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Effective description of resonances at low energy region

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In Prague, July 13, 2022 ....

Author's signature

This thesis is dedicated to my parents

Title: Effective description of resonances at low energy region

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Abstract:

In this doctoral thesis, we study all the relevant two- and three-point Green functions of the chiral currents and densities. Specifically, in the first part of the thesis, we present the leading-order contributions of the QCD condensates up to dimension six to these Green functions, obtained within the framework of the Operator product expansion. These consist of the perturbative contribution followed by the contributions of the quark, gluon, quark-gluon and four-quark condensates.

In the second part, we restrict ourselves to the order parameters of the chiral symmetry breaking in the chiral limit. We investigate them within the Chiral perturbation theory and Resonance chiral theory and, in order to obtain constraints on the parameters of the effective Lagrangians, we require their high-energy behaviour to match OPE. As it turns out, the duplication of the lowest vector, axial-vector, scalar and pseudoscalar resonance multiplets in the corresponding Lagrangians is necessary.

As a special case, we study the $\langle VVP \rangle$ Green function with three vector and three pseudoscalar resonance multiplets taken into account — needless to say, this investigation is performed on an algebraic level only. We also study the correlation of the pion-pole contribution to the muon $g-2$ factor and the effective parameter $\chi^{(\mathrm{r})}$.

Keywords: Quantum chromodynamics, Chiral perturbation theory, Resonance chiral theory, Green functions, Operator product expansion.

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Contents

Introduction

Quantum chromodynamics (QCD) is thought to be the correct theory for a description of the strong interactions in terms of the quarks and gluons — at least the decades of undergoing various experimental checks incline us to such a conclusion. Its nonabelian structure, however, leads to the fact that QCD can not be treated perturbatively at low energies and one is thus forced to proceed to use the effective theories — the Chiral perturbation theory (ChPT) and Resonance chiral theory (RChT).

These methods are based on effective Lagrangians that, as the field content, take into account lowest hadronic states instead of the quarks and gluons. On top of that, the Lagrangians usually contain a large number of unknown parameters and these need to be obtained in order to secure the predictability of one's model. To this end, one may construct the respective Green functions that can be associated with physical processes. Using then either experimental inputs or an expected theoretical behaviour of these objects, such as their high-energy limit obtained within the framework of Operator product expansion (OPE), one may be eventually able to extract some unknown parameters of corresponding effective Lagrangians.

This doctoral thesis, based on two journal articles, [1] and [2], deals with the above-mentioned approach. Firstly, the OPE of all the relevant two- and three-point Green functions was studied in ref. [1]. Then, we have restricted ourselves only to a small part of the Green functions in ref. [2], namely to $\langle VVP \rangle$, $\langle VAS \rangle$ and $\langle AAP \rangle$. Then, the contributions of resonance multiplets to these functions have been calculated therein. Subsequently, these resonance contributions have been then matched onto the respective OPE. Such a procedure then led to constraints for some of the unknown coupling constants of the effective ChPT and RChT Lagrangians.

Outline. This thesis consists of four chapters and four appendices. After the Introduction, a rudimentary chapter 1 follows. In section 1.1, we firstly portray a brief historical background behind the theory of strong interactions. Then, we follow with a qualitative description of the Quantum chromodynamics, Chiral perturbation theory and Resonance chiral theory in sections 1.2, 1.3 and 1.4, respectively.

Chapter 2 deals with the framework of Green functions. In detail, the concepts of Green functions of chiral currents is introduced in section 2.1. Then, we describe their Ward identities in 2.2, classification in 2.3 and tensor decomposition in 2.4. It is important to notice that throughout this chapter, we provide a set of useful informations needed for forthcoming parts of the thesis, nevertheless, we tacitly assume and acquaintance with section 2 and appendices E and F of our paper [1].

In chapter 3, we comment on the well-known properties of the vacuum structure of QCD and add several new observations obtained in our article [1]. Specifically, in sections 3.1, 3.2 and 3.3, we discuss the presence of condensates in the QCD vacuum, introduce the concept of Operator product expansion and comment on a greatly useful framework of the Fock–Schwinger gauge. Then, in section 3.4, we discuss the quark propagator in external gluon field in both the coordinate and momentum representations. Finally, section 3.5 summarizes the propagation formulae that we have derived in article [1] and that were used to study the OPE of all the relevant two- and three-point Green functions therein.

