cf. Eq. (5.5)].

Similar models have already been studied in the literature, usually considering the cases equivalent to $(k_L, k_R) = (3, 0)$ [73], $(0, 3)$ [172] or $(3, 3)$ [170]. In this work, we focus on the more economical models with $k_{L,R} < 3$, which are less challenging if one aims to capture all the possible NP signals in the model, but more restrictive if parameters leading to a chosen signal (such as the $b \to s\mu\mu$ anomalies) are searched for.

Note that enlarging the dimension of $U_{L,R}$ is indeed the only considered consequence of extending the theory of quark-lepton unification in Section 5.1: we assume that the extra leptons are too heavy to be observed and, like in the first part of this chapter, we ignore the details of the scalar sector responsible for the mixing. Note, however, that a construction of the scalar sector leading to a chosen form of $U_{L,R}$ is not a trivial task (see, e.g. [57]).

5.2.2 A note on the Z' boson

Although one can simply assume that the low-energy effects of the gauge LQ are stronger than those of the BSM Yukawas, another heavy field should be cared of more deeply: the Z' boson. Recall from Section 2.5 that Z' can never be much heavier than U_1 as it gains mass only after the $U(1)_{[B-L]}$ breaking which is a subgroup of $SU(4)_C$, and that the Z' coupling [see Eq. (2.30)] is proportional to $[B-L] - 2Y \sin^2\theta'$.

In the models of QLU, the Z' has flavour-diagonal interactions with both leptons and quarks. With the $SU(4)_C$ breaking scale about 100 TeV or higher, the corresponding flavour-conserving 4-fermion operators are safely negligible. On the other hand, in what follows we are going to study also the situations with a much lighter LQ, i.e., with a lower $SU(4)_C$ breaking scale.

Lepton-flavour conserving effective semileptonic interactions mediated by Z' could interfere with the SM amplitudes in the $q\overline{q} \xrightarrow{Z^*,Z'^*} l^+l^-$ production in the $s \gg m_Z$ kinematic region. NP contributions to these processes are constrained by the high-p_T dilepton spectra measurements by ATLAS and CMS, leading to limits around $m_{Z'} < 5$ TeV (depending on the Z' coupling assumed) [227, 228]. As noted in Ref. [59], these limits also indirectly constrain the mass of the gauge LQ.

Ref. [59] further claims that "*the couplings of the Z' to SM fermions are necessarily flavour universal*" and "*proportional to the identity matrix in flavour space*" even in the models with extra fermions because the relevant charged lepton mixing *"necessarily involve states with the same $B-L$ charge"*. This is, however, a misconception arising from not-distinguishing between the gauge symmetry generator $[B-L]$ and the difference of the accidental global symmetries $\mathcal{B}-\mathcal{L}$, as we have discussed in detail in Section 2.4. All the fermionic fields ${}^4\ell_L$, ${}^4e_R$, ${}^1\ell_{L,R}$, ${}^1e_{L,R}$ are fully justified to be called *leptons* and carry the lepton number $\mathcal{L}$, which is conserved by the gauge interactions. On the other hand, only the fields ${}^4\ell_L$ and ${}^4e_L$, which stem from $SU(4)_C$ quadruplets, are also charged with respect to $[B-L]$, the diagonal generator of the $SU(4)_C$ group. As a consequence of this, applying flavour rotations on the left-handed $([B-L]-2Y\sin^2\theta')$ lepton currents yields

$$\begin{aligned}
&\begin{pmatrix}\overline{{}^4\ell_L} & \overline{{}^1\ell_L}\end{pmatrix}\begin{pmatrix}-1+\sin^2\theta' & 0\\ 0 & \sin^2\theta'\end{pmatrix}\gamma^\mu\begin{pmatrix}{}^4\ell_L\\ {}^1\ell_L\end{pmatrix}\\
&\qquad = \begin{pmatrix}\overline{\hat{\ell}_L} & \overline{L_L}\end{pmatrix}V_L^e\begin{pmatrix}-1+\sin^2\theta' & 0\\ 0 & \sin^2\theta'\end{pmatrix}V_L^{e\dagger}\gamma^\mu\begin{pmatrix}\hat{\ell}_L\\ L_L\end{pmatrix}
\end{aligned} \tag{5.23a}$$

and similarly for the right-handed currents:

$$\begin{aligned}
&\begin{pmatrix}\overline{{}^4e_R} & \overline{{}^1e_R}\end{pmatrix}\begin{pmatrix}-1+2\sin^2\theta' & 0\\ 0 & 2\sin^2\theta'\end{pmatrix}\gamma^\mu\begin{pmatrix}{}^4e_R\\ {}^1e_R\end{pmatrix}\\
&\qquad = \begin{pmatrix}\overline{\hat{e}_R} & \overline{E_R}\end{pmatrix}V_R^e\begin{pmatrix}-1+2\sin^2\theta' & 0\\ 0 & 2\sin^2\theta'\end{pmatrix}V_R^{e\dagger}\gamma^\mu\begin{pmatrix}\hat{e}_R\\ E_R\end{pmatrix}.
\end{aligned} \tag{5.23b}$$

Finally, using the block notation of Eq. (5.22), one arrives to the following formula for the Z' couplings with the SM fermions:

$$\begin{aligned}
\mathcal{L}^{Z'll} = \frac{g_{BL}}{\cos\theta'}\Big[\,&\overline{\hat{\ell}}_{Li}\left(s'^2\mathbb{1}-(U_L)^\dagger_{ff}(U_L)_{ff}\right)\gamma^\mu\hat{\ell}_L^{\,i}\\
&+\overline{\hat{e}_R}\left(2s'^2\mathbb{1}\sin^2\theta'-(U_R)^\dagger_{ff}(U_R)_{ff}\right)\gamma^\mu\hat{e}_R\\
&+\frac{1+s'^2}{3}\,\overline{\hat{q}}_{Li}\gamma^\mu\hat{q}_L^{\,i}+\frac{1-4s'^2}{3}\,\overline{\hat{u}_R}\gamma^\mu\hat{u}_R+\frac{1+2s'^2}{3}\,\overline{\hat{d}_R}\gamma^\mu\hat{d}_R\Big]Z'_\mu\,,
\end{aligned} \tag{5.24}$$

where $s'^2 \equiv \sin^2\theta'$ amounts to 0.08 at the 2 TeV scale or to $s'^2 \simeq 0.12$ in the 200 TeV ballpark (assuming SM-like gauge coupling running up to $m_{Z'}$). Thus, the Z' interactions with leptons in the extended $SU(4)_C$ models are not necessarily flavour-universal and, in general, the diagonal couplings could actually be strongly suppressed.

Thus, the limits on the BSM gauge field masses from the high-energy dilepton spectra may be considerably weakened for certain patterns of $(U_{L,R})_{ff}$. The simplified reasoning of Ref. [59] mentioned above has been used as the argument for abandoning the models with the G_{421} gauge group and focusing on G_{4321}-based models instead when attempting to accommodate $\mathrm{R}_{D^{(*)}}$ which call for large NP effects. In this respect, we note that achieving the form of $(U_{L,R})_{ff}$ from Eq. (5.3) in the G_{421} framework would imply that the Z' couplings to the light leptons are suppressed while its couplings to $\tau^+\tau^-$ are experimentally less constrained [229]. Nevertheless, the scenarios with the $SU(4)_C$-breaking scale as low as 2 TeV require full model specification as the effects of the new scalar and fermionic degrees of freedom would be important.

In any case, this study is focusing on the extended $SU(4)_C$ models with $k_L + k_R \leq 2$. Such frameworks can not accommodate the $\mathrm{R}_{D^{(*)}}$ anomalies even if the Z' is completely ignored due to the residual constraints on the GLQ interaction matrices $(U_{L,R})_{ff}$ from the unitarity of $U_{L,R}$. As will be discussed in the next subsection, m_{U_1} is globally bounded from below to about 18 TeV in the considered class of models. For many particular forms of U_L and U_R, the lower LQ mass limit is parametrically higher. For such cases, the vector-type effective operators induced by Z' are generally less restrictive than those originating from interactions of the U_1 leptoquark which are both vector-type ($\mathcal{O}_{ed}, \mathcal{O}_{\ell q}$) and scalar-type ($\mathcal{O}_{\ell edq}$).

