Report on "Vector-valued integral representation"

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The main objective of this thesis is to study properties of (the space of) functions $f : X \to E$ based on their behaviour on the set of extreme points $\operatorname{ext} X$, where X is a convex compact set in a locally convex space and E is a Banach space. This is a research topic on the boarder of the theory of $C(K, E)$ spaces and Integral representation theory as described in the monograph [1].

The thesis consists of 7 research papers, 6 of which are published in good journals and 1 of which is submitted. The first two papers deal with maximum principle of convex functions, in the remainder the author of this thesis joins the active stream of research trying to find all the possible generalizations/variants of the Banach-Stone theorem which asserts that two compact spaces K and L are homeomorphic if and only if the Banach spaces $C(K)$ and $C(L)$ are linearly isometric. This is nowadays quite an active field of research, probably mostly among researchers from Brazil and Poland.

I find the presented results very complex and deep. Topic of the thesis is considered from several deep perspectives and the author had to study many research areas (most notably very deep aspects of the isometric and isomorphism theory of Banach spaces, measure theory, descriptive set theory and topology) in order to find the right context and formulation of the problems he considers. New directions of research are established by considering new and highly non-trivial context of abstract Choquet theory, which enables the author not only to generalize what was previously known to this new context, but also find new results even in the classical case of $C(K, E)$ spaces which was previously considered by other authors. An example of this is the last paper included in the thesis, where in Theorem 1.1 and Theorem 1.2 the author proves an abstract result (in the context of Choquet theory) and he observes in Corollary 1.5 that this has consequences to the theory of $C(K, E)$ spaces, where he obtains a new and very general result in this classical setting improving several results proved previously by other authors.

The topic of the thesis, used methods and their novelty is very nicely explained in the Introduction with a very rich source of references given. The thesis is pleasant to read.

Conclusion: The Dissertation has a very high level. It both deals with established parts of mathematics and natural questions considered by other authors as well as it introduces new notions and formulate new problems which might stimulate further research. The author has done an excellent job, contributing towards the development of a well-established research area. The thesis is pleasant to read. It explains the results and their context very well. The author clearly deserves the degree of Doctor of Philosophy.

References

[1] Lukeš, J., Malý, J., Netuka, I., and Spurný, J. *Integral representation theory*, vol. 35 of *De Gruyter Studies in Mathematics*. Walter de Gruyter & Co., Berlin, 2010. Applications to convexity, Banach spaces and potential theory.