In the last chapter 4, we build upon our paper [2] and discuss results obtained therein. In detail, sections 4.1 and 4.2 contain a description of the procedure of extending the respective Lagrangians for multiplets of resonances with higher mass and, consequently, the corresponding results of resonance contributions to the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions are presented. Then, the matching of these results onto the respective OPE and ChPT contributions are given in sections 4.3 and 4.4, respectively. Finally, section 4.5 deals with a study of a situation with three vector and three pseudoscalar resonances taken into account in the case of the $\langle VVP\rangle$ Green function, and refers to several phenomenological examples studied in detail in [2].

The four appendices then follow. In appendix A, we provide a short note on the Fourier transform used in ref. [1], while appendix B extends the discussion by providing a look beyond the chiral limit within the OPE. Finally, appendices C and D refer to the full versions of articles [1] and [2] attached to this thesis.

Original results. Results presented in this thesis are based on author's journal articles [1, 2], accompanied by the conference proceedings [3, 14], to which we refer throughout this thesis extensively.[1]

Notation. The notation used throughout this thesis corresponds to the one used in refs. [1, 2]. For clarity, we present below only some of the definitions or conventions that truly needs to be reminded. If necessary, a ref. [16] shall be consulted.

- Einstein's summation convention is employed throughout this thesis.
- SU(3) generators T^a ($a = 1, \ldots, 8$) are defined as halves of the Gell-Mann matrices λ^a,
$$T^a = \frac{1}{2}\lambda^a\,, \qquad \mathrm{Tr}\,(T^aT^b) = \frac{1}{2}\delta^{ab}\,.$$
These generators satisfy the following (anti)commutation relations:
$$[T^a, T^b] = if^{abc}T^c\,, \qquad \{T^a, T^b\} = \frac{1}{3}\delta^{ab} + d^{abc}T^c\,,$$
where d^{abc} is the totally symmetric SU(3) group invariant and f^{abc} is the totally antisymmetric SU(3) structure constant.
- Dirac representation of gamma matrices is considered. We use
$$\varepsilon^{0123} = +1\,,$$
the fifth Dirac matrix defined as
$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

[1] To complete the author's contributions, a study text [15] can be mentioned as well. This, however, is not related to the concepts studied in this thesis whatsoever.

and the commutator is taken to be

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

- A short-hand designation for the contractions of Levi-Civita tensor with the components of momenta is used:

$$\varepsilon^{\mu\nu\alpha(p)} \equiv \varepsilon^{\mu\nu\alpha\beta} p_\beta\,, \qquad \varepsilon^{\mu\nu(p)(q)} \equiv \varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta\,.$$

- Feynman's "slash notation" for an arbitrary four-vector p_μ is employed:

$$\not{p} \equiv p_\mu \gamma^\mu\,.$$

1. Theory of strong interactions

Without the four fundamental interactions, the universe would not be able to exist. One may thus argue that the gravitational, weak, electromagnetic and strong interactions are the pillars that not only support but actually allow an existence of nature and life in it as we know it. It would be meaningless to dispute about what interaction is the most significant one — they all are irreplaceable and equally important, although each of them dominates on its own domain, be it either a typical length scale of relevance or a type of particles they act on.

Nevertheless, one can safely say that the strong interaction represents a force which behaviour is rather specific in comparison to the others. Such a peculiarity is given by the fact that the strength of the strong interaction grows with the distance. Such a feature confines the quarks into hadrons and, consequently, allows atomic nuclei to hold together.

In this section, we briefly describe the history behind the theory of strong interactions and then discuss the framework of quantum chromodynamics. Then, we present a pedagogical introduction into effective field theories of strong interactions at low energies, i.e. the Chiral perturbation theory and Resonance chiral theory.