In this analysis, the contributions of Z' to the WC's are not calculated. Including them could be a part of a future study.

5.2.3 First signals of gauge leptoquark in slightly extended $SU(4)_C$ models

We have performed a a similar analysis to that of Section 5.1 for the extended $SU(4)_C$ models with $(k_L, k_R) = (1,0), (0,1), (2,0), (0,2)$ and $(1,1)$.

With growing number of free parameters, more couplings in $(U_{L,R})_{ff}$ can be "rotated away" to the other parts of $U_{L,R}$. New interaction patterns become allowed, with lower lower limits on m_{U_1}. For some cases, the bounds follow from measurements which have not been identified as constraining the parameter space in the basic model of quark lepton unification. Hence, naturally, the catalogue of observables which currently constrain m_{U_1} for some form of $U_{L,R}$ grows with the growing dimensions of these unitary matrices. This is captured in Table 5.3.

While a large effort has been spent to fully capture the parametric freedom in the cases $(k_L, k_R) = (1,0)$ or $(0,1)$, the number of parameters for $k_L + k_R = 2$ is quite high and we admit that the corresponding lists in Table 5.3 may not be complete.

5.2.4 Addressing neutral current B anomalies

In order to accommodate the neutral current anomalies in B-meson decays, the elements $u^L_{s\mu}$ and $u^L_{b\mu}$ need to be non-negligible. To avoid the scalar-type operators $\mathcal{O}_{\ell edq}$ involving electrons or muons, which are responsible for the most severe

Model	$k_L = 0$ $\dim U_L = 3$	$k_L = 1$ $\dim U_L = 4$	$k_L = 2$ $\dim U_L = 5$
$k_R = 0$ $\dim U_R = 3$	see Table 5.1	$\mathrm{BR}(B^0 \to ee)$ $\mathrm{BR}(B_s \to ee)$ $(\varepsilon'/\varepsilon)_{K^0}$	$\mathrm{BR}(B^+ \to K^+\mu^+e^-)$ $\mathrm{BR}(B^+ \to K^+\mu^-e^+)$ ε_{K^0} ...
$k_R = 1$ $\dim U_R = 4$	$\mathrm{BR}(B^0 \to ee)$ $\mathrm{BR}(B_s \to ee)$ $(\varepsilon'/\varepsilon)_{K^0}$	ε_{K^0} ...	
$k_R = 2$ $\dim U_R = 5$	$\mathrm{BR}(B^+ \to K^+\mu^+e^-)$ $\mathrm{BR}(B^+ \to K^+\mu^-e^+)$ ε_{K^0} $\mathrm{R}_{K^{(*)}}$ ...		

Table 5.3: Possible dominant signals of the gauge LQ for some form of $U_{L,R}$ in extended $SU(4)_C$ models featuring k_L extra lepton doublets and k_R extra charged-lepton singlets. For a given cell, all observables from the cells above and to the left are implicitly assumed to be included. The ellipses indicate that the catalogues in the relevant cell may not be complete.

constraints discussed in Section 5.1, $n_R = 2$ generations of extra lepton $SU(2)_L$-singlets are required.

The model with $\dim(U_L) = 3$ and $\dim(U_R) = 5$ allows for the following setup:

$$U_L = \begin{pmatrix} 0 & 0 & e^{i\delta_L} \\ e^{i\delta_1}\cos\gamma & -e^{i\delta_2}\sin\gamma & 0 \\ e^{i\delta_2}\sin\gamma & e^{i\delta_1}\cos\gamma & 0 \end{pmatrix}, \qquad (U_R)_{ff} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e^{i\delta_R} \end{pmatrix}. \tag{5.25}$$

Note that a similar pattern for U_L has been suggested in Ref. [172] and also in Ref. [62] within the $SU(4)_{C_L} \times SU(4)_{C_R} \times SU(2)_L \times U(1)_R$ framework where the couplings to the right-handed fermions are suppressed globally.

Adopting Eq. (5.25), the best fit is close to the simple case[3]

$$\gamma \simeq \pi/4\,, \qquad \delta_1 \simeq \delta_2 \simeq \delta_L \simeq \delta_R \simeq 0\,, \qquad m_{U_1} = 22~\text{TeV}, \tag{5.26}$$

which improves the global log-likelihood function `smelli` [194] by more than 14 units compared to the SM. Such a scenario accommodates well the $\mathrm{R}_{K^{(*)}}$ anomaly and also significantly mitigates the tension in the additional $b \to s\mu\mu$ observables. Using the normalization factor $\mathcal{N} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2}$ [see (3.9)], Eqs. (5.25) and (5.26) imply

$$C^{\rm NP}_{9\_bs\mu\mu} = -C^{\rm NP}_{9\_bsee} = +C^{\rm NP}_{9\_bs\mu e} = -C^{\rm NP}_{9\_bse\mu} = -0.24\,, \tag{5.27a}$$

$$C^{\rm NP}_{10\_bsll'} = -C^{\rm NP}_{9\_bsll'} \tag{5.27b}$$

at the 5 GeV scale.

[3] The need of much lighter scalar leptoquark R_2 encountered in Chapter 4 follows from the non-interference between the NP and SM amplitudes and from the smallness of the y_{se} coupling in Eq. (4.13).

Observable	Model prediction	Experiment	
$\mathrm{R}_K\,[(1.1;6)\ \mathrm{GeV}^2]$	0.79	0.85 ± 0.06	[128]
$\mathrm{R}_{K^*}[(1.1;6)\ \mathrm{GeV}^2]$	0.79	0.68 ± 0.12	[22]
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	3.2×10^{-9}	$(3.0 \pm 0.4) \times 10^{-9}$	[54]
$\mathrm{BR}(B^+ \to K^+\mu^+e^-)$	2.1×10^{-9}	$< 6.4 \times 10^{-9}$	[183]
$\mathrm{BR}(B^+ \to K^+e^+\mu^-)$	2.1×10^{-9}	$< 7.0 \times 10^{-9}$	[183]
$\mathrm{BR}(\mu \to e\gamma)$	1.9×10^{-13}	$< 4.2 \times 10^{-13}$	[96]
$\mathrm{BR}(B^0 \to \tau^+\tau^-)$	9×10^{-7}	$< 1.6 \times 10^{-3}$	[222]
$\mathrm{BR}(B_s \to e^\pm\tau^\mp)$	6.4×10^{-7}	N/A	[54]
$\mathrm{BR}(B_s \to \mu^\pm\tau^\mp)$	6.4×10^{-7}	$< 3.4 \times 10^{-5}$	[206]

Table 5.4: Predictions for several observables with important NP contribution for the benchmark case of Eqs. (5.25) and (5.26).

In comparison, the benchmark one-dimensional effective scenario with only $C^{\mathrm{NP}}_{9\_bs\mu\mu} = -C^{\mathrm{NP}}_{10\_bs\mu\mu} = -0.53$ [154] improves the log-likelihood by 18 units; the simplified vector LQ setup in Eq. (5.3) improves the log-likelihood by 30 units as it also accommodates $\mathrm{R}_{D^{(*)}}$. Notice that translating the likelihoods to the *number of sigmas* depends on the number of free parameters.

Predictions for several important observables following from Eqs. (5.25) and (5.26) are given in Table 5.4. As outlined in Section 5.1.3, the LQ has been integrated out at the tree level and the calculated LFV dipole operators responsible for $\mu \to e\gamma$ arise solely from the one-loop RGE running of the Wilson coefficients. Thus, the predictions for the loop processes should be interpreted with caution.

