matfyz

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Advisor's statement to a doctoral dissertation
" Mathematical Analysis of Selected Problems for Complex Fluids"
by
Tomáš Los

The doctoral thesis by Tomáš Los is based on three complete papers (two of them are published, the revision of the third one is reviewed), namely:

(1) A. Abbatiello, T. Los, J. Málek, O. Souček: On unsteady flows of pore pressure-activated Bingham fluids. *Mathematical Models and Methods in Applied Sciences,* 29(11): 2089–2125, 2019.
(2) A. Abbatiello, M. Bulíček, T. Los, J. Málek, O. Souček: On unsteady flows of pore pressure-activated granular materials. *Zeitschrift für Angewandte Mathematik und Physik*, 72(1):1–18, 2021.
(3) M. Bulíček, T. Los, Y. Lu and J. Málek: On planar flows of viscoelastic fluids of the Giesekus type. The revision considered for publication in *Nonlinearity*, 2022.

The first two articles build on and significantly extend the candidate's MSs thesis focused on thermodynamic derivation, analysis and even some computation of steady flows relevant to Bingham-Korteweg fluids with the activated (threshold) stress depending on the pore-pressure driven by an additional convection-diffusion equation.. The MSs thesis is inspired and uses in the analytical part the approach of Chupin&Mathé (2017). During his first years of PhD studies,

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Tomáš Los was pushed to cooperate with Anna Abbatiello (a PhD student from University in Caserta spending two semesters in our group) and extend the results by Chupin&Mathé (2017) in several directions. Firstly, the model proposed to describe time-dependent processes in water-saturated granular materials is derived/deduced from general principles of the theory of interacting continua (here the team made use of Ondřej Souček's contributions). Second, the constitutive equation for Bingham fluid with a yield stress active by a pore pressure is embedded into the framework of implicitly constituted fluids in several different but equivalent forms. These equivalences are proved in the paper. In addition, the analysis is carried out for a slightly modified system, but one then requires different tools (realistic data are treated in the second paper; an approach how to avoid the apparent need to prove boundedness for pore pressure is proposed by Miroslav Bulíček).

During his PhD studies, Tomáš Los spent one semester at the Hausdorff Research Institute for Mathematics at University of Bonn (spring 2019). He has been also visiting Dr. Yong Lu at Nanjing University (Sept 2019), where we began to understand a sketchy proof published in Nader Masmoudi's paper on long-time and large-data analysis for unsteady flows of a Giesekus fluid, with the aim of understanding appropriate tools for analysis of viscoelastic rate-type fluid models with multiple stress relaxations (which can be viewed as a generalization of the second order model due to Burgers). This initiated a long-term effort (with significant contributions from the candidate) that resulted in a complete proof for the analysis of the evolutionary Giesekus model in two dimensions, which is the third paper cited above.

On the overall assessment, I must admit that I have not been able to change the candidate's approach to writing mathematical texts, teaching students, preparing for exams, etc. Tomáš Los is often focused too much on an important detail (in the study, teaching, social context), but completely loses the understanding of its importance in the broader context. However, this has no negative impact on the thesis under review. However, it could have resulted in other achievements. In conclusion, **I am pleased to recommend Tomáš Los' thesis for defence.**

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