1.1 Historical introduction

The 1950s and 1960s represented the golden age of particle physics due to a tremendous progress in producing and detecting new particles — the hadrons, as they were named later by Lev Borisovich Okun in 1962. Such a progress was possible not only because of the invention of the bubble chamber in 1952 by Donald A. Glaser [17] but also because of the significant improvements made on the spark chamber in 1955 by Paul-Gerhard Henning.[2]

Observation of a large quantity of particles indicated that not all of them can be fundamental, i.e. elementary — that is, further indivisible. Physicist thus began to sort the particles. Firstly, they were classified according to their charge and isospin by Eugene Wigner and Werner Heisenberg, whilst Tadao Nakano, Kazuhiko Nishijima and Murray Gell-Mann proposed to categorize them according to their strangeness [18, 19, 20]. The observed particles were divided into groups with similar properties. Such a classification, named the “eightfold way”, was proposed by Gell-Mann and Yuval Ne’eman in 1961 [21, 22]. In detail, such a scheme is based on the symmetry group SU(3) and the hadrons are members of specific representations — multiplets — of such a group. In detail, the baryons occupy octets and decuplets, while the mesons exist in octets and singlets.[3]

[2] Improvements of what is now called a spark chamber have been made, however, already since the late 1940s and, in a way, were a variation of the first particle detector constructed by Hans Geiger and Walther Müller in 1928.

[3] Let us note that the name “baryon” was introduced by Abraham Pais in 1953. On the other hand, the term “meson” was introduced already in 1934 by Hideki Yukawa who used it as a denomination for a particle that was believed to be a carrier of strong force inside atomic nuclei [23]. The first candidate for such a particle was observed in 1936 by Carl David Anderson and Seth Henry Neddermeyer [24]. However, although this “μ-meson” had a similar mass as the one expected by Yukawa, it turned out that it does not interact strongly and, therefore, could

Indeed, nine baryon resonances with spin 3/2 were already observed as of 1961. It was obvious that they can not form an octet — rather, as it was suggested by Gell-Mann and Ne'eman, they should belong to the decuplet. However, one particle was missing. Based on the mass differences inside the hypothetical decuplet, they predicted the existence of the Ω^- baryon with a mass approximately 1680 MeV. Such a missing piece was indeed found in 1964 [26] and the discovery was an incredible triumph of particle physics of that age.

In 1964, Gell-Mann and George Zweig tried independently to explain the structure of the hadron multiplets by assuming that the baryons and mesons are bound states of three hypothetical particles that form an SU(3) triplet [27, 28, 29]. Whilst Gell-Mann used the term "quarks" for such elementary particles, Zweig called them "aces" — as we now know, the former denomination got generally accepted and ensured its place in history.[4]

As we have already mentioned, the Gell-Mann's model postulated the existence of three fundamental quarks: u ("up"), d ("down") and s ("strange") with electric charges 2/3, $-1/3$ and $-1/3$ of the one of the positron, respectively. The internal structure of the proton could thus be explained as a bound state schematically denoted as uud, the one of the neutron as udd etc.

There was, however, problem with the above-mentioned Ω^- baryon since it was assumed to be a quark combination of the type sss. The Ω^- baryon is the lightest particle with three strange quarks and, therefore, the ground state of such a combination. Since its spin is 3/2, the spins of the three quarks are parallel. The wave function of the Ω^- baryon is then given by a product of its spin and quark parts and thus seems to be fully symmetrical — which, however, contradicts the Pauli's exclusion principle![5]

Obviously, there must be another quantum number the quarks carry and that acquire three different values in order to have the wave function antisymmetric. Such a problem has been studied extensively between 1964 and 1971 by Oscar Greenberg, Moo-Young Han, Yoichiro Nambu, Gell-Mann and Harald Fritzsch [32, 33, 34] and the solution was eventually found — it was proposed that quarks have an additional SU(3) gauge degree of freedom, the so-called colour charge that acquires the "values" of r ("red"), g ("green") and b ("blue"). Further, it was concluded that the wave functions of hadrons are singlets of the colour group.[6]

A concept of colour charge has been experimentally confirmed beyond reasonable doubt since then. One of the evidences is the measurement of the cross section of the e^+e^- annihilation at high energies. In fact, it can be shown that

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not be the particle responsible for holding nuclei together. Later on, it was realized that the Anderson's particle is actually the now-called muon. The first genuine meson, the "π-meson" or simply "pion", was finally observed in cosmic rays in 1947 by Cecil Powell, Hugh Muirhead, César Lattes and Giuseppe Occhialini [25].

[4] To be thorough, one must note that there was another effort to explain the internal structure of some of the hadrons preceding the one of Gell-Mann and Zweig. It is the Sakata model, introduced by Shoichi Sakata in 1956 [30]. Needless to say, the model proved to be incorrect since the proton, neutron and Λ-baryon were assumed to be fundamental particles that the other hadrons were consisted of.

[5] Needless to say, such a problematic property concerned also the doubly-charged Δ^{++} baryon, which is the state of three u-quarks and that was observed already in 1951 [31].