In the scenarios with nonzero couplings $u^L_{se}, u^L_{be}, u^L_{s\mu}$ and $u^L_{b\mu}$ of a vector leptoquark addressing the neutral-current B anomalies, the strongest bounds arise from $B^+ \to K^+\mu^\pm e^\mp$ and from the the LFV loop processes like $\mu \to e\gamma$ (see Ref. [166] for a dedicated study). Generally, the constraints from the latter are quite strong. However, in the chiral leptoquark models with unitary interaction matrix, $\mu \to e\gamma$ is suppressed by an analogue of the GIM mechanism. As the only non-vanishing element of $(U_R)_{ff}$ in (5.25) is essentially irrelevant for $\mu \to e\gamma$, the same applies also to our case. Note that Ref. [166] did not consider the subleading terms and hence found exactly zero contributions to $\mu \to e\gamma$ for the case $u^L_{se}u^{L\,*}_{s\mu} = -u^L_{be}u^{L\,*}_{b\mu}$. Ref. [62] considered the case equivalent to U_L from (5.25) and $U_R = 0$, finding the constraint $m_{U_1} > 10$ TeV based on the BaBar search [230] for $B \to Ke\mu$. The very recent measurement by LHCb [183] has pushed the limit to 17 TeV for the considered interaction pattern.

Finally, let us note that although the Z' interactions are not lepton-flavour universal, the couplings in the particular case of Eq. (5.25) are lepton-flavour diagonal and, hence, the Z' does not mediate any LFV processes. At the same time, with a mass above 20 TeV, Z' is also safely hidden to the high-energy searches at LHC.

To conclude, the interactions of the $SU(4)_C$ gauge leptoquark in a model with two extra weak-isosinglet charged leptons can accommodate the neutral-current B-meson anomalies to a large extent. The setup can be excluded by future negative searches for $B \to Ke\mu$ at LHCb or Belle II [231].

Conclusion

Let us recapitulate the outcomes of this work in three layers.

The first layer consists in offering new views on known concepts. In this respect, we have suggested a perspective (the "*SU2U* approach") relating accidental and imposed symmetries. We have argued that the relation of fermion number $\mathcal{F}$ to the Lorentz symmetry is the same as the relation of baryon number $\mathcal{B}$ to $SU(3)_C$. It has been shown that, similarly to the fact that models with a Lagrangian free of explicit fermion conjugation matrix $\mathcal{C}$ conserve fermion number, Lagrangians without the color Levi-Civita tensor $\varepsilon_{\alpha\beta\gamma}$ conserve baryon number [25]. Similar relations have been found for other symmetries. With this formalism, we have explained comprehensively why both baryon and lepton numbers may remain exactly (perturbatively) conserved even in some models with the Pati-Salam type $SU(4)_C$ quark-lepton symmetry which necessarily feature a spontaneously broken symmetry usually denoted by $B - L$.

Furthermore, identifying *lepton flavour universality* with the permutation group S_3, we have pointed out the complementary roles of *lepton flavour violation* and *lepton flavour universality violation* in effective theories as well as in models with leptoquarks.

The second layer of results consists in analysing general properties of BSM models. Focusing on theories with extended gauge symmetry $SU(4)_C \times SU(2)_L \times U(1)_R$, we have identified the most general scalar potential in the MQLS/FPW models of quark-lepton unification and found the resulting relations between the masses of BSM scalars masses [25] in Chapter 2. Furthermore, we have discussed the number of physical phases in the quark-lepton mixing matrices.

Thirdly but most importantly, two phenomenological analyses of leptoquark effects have been presented, adopting models with $SU(4)_C$ gauge symmetry of the Pati-Salam type.

The study presented in Chapter 4 and published in Refs. [25, 29, 30] searched for a possible accommodation of the neutral-current B anomalies within the MQLS and FPW models. We have found that the MQLSM is incapable of explaining the anomalies, and also its extension – the FPW model, is at (or slightly past) the edge of exclusion. The only perhaps applicable scenario consistent with the measured values of $\mathrm{R}_{K^{(*)}}$ relies on a light scalar leptoquark R_2, predicting that $\tau \to e\pi^+\pi^-$ should be observed in the future at the level slightly *above* the current experimental limit, and also that $\tau \to e\gamma$ should be found at Belle II. We have argued that such a setup also naturally provides an opportunity to partially account for the deviations in $\mathrm{R}_{D^{(*)}}$.

In Chapter 5, possible current or future effects of gauge leptoquarks have been studied in the theory of quark-lepton unification as well as in the similar $SU(4)_C$ models assuming extra generations of leptons [31]. For the former case, the catalogue consisting of measurements which currently set the border of the excluded part of the parameter space has been compiled. The involved observables have a potential to discover the gauge LQ signal even with a small improvement of the experimental sensitivity. For the decays $P^0 \to l_1^+ l_2^-$ not listed in this catalogue, we have found the future experimental bounds needed in order to further probe the considered model.

In Section 5.2, the gauge LQ has been studied in the models with vector-like BSM leptons. The catalogue of constraining observables from Section 5.1 has been extended to include the scenarios with k_L extra leptonic $SU(2)_L$ doublets and k_R extra lepton singlets. In particular, the cases with $k_L + k_R \leq 2$ have been considered. Finally, it has been shown that the $SU(4)_C$ gauge LQ model with 2 extra weak-isosinglet charged leptons can significantly alleviate the tension in the $b \to s\mu\mu$ data.

A. Single-boson SM extensions

In this appendix the catalogue of all bosons which could interact with the SM-fermion pairs is provided. Our results are compatible with, e.g., Refs. [52, 232, 233, 234] (but not the same due to different assumptions). On the other hand, several papers which reached different conclusions are also mentioned below.

The scalar bosons which could interact with the SM fermions have been collected in Table 1.2 on page 15. Here the analogue for *vector* bosons is given in Table A.1. If right-handed neutrinos are added to the SM, these tables should be extended by Table A.2 (left and right). For the scalar fields, the same information as in Table 1.2 is presented in a different format in Table A.3. Throughout this appendix, the Lorentz contractions are implicit.

Concerning the BSM scalars in Tables 1.2 and A.3, their renormalizable interactions with the SM Higgs ϕ would not induce new sources of $\mathcal{B}$ or $\mathcal{L}$ violation, up to two exceptions: $\Delta \sim (1,3,1)$ and $\varphi^0 \sim (1,1,0)$. Their lepton number should be $\mathcal{L} = -2$ according to their Yukawa couplings (with $\ell_L \ell_L$ and $\nu_R \nu_R$, respectively), but their interactions with the Higgs call for $\mathcal{L} = 0$. Furthermore, these fields could acquire a VEV, leading to spontaneous lepton number violation in the case the explicit LNV was suppressed.

Note that there is a fake candidate for a baryon number violating interaction of the $\tilde{R}_2 \sim (3, 2, +1/6)$ field, which must carry $\mathcal{B} = +1/3$ due to its Yukawa interactions. This field the seems to violate $\mathcal{B}$ by

$$\tilde{R}_2{}^{\alpha i} \tilde{R}_2{}^{\beta j} \tilde{R}_2{}^{\gamma k} \phi_k^\dagger \, \varepsilon_{ij} \, \varepsilon_{\alpha\beta\gamma} \,. \tag{A.1}$$

However, this term is identically zero and hence $\tilde{R}_2$ is safe concerning proton decay.

Misidentification of the above term has compromised the results of Ref. [235]. This has been corrected in Ref. [236]; nevertheless, the *catalogue of baryon-number-violating scalar interactions* is still not complete there: the scalar $\tilde{S}_1 \sim (\bar{3}, 1, 4/3)$ is missing since its two Yukawa interactions (with $\overline{u_R}\,\overline{u_R}$ and $d_R \, e_R$) are formally ascribed to two different fields. Finally, the correct conclusion in this respect has been made in [234]. In Ref. [237], two-scalar extensions of the SM are investigated, listing those leading to $|\Delta\mathcal{B}| = 2$ but no proton decay. This work also suffers from misinterpreting the (A.1) catch. Nevertheless, it is straightforward to verify that the main results of this work are unaffected by this issue.

Refs. [123, 238] misidentified the U_1 leptoquark as a potentially baryon number violating field. In fact, it cannot have a diquark coupling since the claimed interaction with two d-type quarks can not be written in a Lorentz-invariant way.

