MATEMATICKO-FYZIKÁLNÍ
FAKULTA
Univerzita Karlova

Bachelor thesis review

Review of opponent

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Thesis title:	Coloring of triangle-free graphs on torus
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The purpose of this thesis is to provide implementation of algorithms for 3-coloring recently discovered by Dvořák and Pekárek. They use the flow-coloring duality and network flows algorithms to get efficient solutions in particular for graphs that are near-quadrangulations. The thesis starts by description of the algorithm in the planar case and then progresses to more complicated contexts: coloring extensions and, mainly, toroidal graphs.

The theoretical description of the algorithms is mostly easy to read and correct (I will mention some comments later). This is quite an achievement, as the algorithms are rather complex and require understanding of advanced graph theory concepts. I have not read the implementation in detail, but I verified that it compiles and, apparently, passes all the tests.

There are occasional typos and inconsistencies:

- b_H instead of b_{G^*} on page 8 (four lines above Theorem 2.2),
- Lemma 1 instead of Lemma 2.1 on the same page, second line of the proof,
- on page 12, definition of H' contains $dual(\phi)$ on both sides,
- on page 15, the flow called f_{avg} is more a difference, than an average,
- on page 17, in the first line of the proof of Theorem 3.9, we probably need to assume not only $\Delta f_{max} = \Delta f$, but also a maximality property along the path P.

For a more substantial question: you mention that the implemented algorithms only check colorability, they don't actually provide a coloring. Is there any obstacle to this, or is it just because of lack of time? It seems the duality should give you an explicit color for each vertex, once you find the flow?

Finally, I must acknowledge that the extent of the thesis is substantially larger than usual for bachelor theses: definitely in the theoretical part and probably also in the implementation part.

Recommendation: In accordance with the above, I recommend to accept the thesis as a bachelor thesis and to classify it by the best grade.

In Liberec on Aug 30, 2022

Robert Šámal