[6] The problem of an additional quantum number of quarks was also discussed already in 1965 by Nikolay Bogolyubov, Boris Struminsky and Albert Tavkhelidze [35, 36].

the following relation holds:

$$\frac{\sigma_{\mathrm{tot}}(E_{\mathrm{c.\,m.}}; e^+e^- \to \mathrm{hadrons})}{\sigma_{\mathrm{tot}}(E_{\mathrm{c.\,m.}}; e^+e^- \to \mu^+\mu^-)} \xrightarrow{E_{\mathrm{c.\,m.}} \gg m_f} 3 \sum_f e_f^2 \,, \tag{1.1}$$

where the sum runs over all the quark flavours (f being u, d, s etc.) with masses m_f and electric charges e_f, that are kinematically accessible at the centre-of-mass energy $E_{\mathrm{c.\,m.}}$, and where the factor of 3 in front of the sum stands for three colours of each quark flavour. The validity of the relation above has been indeed confirmed by many experiments for various regions of available energy.

Naturally, experimentalists kept searching for a direct observation of the quarks themselves — to no avail, nonetheless. It was often pointed out by Gell-Mann that quarks might be purely mathematical constructs and not real particles. On the other hand, Richard Feynman claimed that high-energy experiments indeed show quarks as real particles — even though he called them "partons" [37].[7]

In 1972 and 1973, Fritzsch, Gell-Mann and Heinrich Leutwyler interpreted the colour group as a gauge group [38, 39]. The corresponding gauge theory, the quantum chromodynamics (QCD), introduces an octet of massless colour gauge vector bosons, the gluons, that generate the interactions. The framework of QCD is based on the Yang–Mills theory developed in 1954 by Chen Ning Yang and Robert Mills [40]. Because of this, the gluons can interact also with each other, which was a bit unusual at the time — indeed, recall that photons in QED do not posses such a property.

The above-mentioned self-interaction of gluons has a major consequence. It makes the QCD to be asymptotically free, which means that the value of the strong coupling constant decreases as the energy increases. In other words, the quarks and gluons behave as free particles at high energies and, on the other hand, are confined inside the hadrons at low energies. Such a behaviour was discovered by David Gross, Frank Wilczek and David Politzer in 1973 [41, 42].[8]

"Show must go on", as would classic say — and it has gone. The discovery of the asymptotic freedom commenced a rapid development not only in QCD but in the quantum field theory as well. Here, however, we can not spend much time and space by the other fascinating stories that followed (or slightly preceded to it) — theoretical predictions and the discoveries of the heavy c, b and t quarks, observations of jets etc. Nevertheless, to make the tale complete as much as possible for our purposes, let us finally remark that not only the quarks, but also the gluons have been experimentally found as well — their existence was proven by the observation of the three-jet event by the TASSO experiment at the PETRA accelerator at DESY [45].

By this we conclude our short historical introduction and move onto the theoretical description of QCD in detail.

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[7]The distinction between the quarks and partons can now be understood only as a subtlety in nomenclature, however, both Gell-Mann and Feynman viewed the concept of "particles" and their fundamental properties slightly differently.

[8]Interestingly enough, the asymptotic freedom of the Yang–Mills theory was discovered even before that — by Iosif Benzionovich Khriplovich in 1969 [43] and Gerard 't Hooft in 1972 [44].

1.2 Quantum chromodynamics

From the historical entrée presented above, one can be sure of the fact that the quarks and gluons are real physical objects. Constructing a theory, governing interactions between them, is the natural next step. As we know already, such a theory indeed exists — the Quantum chromodynamics, which validity has been proved by many experimental verifications. In what follows, we shortly comment on the ideas leading to the QCD Lagrangian and consequences following from the nonabelian nature of this theory.[9]

Quarks and gluons. The Quantum chromodynamics (QCD) is a local nonabelian gauge theory of strong interactions. It is based on the colour SU(3) symmetry group as the underlying gauge group, with the fundamental degrees of freedom being the quarks and gluons.