Ref. [239] is missing the interaction $(u_R{}^\alpha d_R{}^\beta)\chi_{\alpha\beta}$ of the $(\overline{6}, 1, -1/3)$ scalar but this is irrelevant for the topic of its interest – the BNV $d = 5$ operators built from the SM fields and a single BSM scalar. We confirm that the list cast in [239] is correct up to a single missing term $H_i^\dagger (u_R{}^\alpha d_R{}^\beta) \tilde{R}_2{}^{\gamma i} \varepsilon_{\alpha\beta\gamma}$.

Similarly, findings of Ref. [240] regarding minimal flavour violation have not been affected by omitting R_2 and $\tilde{R}_2$ in the considerations.

$\Psi(Y)^{3\mathcal{B}}_{\mathcal{L}}$	$\overline{e_R}\,(1)^{0}_{-1}$	${\ell_L}^{j}\,(-\frac{1}{2})^{0}_{1}$	$\overline{d_{R\beta}}\,(\frac{1}{3})^{-1}_{0}$	$\overline{u_{R\beta}}\,(-\frac{2}{3})^{-1}_{0}$	${q_L}^{\beta j}\,(\frac{1}{6})^{1}_{0}$
$\overline{q_{L\alpha i}}(-\frac{1}{6})^{-1}_{0}$	${V_2}^{\dagger\alpha i}\,(-\frac{5}{6})^{1}_{1}$	${U_1}^{\alpha}\delta^{i}_{j}$ $U_{3j}^{\alpha i}$ $(\frac{2}{3})^{1}_{-1}$	$\tilde{V}_{2\gamma j}\varepsilon^{\alpha\beta\gamma}\varepsilon^{ij}$ $\tilde{\Omega}^{\alpha\beta i}$ $(-\frac{1}{6})^{2}_{0}$	$V_{2\gamma j}\varepsilon^{\alpha\beta\gamma}\varepsilon^{ij}$ $\Omega^{\alpha\beta i}$ $(\frac{5}{6})^{2}_{0}$	$B\,\delta^{\alpha}_{\beta}\delta^{i}_{j}$ $W^{i}_{j}\delta^{\alpha}_{\beta}$ $G^{\alpha}_{\beta}\delta^{i}_{j}$ $G_{3\beta j}^{\alpha i}$ $(0)^{0}_{0}$
${u_R}^{\alpha}\,(\frac{2}{3})^{1}_{0}$	$\tilde{U}_{1\alpha}^{\dagger}\,(-\frac{5}{3})^{-1}_{-1}$	$\tilde{V}_{2\alpha j}\,(-\frac{1}{6})^{-1}_{-1}$	$W_R^{-}\,\delta^{\beta}_{\alpha}$ $G^{-\beta}_{\alpha}$ $(-1)^{0}_{0}$	$B\,\delta^{\beta}_{\alpha}$ G^{β}_{α} $(0)^{0}_{0}$	
${d_R}^{\alpha}\,(-\frac{1}{3})^{1}_{0}$	$U^{\dagger}_{1\alpha}\,(-\frac{2}{3})^{-1}_{1}$	$V_{2\alpha j}\,(\frac{5}{6})^{-1}_{-1}$	$B\,\delta^{\beta}_{\alpha}$ G^{β}_{α} $(0)^{0}_{0}$		
$\overline{\ell_{Li}}\,(\frac{1}{2})^{0}_{-1}$	${A_2}^{i}\,(-\frac{3}{2})^{0}_{2}$	$B\,\delta^{i}_{j}$ W^{i}_{j} $(0)^{0}_{0}$			
$e_R\,(-1)^{0}_{1}$	$B\,(0)^{0}_{0}$				

Table A.1: Table of all possible interactions of the SM-fermion bilinears with vector fields. G, W and B correspond to the gauge fields of the SM.

$\Psi(Y)^{3\mathcal{B}}_{\mathcal{L}}$	$\overline{\nu_R}\,(0)^{0}_{-1}$
${q_L}^{i\alpha}\,(\frac{1}{6})^{1}_{0}$	$\tilde{R}^{\dagger}_{2\alpha i}\,(-\frac{1}{6})^{-1}_{1}$
$\overline{u_{R\alpha}}\,(-\frac{2}{3})^{-1}_{0}$	$\bar{S}_1^{\dagger\alpha}\,(\frac{2}{3})^{1}_{1}$
$\overline{d_{R\alpha}}\,(\frac{1}{3})^{-1}_{0}$	$S_1^{\dagger\alpha}(-\frac{1}{3})^{1}_{1}$
${\ell_L}^{i}(-\frac{1}{2})^{0}_{1}$	$\phi^{j}\,\varepsilon_{ij}\,(\frac{1}{2})^{0}_{0}$
$\overline{e_R}\,(1)^{0}_{-1}$	$\varphi^{-}\,(-1)^{0}_{2}$
$\overline{\nu_R}\,(0)^{0}_{-1}$	$\varphi^{0}\,(0)^{0}_{2}$

$\Psi(Y)^{3\mathcal{B}}_{\mathcal{L}}$	$\overline{\nu_R}\,(0)^{0}_{-1}$
$\overline{q_{Li\alpha}}\,(-\frac{1}{6})^{-1}_{0}$	$\tilde{V}_2^{\dagger i\alpha}\,(\frac{1}{6})^{1}_{1}$
${u_R}^{\alpha}\,(\frac{2}{3})^{1}_{0}$	$U^{\dagger}_{1\alpha}\,(-\frac{2}{3})^{-1}_{1}$
${d_R}^{\alpha}\,(-\frac{1}{3})^{1}_{0}$	$\bar{U}^{\dagger}_{1\alpha}\,(\frac{1}{3})^{-1}_{1}$
$\overline{\ell_{Li}}\,(\frac{1}{2})^{0}_{-1}$	$\tilde{A}_2^{\ i}\,(-\frac{1}{2})^{0}_{2}$
$e_R\,(-1)^{0}_{1}$	$W_R^{+}(+1)^{0}_{0}$
$\nu_R\,(0)^{0}_{1}$	$B\,(0)^{0}_{0}$

Table A.2: Columns to be added to Tables 1.2 and A.1 if right-handed neutrinos are considered.

