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Subject: *Report on the doctoral thesis "Structure of flow-continuous Mappings in Algebraic Context" by Radek Hušek*

To whom it may concern,

The doctoral thesis lies at the intersection of Computer Science and Mathematics, considering questions in graph theory and solving them borrowing tools from both sides. Radek Hušek focuses on group flows, group connectivity of graphs and the number of cycle[1] double covers in a graph. Most but not all of his work deals with cubic graphs and in particular snarks.

Chapter 1 is an introduction that contains all the useful definitions and theorems for the rest of the thesis. The reader cannot help but wonder why the proof (by the author and two others) that a hypothetical minimal counter-example to the 5-flow conjecture has girth at least 12 remains unpublished 6 years after its first public announcement.

Chapter 2 discusses how the author obtained (in collaboration) a computer-assisted solution to challenging 1992 question by Jaeger et al. They exhibited two small graphs that fully distinguish $\mathbb{Z}_2^2$- and $\mathbb{Z}_4$-connectivity. I enjoyed the discussions of how they designed an algorithm that was efficient enough, and particularly like Conjecture 2.17.

Chapter 3 is rightfully presented as partial results regarding a conjecture that homomorphisms between Cayley graphs should guarantee existence of group flows. While the results are certainly remarkable, it feels like the framework and proof techniques have a lot of potential – I hope that this research direction will be continued.

Chapter 4 considers upper and lower bounds on the number of cycle double covers in cubic graphs. This chapter is extremely pleasant to read, the tools being basic but elegant. Chapter 5 introduces more technical, algebraic tools, which really prove their worth in Chapter 6, where exponential lower bounds are provided for a special family of interesting cubic graphs, called Flower snarks, as well as for planar cubic graphs. The results for Flower snarks are remarkably precise, while the exact behaviour for planar cubic graphs is left as a conjecture.

Finally, Chapter 7 further exploits the tools of Chapter 5 to generate snarks in a new fashion,

[1]Called circuits in his thesis – cycles being another object – but I prefer the names that I am most used to.

and the thesis concludes with an appropriate selection of the most striking open questions left from the thesis. Since I already mentioned Conjecture 2.17, let me highlight the bold Conjecture 6.10 – studying it might indeed give some insight in the elusive Cycle Double Cover conjecture.

Overall, this thesis shows an interesting span of research interests and competences, a search for elegance and a keen scientific mind. I note with satisfaction that Radek Hušek published high-quality papers with and without his PhD advisor. The research topics are well motivated, deep, and well established in the community. All things considered, I fully support, based on this thesis, that Radek Hušek be awarded the doctoral degree.

Yours sincerely,

Dr Marthe Bonamy