A Report on
"Minimal Taylor Clones on Three Elements"
by Filip Jankovec.

Clones are sets of operations containing all projections and closed under composition. The least clone is trivial and contains only projections. The author studies minimal clones that avoid such a trivial behaviour. Formally speaking, minimal Taylor clones are minimal idempotent clones containing an operation satisfying an identity that cannot be satisfied by projections.

The author studies minimal Taylor clones on three elements, which were completely described by Brady. For 12 of them the author described operations of the clone and gave its relational definition. For most cases the obtained description is based on the characterization of minimal clones on $\{0, 1\}$, which is also given in the thesis. It is not always clear what a complete description of all the operations is. For example, the clone generated by the majority operation on $\{0, 1\}$ is described as the clone of all self-dual and monotone operations, and I don't see a better description if this clone. Thus, I am convinced that the characterization given for these 12 clones is final and cannot be significantly improved.

Notice that minimal Taylor clones have a lot of nice properties, which makes it possible to characterize all their operations. By the same reason, the obtained results are important for the study of the lattice of clones and finite algebras on a 3-element domain.

Even though I found quite a lot of inaccuracies and misprints (see below), the quality of the mathimatical text is rather high, all statements are provided with a detailed proof, and the definitions are precise. Nevertheless, closer to the end of the paper proofs are not that good, some implications are not explained, and numerous inaccuracies make it difficult to understand the proof. Additionally, some proofs repeat the same idea, which can definitely be generalized. For example, the author checks many times that an operation preserves the relation T_S. Instead, he could prove another direction of Lemma 1.6 and use it everywhere.

Another thing that could make the work sronger is a description of all relations that are invariants of the corresponding clone. For clones generated by majority operation it follows from the Baker-Pixley Theorem, but I believe a similar discription could also be obtained for some other clones. Neverthellesss, even current work contains very interesting and highly nontrivial resuts, that is why, I recommend this work be accepted as a Master thesis.

Some remarks are listed below.

1. Why don't you prove Lemma 1.6 in the form "$f \in \mathrm{Pol}(T_S)$ if and only if ...". As a result you will not need to check compatibility with T_S in many lemmas.

2. References in the paper look strange. The author refers to the first author of the paper and the year instead of the index in the bibliography. As a result names are mixed with the text which is hard to read. Also, it is a

bad style to mention only the first author, either you mention all of them, or none of them.

3. Definition 1.4. Write $ar(f)$ instead of $ar(f^A)$.
4. Definition 1.4. Restriction to B^n was never defined.
5. Definition 1.13. i was from $[m]$ before, better change i to j in the formula.
6. Definition 1.23. It is difficult (or even impossible) to understand if you don't know what it means.
7. All minimal Taylor clones but $\mathbf{T}_4^N$ are defined. I would definitely define it here.
8. Theorem 2.1. It is too informal to write Baker-Pixley Theorem in this form without any explanation. When you write projection, you get binary relations, and it is not clear what you mean by conjunction of binary relations.
9. Lemma 2.5. I think it should be $T^{\mathbf{A}}$ instead of $T^{\mathbf{M}}$ (also in the proof).
10. Proof of Lemma 2.5. You talk about majority operation m in the statement, but here you consider some maj. Moreover, you cannot just put maj in the formula, you should put the term defining m.
11. page 17, line -11. Remove extra }.
12. Proof of Theorem 2.6. n is the size of your domain, you cannot use it in the definition of S. What is I in the sum? Should it be sum from 1 to m? Please exmplain the induction (first line of page 18).
13. Page 18, line -14. In my opinion, $S_{0,1}^2$ is a generalization of $S_{0,1}$ and not vice versa.
14. page 19, line 2. It should be m instead of n.
15. Page 19, line 11-12. I would explain for the first time how to apply relations C_0 and C_1 in pp-definition.
16. It was a bit confusing that upper index always means arity but in aff^n it means the size of the domain.
17. page 33, line 4. Change "fist" to "first".
18. page 33, lines -3. I would explain how to pp-define this (at least for the first time).
19. Lemma 5.8. The implications inside and outside brackets should be switched.
20. page 36, line -10. It should be $\in \mathfrak{F}$ instead of $= [n]$.

21. Proof of Lemma 5.12. The proof is too twisted. To prove that a relation is preserved you apply an operation to some tuples, assume that the result is outside of the relation and prove that some of the original tuples are outside. In each of the items 1 and 2 you assume that the original tuples are in the relation and prove that they are really outside. Why don't you just prove by contradiction assuming that the original tuples are inside?

22. Item 4 of the proof of Lemma 5.14 was not clear to me.

23. Definition 5.5. I think it should be $V \subseteq S$ or $V \setminus S = \varnothing$ instead of $V \cap S = \varnothing$. The same is written in the proof, which does not make much sense.

24. Line 3 of the proof of Teorem 5.16. Should it be $A^{|S|}$ instead of A^{m_s}?

25. $\mathfrak{F}$ is an ms-collection by definition. Probably you wanted to say bms-collection in "This shows that $\mathfrak{F}$ is an ms-collection".

26. The beginning of the proof of Theorem 5.16. What is $\mathfrak{F}_{f^S}$? What do you mean by $f \in \mathcal{M}^{1,2}$?

27. page 40, line -10. It should be $=$ instead of $\subseteq$.

28. page 41, line 4. You should use the last property of a bms-collection here.

29. page 41, line 6. Add "$= a$".

30. page 41, line 4. There should be $S \in \mathfrak{F}$.

31. page 41, item 1. The explanation is not good. You don't really care whether $\{i \mid a_i = 1\} \in \mathfrak{F}$. Explain where you use self-duality, where you use monotonicity. Just rewrite it completely.

32. page 43, line 3. Should it be $\{2, b\}$ instead of $\{0, a\}$?

33. page 43, line 11. It should be $d = 0$ instead of $d = 2$.

34. page 43, line 14, line 15. It should be $\notin \{b, 2\}$.

35. page 43, it should be G_a instead of D_a.

36. page 43, item 3 (the second one). It is not true that $c = d$ if $c_i = d_i = c_j = d_j = 2$.

37. page 44, line 4. It is very confusing when you prove $f(2, 2, \ldots, 2) = 2$ by compatibility with C_2, which follows from idempotency. This just follows from idempotency.

38. The second half of page 44 is difficult to understand.

39. page 45, line -1. Should it be $F^2_{1,0}$ instead of $F^2_{1,2}$. The same is in the next line.

40. page 46. There should be $S \subseteq [n]$ instead of $S \in [n]$ several times.
41. page 46. In the definition of $\mathfrak{F}_S$ explain how you quantify i.
42. page 46, line -3. b was never defined.
43. page 48. Definition 6.4. Why don't you define $\oplus$ as an operation that returns 2 on $(2, 2)$. This would simplify everything.
44. Definition 6.5. It should be $\{i\}$ instead of $\{j\}$.
45. page 48, line -4. Replace comma by $\star$
46. page 49, line 2. You apply this property for $f_{V,i,k}(e_{V,j})$ for different i, j, k.
47. The middle of page 49. $V \subseteq [n]$ is not just some V, you should set $V = V_{\mathbf{a}}^2$.
48. page 49. Remove m after 6.5.
49. page 50, line 11. Explain how we get an equation starting with "We know".