Scalar field	Yukawa and relevant Higgs interactions	$\mathcal{B}$	$\mathcal{L}$	$\mathcal{F}$
Leptoquarks:				
$R_2 \sim (3, 2, +7/6)$	$(\overline{u_{R\alpha}}\, \ell_L{}^j)\varepsilon_{ji} R_2{}^{i\alpha} + (\overline{q_{L\alpha i}}\, e_R) R_2{}^{\alpha i}$	$+1/3$	-1	0
$\tilde{R}_2 \sim (3, 2, +1/6)$	$(\overline{d_{R\alpha}}\, \ell_L{}^j)\varepsilon_{ji} \tilde{R}_2{}^{i\alpha} + (\overline{q_{L\alpha i}}\, \nu_R) \tilde{R}_2{}^{i\alpha}$	$+1/3$	-1	0
Diquark – leptoquarks:				
$S_3 \sim (\bar{3}, 3, +1/3)$	$(\overline{q_{L\alpha i}}\, \overline{q_{L\beta j}})\varepsilon^{\alpha\beta\gamma} S_{3\gamma ij}$	$+2/3$	0	$+2$
	$(q_L{}^{\alpha i} \ell_L{}^j) S_{3\alpha ij}$	$-1/3$	-1	-2
$S_1 \sim (\bar{3}, 1, +1/3)$	$(\overline{q_{Li\alpha}} \varepsilon^{ij} \overline{q_{Lj\beta}})\varepsilon^{\alpha\beta\gamma} S_{1\gamma} + (\overline{u_{R\alpha}} \overline{d_{R\beta}})\varepsilon^{\alpha\beta\gamma} S_{1\gamma}$	$+2/3$	0	$+2$
	$(q_L{}^{\alpha i} \varepsilon_{ij} \ell_L{}^j) S_{1\alpha} + (u_R{}^\alpha\, e_R) S_{1\alpha} + (d_R{}^\alpha \nu_R) S_{1\alpha}$	$-1/3$	-1	-2
$\tilde{S}_1 \sim (\bar{3}, 1, +4/3)$	$(\overline{u_{R\alpha}}\, \overline{u_{R\beta}})\varepsilon^{\alpha\beta\gamma} \tilde{S}_{1\gamma}$	$+2/3$	0	$+2$
	$(e_R\, d_R{}^\alpha) \tilde{S}_{1\alpha}$	$-1/3$	-1	-2
Diquark – leptoquark:				
$\bar{S}_1 \sim (\bar{3}, 1, -2/3)$	$(\overline{d_{R\alpha}}\, \overline{d_{R\beta}})\varepsilon^{\alpha\beta\gamma} \bar{S}_{1\gamma}$	$+2/3$	0	$+2$
	$(\nu_R u_R{}^\alpha) \bar{S}_{1\alpha}$	$-1/3$	-1	-2
Diquarks:				
$\chi \sim (\bar{6}, 1, -1/3)$	$(q_L{}^{\alpha i} \varepsilon_{ij}\, q_L{}^{\beta j}) \chi_{\alpha\beta} + (u_R{}^\alpha\, d_R{}^\beta) \chi_{\alpha\beta}$	$-2/3$	0	-2
$\bar{\chi} \sim (\bar{6}, 1, +2/3)$	$(d_R{}^\alpha\, d_R{}^\beta) \bar{\chi}_{\alpha\beta}$	$-2/3$	0	-2
$\tilde{\chi} \sim (\bar{6}, 1, -4/3)$	$(u_R{}^\alpha\, u_R{}^\beta) \tilde{\chi}_{\alpha\beta}$	$-2/3$	0	-2
$X \sim (\bar{6}, 3, -1/3)$	$(q_L{}^{\alpha i}\, q_L{}^{\beta j}) X_{\alpha\beta ij}$	$-2/3$	0	-2
Dileptons:				
$\varphi^+ \sim (1, 1, 1)$	$(\ell_L{}^i \varepsilon_{ij} \ell_L{}^j) \varphi^+$	0	-2	-2
$\varphi^{++} \sim (1, 1, 2)$	$(e_R\, e_R) \varphi^{++}$	0	-2	-2
Dileptons – $\mathcal{B}$&$\mathcal{L}$-uninvolved:				
$\Delta \sim (1, 3, 1)\,!$	$(\ell_L{}^i\, \ell_L{}^j) \Delta_{ij}$	0	-2	-2
	$\phi_i^\dagger \varepsilon^{ij} \Delta_{jk} \varepsilon^{kl} \phi_l^\dagger$	0	0	0
$\varphi^0 \sim (1, 1, 0)\,!$	$(\nu_R \nu_R) \varphi^0$	0	-2	-2
	$\phi^i \phi_i^\dagger \varphi^0$	0	0	0
$\mathcal{B}$&$\mathcal{L}$-uninvolved:				
$G_2 \sim (8, 2, +1/2)$	$(\overline{q_{L\alpha i}} d_R{}^\beta) G_2{}^{\alpha i}_{\beta} + (\overline{u_{R\beta}}\, q_L{}^{\alpha i}) G_2{}^{\beta j}_{\alpha} \varepsilon_{ij}$	0	0	0
$\phi \sim (1, 2, +1/2)\,!$	$\overline{q_{L\alpha i}} d_R{}^\alpha \phi^i + \overline{u_{R\alpha}} q_L{}^{\alpha j} \varepsilon_{ji} \phi^i + {}$ $+ \overline{\ell_{Li}} e_R \phi^i + \overline{\nu_R} \ell_L{}^j \varepsilon_{ji} \phi^i$	0	0	0

Table A.3: List of all possible spinless bosons which may interact with the SM fermion bilinears. Interactions with right-handed neutrinos are also included but denoted by gray color. The Lorentz contractions are implicit. Fields labeled by '!' contain a neutral component which can acquire a VEV.

B. Accidental symmetry in the $SU(5)$ grand unified theory

By definition, grand unified theories (GUTs) are build around a simple gauge group. Thus, there is only a single candidate for an accidental $U(1)$ symmetry suggested by the *SU2U* approach developed in Chapter 1. Accordingly, the $\mathcal{B}-\mathcal{L}$ conserving proton decay is a well known prediction of the $SU(5)$ GUTs.

As an example, let us consider the simple Georgi-Glashow model [241], field content of which is summarized in Table B.1. Suppressing the coupling constants for notational simplicity, the most interesting interaction terms read

$$\begin{aligned}\mathcal{L}_{\text{GG}} = \text{kinetic terms} &+ (\Psi^{\underbrace{ab}}\psi_a)H^\dagger_b + (\Psi^{\underbrace{ab}}\Psi^{\underbrace{cd}})H^e\varepsilon_{abcde} + \text{h.c.} \\ &+ H^\dagger_a\Phi^a_bH^b + H^\dagger_a\Phi^a_b\Phi^b_cH^c + \Phi^a_b\Phi^b_c\Phi^c_a + \Phi^a_b\Phi^b_c\Phi^c_d\Phi^d_a \\ &+ (H^\dagger_aH^a)^2 + (\Phi^a_b\Phi^b_a)^2 + (\Phi^a_b\Phi^b_a)(H^\dagger_cH^c)\,.\end{aligned} \tag{B.1}$$

The fermion number is violated, but the model features a global symmetry $U(1)_\mathcal{Z}$ whose charges are also specified in Table B.1. The relation between $SU(5)$ and $\mathcal{Z}$ is established by observing that, for each multiplet,

$$\mathcal{Z} \overset{\text{mod}\,5}{=\!=} c^{SU(5)} \overset{\text{mod}\,5}{=\!=} \Big(\#\text{ upper } SU(5) \text{ indices}\Big) - \Big(\#\text{ lower } SU(5) \text{ indices}\Big) \tag{B.2}$$

where $c^{SU(5)}$ denotes the *quintility* of the representation (see Section 1.2.3).

$\mathcal{Z}$ remains a good symmetry after the first stage of symmetry breaking by $\langle\Phi\rangle$ but it gets broken together with the weak hypercharge during the EWSB by $\langle H\rangle$. In order to obtain a global symmetry respected by the vacuum of the theory, the combination proportional to $\langle Y\rangle\mathcal{Z} - \langle\mathcal{Z}\rangle Y$ must be considered, which yields the anticipated result

$$\mathcal{B}-\mathcal{L} = \frac{1}{10}\mathcal{Z} + \frac{4}{5}Y. \tag{B.3}$$

Notice that $\mathcal{Z}$ is *not* directly given by the differences between upper and lower indices used in Eq. (B.1) and that the Levi-Civita tensor is present in there. Formally, resolving this apparent non-compatibility with the intuitive picture of the *SU2U* approach would require redefining

$$H^e\varepsilon_{abcde} \equiv H_{\underbrace{abcd}} \qquad \Leftrightarrow \qquad H^e = \frac{1}{4!}H_{\underbrace{abcd}}\,\varepsilon^{abcde}, \tag{B.4}$$

$$\psi_a\,\varepsilon_{bcdef} \equiv \psi_{a\,\underbrace{bcdef}} \qquad \Leftrightarrow \qquad \psi_a = \frac{1}{5!}\psi_{a\,\underbrace{bcdef}}\,\varepsilon^{bcdef}, \tag{B.5}$$

in which case the relevant Lagrangian terms take the form[1]

$$\begin{aligned}\mathcal{L}_{\text{GG}} = \text{kinetic terms} &+ \frac{1}{4!}(\Psi^{\underbrace{ab}}\psi_{a\,\underbrace{bcdef}})H^{\underbrace{cdef}} + (\Psi^{\underbrace{ab}}\Psi^{\underbrace{cd}})H_{\underbrace{abcd}} + \text{h.c.} \\ &- \frac{1}{(3!)^2}H^{\dagger\,\underbrace{bcde}}\Phi^a_bH_{\underbrace{acde}} + \ldots\end{aligned} \tag{B.6}$$

[1]The derivation makes use of $\psi_aH^\dagger_b = \psi_a\delta^g_bH^\dagger_g = \psi_a\frac{1}{4!}\varepsilon_{bcdef}\varepsilon^{gcdef}H^\dagger_g = \psi_{a\,\underbrace{bcdef}}H^{\underbrace{cdef}}$ and $H^\dagger_a\Phi^a_bH^b = \frac{1}{(4!)^2}H^{\dagger\underbrace{cdef}}\,\varepsilon_{acdef}\Phi^a_b\varepsilon^{bghij}H_{\underbrace{ghij}} = \frac{1}{(4!)^2}H^{\dagger\underbrace{cdef}}\Phi^a_b\delta^{bghij}_{acdef}H_{\underbrace{ghij}} = \frac{-4^2}{(4!)^2}H^{\dagger\,\underbrace{bdef}}\Phi^a_bH_{\underbrace{adef}}$ where the relation $\Phi^a_a = 0$ has been used in the last step.