In such a theory, the quarks $q_f(x)$ are Dirac spinors and transform under the fundamental representation of the SU(3) group. On top of that, it is theoretically and experimentally confirmed that there are six flavours of quarks. The respective flavour index f thus acquires the following "values":

$$f = u,\ d,\ s,\ c,\ b,\ t\,, \tag{1.2}$$

with, in addition to the quarks introduced in the preceding section, c ("charm"), b ("bottom") and t ("top") being quarks with electric charges $2/3$, $-1/3$ and $2/3$ of the one of the positron, respectively. To this end, each of the above-mentioned quark flavours is the colour triplet, with the colour indices denoted explicitly as

$$q_f(x) = \begin{pmatrix} q_f^r(x) \\ q_f^g(x) \\ q_f^b(x) \end{pmatrix}, \tag{1.3}$$

and with the spinor indices being tacitly omitted for simplicity. In what follows, however, we shall restrain ourselves from writing down any indices apart from the flavour ones for clarity.[10]

The quarks differ, among other quantum numbers, mainly in their masses. One can thus separate the light and heavy quarks with respect to the typical hadronic scale of $\Lambda_{\mathrm{H}} = 1\,\mathrm{GeV}$:

$$m_u, m_d, m_s \ll \Lambda_{\mathrm{H}} < m_c, m_b, m_t\,. \tag{1.4}$$

For our purposes, we will neglect any contributions from the heavy quarks and restrict ourselves only to the three light ones.

Finally, there are eight gluon fields, denoted as $\mathcal{A}_\mu^a(x)$ for $a = 1, \ldots, 8$, that are Lorentz vectors. For each gluon field there exist the corresponding generator T^a that, in the fundamental representation, is given as a half of the Gell-Mann matrix λ^a.

[9] For a detailed introductory literature on QCD, apart from the original scientific papers cited in the preceding section, one may also rely on a large number of introductory books or review articles. For such a purpose, we refer the reader to [46, 47, 48, 49, 50, 51].

[10] The reason for such a simplification is purely aesthetic. Having to be meticulous, the quark field should be generally denoted as $q_{i,f}^\alpha(x)$ with the spinor ($i = 1, 2, 3, 4$), colour ($\alpha = r, g, b$) and flavour (f) indices shown explicitly. This would, however, make the relations a bit chaotic.

QCD Lagrangian. Having identified the field content of the theory, let us write down the QCD Lagrangian.[11] It reads

$$\mathcal{L}_{\mathrm{QCD}} = \underbrace{\sum_f i\bar{q}_f(x)\not{\nabla} q_f(x) - \frac{1}{4}G^a_{\mu\nu}(x)G^{a\,\mu\nu}(x)}_{\mathcal{L}^{(0)}_{\mathrm{QCD}}} \underbrace{- \sum_f m_f\bar{q}_f(x)q_f(x)}_{\mathcal{L}^{(\mathrm{mass})}_{\mathrm{QCD}}}\,, \tag{1.5}$$

which we have split into two parts — the first one being the Lagrangian in the chiral limit $\mathcal{L}^{(0)}_{\mathrm{QCD}}$ and the second one being the mass term $\mathcal{L}^{(\mathrm{mass})}_{\mathrm{QCD}}$ — and omitted terms that are not relevant for our discussion.[12] The meaning of the symbols in the individual terms is as follows.

a) In order to ensure the QCD Lagrangian to be invariant under the local SU(3) colour gauge symmetry, we have introduced the covariant derivative in the fundamental representation,

$$\nabla_\mu = \partial_\mu + ig_s\mathcal{A}_\mu(x)\,, \tag{1.6}$$

with g_s being the strong coupling constant and $\mathcal{A}_\mu(x) = \mathcal{A}^a_\mu(x)T^a$ stands for the octet of gluons. In (1.5), we have conveniently utilized the Feynman's slash notation, i.e. $\not{\nabla} \equiv \nabla_\mu\gamma^\mu$.

b) The second term is responsible for the dynamics of the gluons. To this end, we have defined the gluon field strength tensor $G_{\mu\nu}(x) = G^a_{\mu\nu}(x)T^a$ as

$$[\nabla_\mu, \nabla_\nu] = ig_sG_{\mu\nu}(x)\,, \tag{1.7}$$

for which one easily obtains

$$G_{\mu\nu}(x) = \partial_\mu\mathcal{A}_\nu(x) - \partial_\nu\mathcal{A}_\mu(x) + ig_s[\mathcal{A}_\mu(x), \mathcal{A}_\nu(x)]\,, \tag{1.8}$$

i.e.