$SU(5) \to G_{\mathrm{SM}}$	Y	$\mathcal{Z}$	$\mathcal{B}-\mathcal{L}$
Field content of the Georgi-Glashow model			
Fermions:			
$\psi_{(\bar{5})} \quad = \begin{pmatrix} d^c_{R\,\alpha} & \ell_L{}^j \varepsilon_{ji} \end{pmatrix}$	$\begin{pmatrix} 1/3 & -1/2 \end{pmatrix}$	-6	$\begin{pmatrix} -1/3 & -1 \end{pmatrix}$
$\Psi_{(10)} \quad = \begin{pmatrix} \varepsilon^{\alpha\beta\gamma} u^c_{R\,\gamma} & -q_L{}^{\beta i} \\ q_L{}^{\alpha j} & \varepsilon^{ij} e^c_R \end{pmatrix}$	$\begin{pmatrix} -2/3 & 1/6 \\ 1/6 & 1 \end{pmatrix}$	2	$\begin{pmatrix} -1/3 & 1/3 \\ 1/3 & 1 \end{pmatrix}$
Scalars:			
$H_{(5)} \quad = \begin{pmatrix} S_1^{\dagger\alpha} \\ \phi^i \end{pmatrix}$	$\begin{pmatrix} -1/3 \\ 1/2 \end{pmatrix}$	-4	$\begin{pmatrix} -2/3 \\ 0 \end{pmatrix}$
$\Phi_{(24)} \quad = \begin{pmatrix} G^\alpha_\beta + \frac{1}{3}\delta^\alpha_\beta \varphi & \varpi^\alpha_j \\ \varpi^{\dagger i}_\beta & w^i_j - \frac{1}{2}\delta^i_j \varphi \end{pmatrix}$	$\begin{pmatrix} 0 & -5/6 \\ 5/6 & 0 \end{pmatrix}$	0	$\begin{pmatrix} 0 & -2/3 \\ 2/3 & 0 \end{pmatrix}$
Gauge fields:			
$A_{\mu(24)} \quad = \begin{pmatrix} G^\alpha_{\mu\beta} + \frac{1}{3}\delta^\alpha_\beta B_\mu & V_{2\,\mu j}^{\dagger\alpha} \\ V_{2\,\mu\beta}^{\;i} & W^{\;i}_{\mu j} - \frac{1}{2}\delta^i_j B_\mu \end{pmatrix}$	$\begin{pmatrix} 0 & -5/6 \\ 5/6 & 0 \end{pmatrix}$	0	$\begin{pmatrix} 0 & -2/3 \\ 2/3 & 0 \end{pmatrix}$

Table B.1: Fields in the simplest $SU(5)$ GUT, weak hypercharges of their components and generators of the global symmetry in the model. For better readability, some field normalization factors have been omitted.

C. Details of the analysis of the gauge leptoquark signals

In Chapter 5, we have presented the aim to fully take into account the available parameter space of the simplified model, keeping in mind that there is no prior (Bayesian) measure on it. This is, however, impossible just with the numerical scanning as some kind of a measure needs to be introduced in order to define either a grid or a random distribution for choosing the points investigated, and one should assure that the scan is "dense enough". Hence, numerical scanning has been augmented by studying the correlations among various predictions analytically.

Note that some measure-dependent statements like "*the LQ mass limit for most forms of* $U_{L,R}$ *is set by* $\mathrm{BR}(K_L^0 \to e\mu)$" correspond well to measure-free claims like "*the highest mass limits stem from* $\mathrm{BR}(K_L^0 \to e\mu)$", as can be seen from Fig. 5.2 on page 73.

C.1 Parametrization and scanning

For easiness, we will afford to refer to "tiny parts in the parameter space" within this appendix, based on the naive measure $\prod_{ij} \mathrm{d}\lambda_{ijL}\mathrm{d}\lambda_{ijR}$ in the notation of the so-called *composite parametrization* of the flavour matrices $U_{L,R}$. The composite parametrization of unitary matrices of arbitrary dimension n has been introduced in Refs. [242, 243] and its implementation in `Wolfram Mathematica` is available [244]. It turned out to be particularly convenient for the current study. Its n^2 parameters λ_{ij} consist of $\frac{1}{2}n(n-1)$ anlges ($i<j$) and $\frac{1}{2}n(n+1)$ phases ($i \geq j$). For $n=3$, the parametrization reads

$$\begin{pmatrix} c_{12}c_{13}e^{i\lambda_{11}} & \left(c_{23}s_{12}-c_{12}s_{13}s_{23}e^{i\lambda_{32}}\right)e^{i\lambda_{22}} & \left(s_{12}s_{23}+c_{12}c_{23}s_{13}e^{i\lambda_{32}}\right)e^{i\lambda_{33}} \\ -c_{13}s_{12}e^{i\lambda_{11}+i\lambda_{21}} & \left(c_{12}c_{23}+s_{12}s_{13}s_{23}e^{i\lambda_{32}}\right)e^{i\lambda_{22}+i\lambda_{21}} & \left(c_{12}s_{23}-c_{23}s_{12}s_{13}e^{i\lambda_{32}}\right)e^{i\lambda_{33}+i\lambda_{21}} \\ -s_{13}e^{i\lambda_{11}+i\lambda_{31}} & -c_{13}s_{23}e^{i\lambda_{22}+i\lambda_{31}+i\lambda_{32}} & c_{13}c_{23}e^{i\lambda_{31}+i\lambda_{32}+i\lambda_{33}} \end{pmatrix} \tag{C.1}$$

where $c_{ij} = \cos\lambda_{ij}$, $s_{ij} = \sin\lambda_{ij}$. For higher dimensions, see the original literature.

Avoiding constraints from $K_L^0 \to l_1 l_2$ decays

In Section 5.1.2, we have presented Smirnov's argument that due to the stringent experimental bounds on NP in $K_L^0 \to ee, \mu\mu, e\mu$, a scan over close neighborhood

λ_{11L}	λ_{12L}	λ_{13L}	λ_{21L}	λ_{22L}	λ_{23L}	λ_{31L}	λ_{32L}	λ_{33L}
$\frac{\pi}{7}$	$\frac{\pi}{16}$	$\frac{\pi}{7}$	$-\frac{\pi}{7}$	$\frac{\pi}{7}$	$(0, 2\pi)$	$\frac{\pi}{7}$	$\frac{\pi}{7}$	$\frac{\pi}{7}$
λ_{11R}	λ_{12R}	λ_{13R}	λ_{21R}	λ_{22R}	λ_{23R}	λ_{31R}	λ_{32R}	λ_{33R}
$\frac{\pi}{7}$	$-\frac{\pi}{16}$	$\frac{47\pi}{96}$	$\frac{\pi}{7}$	$\frac{\pi}{7}$	$\frac{\pi}{2}$	$\frac{\pi}{7}$	$\frac{\pi}{7}$	$\frac{\pi}{7}$

Table C.1: Angles and phases defining the slice of the parameter space used in Fig. 5.2.

of the parameter subspace in which the NP contributions to these processes vanish is essentially sufficient in order to find the other limiting observables. This observation is useful in analyzing the 3×3 matrices $U_{L,R}$ in Section 5.1 as well as the extended models in Section 5.2 where the dimensions of $U_{L,R}$ are $3 + k_{L,R}$. To this end, the complete solution of the four equations (5.14) for unitary $U_{L,R}$ of *arbitrary* dimensions is obtained by fixing

$$\lambda_{23L} \to \frac{\pi}{2}, \qquad \lambda_{23R} \to \frac{\pi}{2}, \qquad \lambda_{12R} \to -\lambda_{12L}, \qquad \lambda_{21L} \to -\lambda_{21R}, \tag{C.2}$$

in obvious notation. For $k_L = k_R = 0$, this essentially coincides with "solution A" from Ref. [175], which employed a different parametrization. Note that (C.2) does not fix the magnitude of any single element of $U_{L,R}$, except for $u^{L,R}_{b\tau} = 0$ in the $k_L = k_R = 0$ case (which is unavoidable, as explained in Section 5.1.2).

In fact, there is also another seemingly independent algebraic solution of (5.14) in the composite parametrization: $\lambda_{13L} = \lambda_{13R} = \pi/2$. Nevertheless, six of the remaining parameters in the obtained structure (which can be also fully defined by fixing $|u^L_{be}| = |u^R_{be}| = 1$) are redundant, and one can show that it actually corresponds to a specific subset of the more-dimensional set obtained by (C.2).[1]

Note that a tricky interconnection among the relevant amplitudes implies that the parameter subspace suppressing only $K^0_L \to ee$ and $K^0_L \to e\mu$ but allowing for contributions to $K^0_L \to \mu\mu$ (which may also be of relevance since NP in the last channel is actually less stringently constrained) is less-dimensional than the set suppressing also the $\mu^+\mu^-$ final state, and is covered by

$$\lambda_{13L} \to \frac{\pi}{2}, \quad \lambda_{21L} \to -\lambda_{21R}, \quad \lambda_{12L} \to \frac{\pi}{2} - \lambda_{12R}, \quad \lambda_{11L}, \lambda_{32L}, \lambda_{23L} \to 0 \tag{C.3}$$

or by (C.3) with $L \leftrightarrow R$.

Avoiding limits from $\mu \to e$ conversion

Another very important constraint stems from the limits on $\mu \to e$ conversion on the nuclei [219]. A leptoquark with $Q = +2/3$ mediates this process at the tree level by an interaction with the d quarks and also the sea s quarks in the nucleons. The calculation in `flavio` is based on Ref. [245]. The scalar-type effective vertices, $(\overline{d_R} d_L)(\overline{e_L}\mu_R)$ and $(\overline{d_R} d_L)(\overline{\mu_L} e_R)$, are predicted to trigger this process even more efficiently than the vector-type ones. Thus, to avoid these constraints when searching for limits from other interesting processes, the following condition must be approximately fulfilled:

$$|u^L_{de} u^R_{d\mu}|^2 + |u^R_{de} u^L_{d\mu}|^2 = 0. \tag{C.4}$$

Even without imposing unitarity, it can be shown that any U_L, U_R pair obeying Eq. (C.4) together with the set of Eqs. (5.14) must necessarily follow one of these

[1] Of course, this hassle is a result of not considering rigorously the domain of the parametrization. An analogy: among all point on the Earth surface lying on the Greenwich meridian there is also the North and South poles. However, on the poles one can formally choose any longitude as it is actually irrelevant.

patterns:

$$(U_L)_{ff} = \begin{pmatrix} \bullet & 0 & \bullet \\ \bullet & 0 & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}, \ (U_R)_{ff} = \begin{pmatrix} \bullet & 0 & \bullet \\ \bullet & 0 & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}; \tag{C.5a}$$

$$(U_L)_{ff} = \begin{pmatrix} 0 & \bullet & \bullet \\ 0 & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}, \ (U_R)_{ff} = \begin{pmatrix} 0 & \bullet & \bullet \\ 0 & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}; \tag{C.5b}$$

$$(U_L)_{ff} = \begin{pmatrix} 0 & 0 & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}, \ (U_R)_{ff} = \begin{pmatrix} 0 & 0 & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}; \tag{C.5c}$$

$$(U_L)_{ff} = \begin{pmatrix} 0 & 0 & \bullet \\ 0 & 0 & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}, \ (U_R)_{ff} = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}; \tag{C.5d}$$

$$(U_L)_{ff} = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}, \ (U_R)_{ff} = \begin{pmatrix} 0 & 0 & \bullet \\ 0 & 0 & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}; \tag{C.5e}$$

where $\bullet$ denotes an unfixed value. Notice that only the patterns (C.5a) – (C.5c) can be obtained for 3×3 unitary matrices, while the zeros in (C.5d) and (C.5e) require that $(U_{L,R})_{ff}$ is a part of a unitary matrix of dimension ≥ 4.

Finding the unitary parametrization fulfilling both Eqs. (5.14) and (C.4) is lengthy but straightforward. The solutions in the composite parametrization must be found for each pair of $(\dim U_L, \dim U_R)$ separately, unlike for Eq. (5.14) alone.

Notable but order-of-magnitude smaller contributions to the coherent $\mu \to e$ conversion arise also from vector-type operators (triggered by $u^L_{de} u^{L\,*}_{d\mu}$ and $u^R_{de} u^{R\,*}_{d\mu}$) and well as the muon conversion on the sea s-quarks in the nucleons (such amplitudes are proportional to $u^L_{se} u^R_{s\mu}$ or $u^L_{se} u^R_{s\mu}$).

Scanning of the parameter space

In what follows, *parameter point* means a particular form of U_L and U_R. Note that for each parameter point, a range of leptoquark masses is allowed and has to be considered (see Section 5.1.3).

For each of the models studied in Chapter 5, the presented results rely on investigating the matrices $U_{L,R}$ obtained in several different ways:

- 10^3 parameter points have been obtained by random scanning of the parameter space with the measure $\prod_{ij} \mathrm{d}\lambda_{ijL} \mathrm{d}\lambda_{ijR}$.
- About $2{\times}10^3$ parameter points have been chosen by random scanning (using the same naive measure) of the parameter space restricted by Eq. (C.2).
- More than 10^4 parameter points have been obtained by random scanning restricted by Eq. (C.2) and by solutions to Eq. (C.4).
- For the basic model ($k_{L,R} = 0$), also many manually chosen and tuned points have been investigated. Considering various special parts of the parameter

space enables one to claim with a high level of confidence that the catalogue in Table 5.1 is *complete*.

Note that considering 10× as many parameter points would require a non-trivial modification of the code since analysis of a single point currently typically takes more than a minute.

C.2 Examples

For each observable whose measurement currently limits the gauge-LQ mass for some parameter point in the basic model of quark-lepton unification $(k_L, k_R) = (0, 0)$ (see Table 5.1), a corresponding example form of the $U_{L,R}$ matrices is provided in Table C.2.

Notice that the form of $U_{L,R}$ for which m_{U_1} is restricted by $\mathrm{BR}(B^0 \to \mu\mu)$ has been highly tuned so that: (i) there is a cancellation of amplitudes for coherent $\mu \to e$ conversion on d-quarks and see s-quarks in the nuclei; (ii) the LQ and SM amplitudes to $B_s \to \mu\mu$ satisfy $\mathcal{M}^{\mathrm{NP}} \simeq -2\mathcal{M}^{\mathrm{SM}}$, yielding $\mathrm{BR}(B_s \to \mu\mu) \simeq \mathrm{BR}^{\mathrm{SM}}(B_s \to \mu\mu)$.

C.3 Observables included in the likelihood

The set of observables taken into account by `smelli` in the analysis of Chapter 5 is listed in Table C.3 In our setting, the global likelihood is a product of 5 partial likelihoods, each of which considers several (tens of) observables. Generally, `smelli` offers 7 more partial likelihoods but we have checked that they essentially do not differ from their SM value in the considered scenarios. Thus, they have been omitted in order to save the computing time.