$$G^a_{\mu\nu}(x) = \partial_\mu\mathcal{A}^a_\nu(x) - \partial_\nu\mathcal{A}^a_\mu(x) - g_sf^{abc}\mathcal{A}^b_\mu(x)\mathcal{A}^c_\nu(x)\,. \tag{1.9}$$

We take the liberty to point out that a contraction of two gluon field strength tensors is not gauge-invariant, however, its colour trace is — which is the reason of the form of such a term.

c) Finally, in accordance with section 1.1, we have denoted m_f as the individual quark masses. For future purpose, it is useful to define the mass matrix $\mathcal{M}$ as

$$\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)\,. \tag{1.10}$$

[11] Strictly formally, $\mathcal{L}_{\mathrm{QCD}}$ should be called the "Lagrangian density", however, we will prefer a commonly used term "Lagrangian" instead — this applies throughout the whole thesis.

[12] We have omitted the gauge-fixing term and the term incorporating the Faddeev–Popov ghosts. In this regard, it is also important to point out that the gauge symmetry permits an addition of another term which we have neglected, that is the so-called theta-term

$$\theta\frac{g_s^2}{64\pi^2}G^a_{\mu\nu}(x)\widetilde{G}^{a\,\mu\nu}(x)\,,$$

where $\widetilde{G}^{a\,\mu\nu}(x) = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}(x)$ is the dual gluon field strength tensor. This term, even though gauge-invariant and renormalizable, violates both P and CP symmetry. In fact, the physical CP-violating angle is $\bar{\theta} = \theta + \arg\det\mathcal{M}$, with $\mathcal{M}$ being the mass matrix (see below), and the experimentally obtained upper limit is $|\bar{\theta}| \leq 10^{-10}$ [52, 53].

Equations of motion. Having the Lagrangian (1.5), the equations of motion can be obtained using the Euler–Lagrange equations. In what follows, we shall consider only the part of the QCD Lagrangian that is relevant in the chiral limit, i.e. $\mathcal{L}_{\mathrm{QCD}}^{(0)}$.

The equation of motion for the quark field corresponds to the well-known Dirac equation:

$$\not{\nabla} q_f(x) = 0\,, \tag{1.11a}$$

$$\overline{q}_f(x)\overleftarrow{\not{\nabla}} = 0\,, \tag{1.11b}$$

with $\not{\nabla}$ acting to the right, as usual, and $\overleftarrow{\not{\nabla}}$ acting to the left.

On the other hand, the equation of motion for the gluon field reads

$$(D_\mu G^{\mu\nu}(x))^a = g_s \sum_f \overline{q}_f(x)\gamma^\nu T^a q_f(x)\,, \tag{1.12}$$

with the T^a matrix acting in the colour space and where the covariant derivative in the adjoint representation is

$$(D_\mu)^{ab} = \partial_\mu \delta^{ab} + g_s f^{abc}\mathcal{A}_\mu^c(x)\,. \tag{1.13}$$

Running coupling. Instead of the strong coupling g_s, one usually considers another parameter, that is

$$\alpha_s = \frac{g_s^2}{4\pi}\,. \tag{1.14}$$

Then, in the perturbative regime of QCD, expressions for observables of the theory are given in terms of the renormalized coupling constant $\alpha_s(\mu_{\mathrm{R}}^2)$, which depends on an unphysical renormalization scale μ_{R}. If the renormalization scale is taken close to the scale of momentum transfer Q for a given process, the parameter $\alpha_s(\mu_{\mathrm{R}}^2 \approx Q^2)$ represents the effective strength of the strong interaction in the said process.

The dependence of the strong coupling constant on the renormalization scale is given by the so-called renormalization group equation, through which one also defines the QCD β-function as

$$\mu_{\mathrm{R}}^2 \frac{\mathrm{d}\alpha_s}{\mathrm{d}\mu_{\mathrm{R}}^2} = \beta(\alpha_s)\,, \tag{1.15}$$

with

$$\beta(\alpha_s) = -\alpha_s^2 \sum_{n=0}^{\infty} \beta_n \alpha_s^n\,, \tag{1.16}$$

where the coefficients β_n are given by the contributions of corresponding Feynman diagrams at $\mathcal{O}(\alpha_s^{n+1})$. For example, at one-loop level, one has [41, 42][13]

$$\beta_0 = \frac{33 - 2N_f}{12\pi}\,, \tag{1.17}$$

[13]For a curious reader, we reproduce here also the coefficient β_1 corresponding to the two-loop contribution:

$$\beta_1 = \frac{153 - 19N_f}{24\pi^2}\,.$$

Starting with the three-loop coefficient β_2, one founds a presence of the dependence of the coefficients on the renormalization scheme. Also, for an extensive overview of the running of the QCD coupling constant, see ref. [54] or [55] and references therein.

where we tacitly assume the existence of three colours of quarks and N_f being the number of their flavours. This then leads to

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\dfrac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}\,, \tag{1.18}$$

where $\Lambda_{\mathrm{QCD}} \approx 200\,\mathrm{MeV}$.