The names of the observables are adopted from `flavio`. Observables in angle brackets are measured in several bins of dilepton invariant mass squared which we do not indicate explicitly. For example, the R_{K^*} ratio, denoted as `<Rmue>(B0->K*ll)` here, is considered in two different bins measured by LHCb and two other bins measured by Belle II. For further details, see the on-line documentation of the `flavio` package [117].

Observable	U_L	U_R
$\mathrm{BR}(K_L^0 \to e\mu)$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$\mathrm{BR}(K_L^0 \to ee)$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$
$\mathrm{BR}(K_L^0 \to \mu\mu)$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$
$\mathrm{BR}(K_S^0 \to \mu\mu)$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$
$\mathrm{BR}(B^0 \to \mu\mu)$	$\begin{pmatrix} 0 & -\frac{1}{\sqrt{21}} & -\sqrt{\frac{20}{21}} \\ 0 & -\sqrt{\frac{20}{21}} & \frac{1}{\sqrt{21}} \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{21}} & 0 & -\sqrt{\frac{20}{21}} \\ \sqrt{\frac{20}{21}} & 0 & -\frac{1}{\sqrt{21}} \\ 0 & -e^{0.8i} & 0 \end{pmatrix}$
$\mathrm{BR}(B_s \to \mu\mu)$	$\begin{pmatrix} 0. & 0. & i \\ -0.26-0.34i & 0.78-0.45i & 0. \\ -0.74-0.52i & -0.29+0.32i & 0. \end{pmatrix}$	$\begin{pmatrix} 0. & 0. & 1 \\ 0.20-0.29i & 0.83-0.43i & 0. \\ -0.14-0.92i & -0.12+0.34i & 0. \end{pmatrix}$
$\mathrm{BR}(B^0 \to e\mu)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$
$\mathrm{BR}(B_s \to e\mu)$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$\mathrm{CR}(\mu \to e, \mathrm{Au})$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$
$\mathrm{R}_{e/\mu}(K^+ \to l\nu)$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathrm{R}_{e/\mu}(\pi^+ \to l\nu)$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Table C.2: Examples of quark-lepton mixing matrices and corresponding dominant signals of the gauge leptoquark.

likelihood_lfu_fccc
BR(Bc->taunu)
FLtot(B0->D*taunu)
 /BR(B->Dtaunu)
 /BR(B->D*taunu)
BR(pi+->enu)
Remu(K+->lnu)

likelihood_lfu_fcnc
<Rmue>(B0->K*ll)
<Rmue>(B+->K*ll)
<Rmue>(B+->Kll)
<Rmue>(B0->Kll)
<Dmue_P4p>(B0->K*ll)
BR(B+->Ktautau)
BR(B0->tautau)
BR(Bs->tautau)

fast_likelihood_leptons
BR(tau->enunu)
BR(tau->mununu)
BR(tau->pinu)
BR(tau->Knu)
a_e
a_mu
a_tau

likelihood_lfv
BR(B+->K*emu)
BR(B+->K*mue)
BR(B0->K*emu)
BR(B0->K*mue)
BR(B+->Kemu)
BR(B+->Ketau)
BR(B+->Kmue)
BR(B+->Kmutau)
BR(B+->Ktaue)
BR(B+->Ktaumu)
BR(B+->pimutau)
BR(B+->pitaue)
BR(B+->pietau)
BR(B+->pitaumu)
BR(B0->emu,mue)
BR(B0->etau,taue)
BR(B0->mutau,taumu)
BR(Bs->emu,mue)
BR(Bs->mutau,taumu)
BR(B0->piemu,mue)
BR(B+->piemu,mue)
BR(KL->emu,mue)
BR(mu->eee)
BR(mu->egamma)
BR(tau->eee)
BR(tau->muee)
BR(tau->mugamma)
BR(tau->mumumu)
BR(tau->emumu)
BR(tau->muemu)
BR(tau->emue)
BR(tau->egamma)
BR(tau->rhoe)
BR(tau->rhomu)
BR(tau->phie)
BR(tau->phimu)
CR(mu->e,Ti)
CR(mu->e,Au)

fast_likelihood_quarks
<FL>(B0->K*mumu)
<P1>(B0->K*mumu)
<P4p>(B0->K*mumu)
<P5p>(B0->K*mumu)
<dBR/dq2>(B+->K*mumu)
<dBR/dq2>(B0->K*mumu)
<dBR/dq2>(Bs->phimumu)
<dBR/dq2>(B+->Kmumu)
<dBR/dq2>(B0->Kmumu)
 (B->Xsmumu)
<AFB>(B0->K*mumu)
<P2>(B+->K*mumu)
<P4p>(B+->K*mumu)
<FL>(B+->K*mumu)
<P5p>(B+->K*mumu)
<P1>(B+->K*mumu)
<dBR/dq2>(Lambdab->Lambdamumu)
<AFBh>(Lambdab->Lambdamumu)
<AFBl>(Lambdab->Lambdamumu)
<AFBlh>(Lambdab->Lambdamumu)
BR(Bs->mumu)
BR(B0->mumu)
BR(Bs->ee)
BR(B0->ee)
BR(B+->K*gamma)
BR(B0->K*gamma)
BR(B->Xsgamma)
ACP(B->Xgamma)
BR(B0->K*gamma)/BR(Bs->phigamma)
BR(Bs->phigamma)
ADeltaGamma(Bs->phigamma)
S_phigamma
S_K*gamma
<FL>(Bs->phimumu)

<S3>(Bs->phimumu)
<S4>(Bs->phimumu)
 (B->Xsee)
<P2>(B0->K*ee)
<FL>(B0->K*ee)
<P1>(B0->K*ee)
<ATIm>(B0->K*ee)
BR(B0->pitaunu)
BR(B+->enu)
BR(B+->munu)
DeltaM_s
x12Im_D
S_psiK
S_psiphi
eps_K
epsp/eps
BR(K+->pinunu)
BR(KL->pinunu)
BR(KL->mumu)
BR(KS->mumu)
BR(KL->ee)
BR(KS->ee)
BR(K+->munu)
BR(KL->pienu)
BR(KS->pienu)
BR(K+->pienu)
BR(KL->pimunu)

BR(KS->pimunu)
BR(K+->pimunu)
lnC(K->pimunu)
RT(K->pimunu)
tau_n
Atilde_n
lambdaAB_n
a_n
atilde_n
Btilde_n
D_n
R_n
Ft(10C)
Ft(14O)
Ft(22Mg)
Ft(26mAl)
Ft(34Cl)
Ft(34Ar)
Ft(38mK)
Ft(38Ca)
Ft(42Sc)
Ft(46V)
Ft(50Mn)
Ft(54Co)
Ft(62Ga)
Ft(74Rb)
Gamma(pi+->munu)

Table C.3: The list of observables included in our version of the `smelli` global likelihood. (The table begins on the previous page.)

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List of author's publications

Regular articles

[25] T. Faber, M. Hudec, M. Malinský, P. Meinzinger, W. Porod, F. Staub
A unified leptoquark model confronted with lepton non-universality in B-meson decays
Physics Letters B **787** (2018) 159

[77] M. Hudec, M. Malinský
Hierarchy and decoupling
Journal of Physics G: Nuclear and Particle Physics **47** (2020) 015004

[29] T. Faber, M. Hudec, H. Kolešová, Y. Liu, M. Malinský, W. Porod, F. Staub
Collider phenomenology of a unified leptoquark model
Physical Review D **101** (2020) 095024

Conference proceedings

[30] M. Hudec
Confronting quark-lepton unification with LFUV
40th International Conference on High Energy Physics (ICHEP2020)
Prodeedings of Science, **390** (2021) 380

The analysis and results of Refs. [25, 29] and [30] have been discussed in this thesis. Ref. [77] is devoted to a different topic which has not been elaborated on here but rather in author's Master thesis [246].

Attachments

The three journal articles [25, 77, 29] listed on page 113 are attached to this thesis.