One can thus notice that at low momentum transfers, the strong coupling grows rapidly and the QCD becomes nonperturbative. On top of that, such a behaviour at low energies leads to the confinement — the effect that does not allow separating single quarks or gluons and, rather, making them hidden inside the colourless states, i.e. the hadrons.[14] On the other hand, as the momentum transfer increases, the strong coupling decreases and the QCD is indeed perturbative, with quarks and gluons then interacting weakly — this is then called the asymptotic freedom.[15]

Chiral symmetry. Let us carry on in the investigation of the QCD Lagrangian in the chiral limit, i.e. for the quark masses set to zero. Then, the Lagrangian $\mathcal{L}^{(0)}_{\mathrm{QCD}}$ is invariant under the SU(3) colour gauge symmetry and, due to the elimination of the mass term, also under the U(3) flavour symmetry.

In order to examine the global symmetries of $\mathcal{L}^{(0)}_{\mathrm{QCD}}$, we introduce the projection operators[16]

$$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\,, \tag{1.19}$$

that decompose the quark field $q_f(x)$ into its chiral components $q_{f_L}(x)$ and $q_{f_R}(x)$,

$$q_{f_{L,R}}(x) = P_{L,R}\, q_f(x)\,, \tag{1.20}$$

with

$$q_f(x) = q_{f_L}(x) + q_{f_R}(x)\,. \tag{1.21}$$

Then, the QCD Lagrangian in the chiral limit can be rewritten to

$$\mathcal{L}^{(0)}_{\mathrm{QCD}} = \sum_f \left(i\overline{q}_{f_R}(x)\not{\nabla} q_{f_R}(x) + i\overline{q}_{f_L}(x)\not{\nabla} q_{f_L}(x)\right) - \frac{1}{4}G^a_{\mu\nu}(x)G^{a\,\mu\nu}(x)\,, \tag{1.22}$$

which is invariant not only under the above-mentioned U(3) flavour symmetry, but also under the independent transformations of the chiral components,

$$q_{f_{L,R}}(x) \to U_{L,R}(x) q_{f_{L,R}}(x)\,, \tag{1.23}$$

[14] In such a context, the parameter Λ_{QCD} is called the characteristic scale of confinement.

[15] On an algebraical level, the asymptotic freedom can be traced back to the minus sign in front of eq. (1.16) and the positivity of the coefficient (1.17) for the number of quark flavours in the Standard model. Equivalently, on the Lagrangian level, it is caused by the presence of the self-interaction of the gluons — upon substituting (1.9) into the QCD Lagrangian (1.5), one obtains the following three- and four-gluon interaction terms:

$$\mathcal{L}^{(0)}_{\mathrm{QCD}} \ni \frac{1}{2} g_s f^{abc}(\partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu)\mathcal{A}^{b\,\mu}\mathcal{A}^{c\,\nu} - \frac{1}{4} g_s^2 f^{abc} f^{ade} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \mathcal{A}^{d\,\mu}\mathcal{A}^{e\,\nu}\,.$$

[16] These projection operators are idempotent ($P_L^2 = P_L$, $P_R^2 = P_R$), orthogonal ($P_L P_R = 0$, $P_R P_L = 0$) and satisfy the completeness relation ($P_L + P_R = 1$).

where $U_{L,R}$ are unitary 3×3 matrices. Then, the Lagrangian (1.22) possesses the classical $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ symmetry.

Chiral currents and densities. The element of the U(3) group can be divided into the SU(3) component and the U(1) phase part. Then, in order to be consistent with the notation chosen in our work [1], let us consider, instead of the colour triplet of a single-flavour quark field (1.3), rather the flavour triplet $q(x)$:

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix} . \tag{1.24}$$

As a consequence of the Noether's theorem applied to the Lagrangian (1.22), there are eighteen conserved currents on the classical level, associated with the above-mentioned transformations of left- and right-handed quark components. The SU(3) currents ($a = 1, \ldots, 8$) are[17]

$$V^a_\mu(x) = \overline{q}(x)\gamma^\mu T^a q(x) \,, \tag{1.25a}$$
$$A^a_\mu(x) = \overline{q}(x)\gamma^\mu \gamma_5 T^a q(x) \,, \tag{1.25b}$$

that transform as vector and axial-vector under parity transformation, respectively, whilst the U(1) singlet currents read

$$V_\mu(x) = \overline{q}(x)\gamma^\mu q(x) \,, \tag{1.26a}$$
$$A_\mu(x) = \overline{q}(x)\gamma^\mu \gamma_5 q(x) \,. \tag{1.26b}$$

At the quantum level, however, the situation is a bit different — after quantization, the conservation of the axial-vector current (1.26b) is spoiled and the symmetry is not preserved anymore because of the presence of the anomaly,

$$\partial^\mu A_\mu(x) = \frac{3 g_s^2}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \,, \tag{1.27}$$

due to which the QCD Lagrangian in the chiral limit (1.22) is invariant under the chiral group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$.

Finally, as the last remark and without further discussion, let us add that it turns out to be useful to define also the SU(3) scalar and pseudoscalar densities,

$$S^a(x) = \overline{q}(x) T^a q(x) \,, \tag{1.28a}$$
$$P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x) \,, \tag{1.28b}$$

and, equivalently, the U(1) singlets

$$S(x) = \overline{q}(x) q(x) \,, \tag{1.29a}$$
$$P(x) = i\overline{q}(x)\gamma_5 q(x) \,. \tag{1.29b}$$

[17] Let us also point out that in (1.25a)-(1.25b) and (1.28a)-(1.28b), the matrix T^a acts in the flavour space.

Explicit symmetry breaking. Let us now consider the mass term $\mathcal{L}_{\text{QCD}}^{(\text{mass})}$ of the QCD Lagrangian (1.5). Although we have neglected it up to this point, its presence is of great interest. Indeed, once the masses of the quarks are considered to be nonzero, the symmetry is explicitly broken.

The symmetry of the theory is affected by the various scenarios of mutual sizes of the quark masses. We present some of them in the following itemized list, inspired by ref. [56].

a) $m_u = m_d = m_s = 0$: the octet of vector and axial-vector currents are conserved and the symmetry group is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$.

b) $m_u = m_d = m_s \neq 0$: vector current is conserved and the Lagrangian is invariant under the symmetry $\mathrm{SU}(3)_V \times \mathrm{U}(1)_V$.

c) $m_u = m_d = 0$: the symmetry is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{SV} \times \mathrm{U}(1)_V$, with $\mathrm{U}(1)_{SV}$ being the conservation of the strangeness.

d) $m_u = m_d \neq 0$: the chiral limit of the lightest quarks with the symmetry $\mathrm{SU}(2)_V \times \mathrm{U}(1)_{SV} \times \mathrm{U}(1)_V$.

e) For general values of the masses m_u, m_d, m_s, there is no flavour symmetry except for $\mathrm{U}(1)_V$, which represents the conservation of the baryon number.

To this end, let us point out that an addition of the quark masses to the Lagrangian makes the currents (1.25a)-(1.26b) not conserved — for details, see (2.9a) and (2.9b).

Spontaneous symmetry breaking. In the chiral limit, the QCD symmetry group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$ is spontaneously broken to $\mathrm{SU}(3)_V \times \mathrm{U}(1)_V$ due to the presence of the order parameter — the quark condensate.

According to the Goldstone theorem, to each generator, which does not annihilate the vacuum state, there corresponds one massless Goldstone boson [57, 58]. Therefore, an octet of these particles appears in the QCD spectrum of QCD. As we will see in the next section, such a consequence turns out to be crucial for our understanding of the behaviour of QCD at low energies.

1.3 Chiral perturbation theory

As we have suggested in the previous section, an approach to the description of the strong interaction with the quarks and gluons as the force carriers is no longer valid at low energies due to the increase of the strong coupling, which thus spoils the perturbative behaviour of QCD. To be able to qualitatively describe the hadronic spectrum below approximately 2 GeV, one is thus required to introduce an effective field theory with another relevant degrees of freedom — the mesons and baryons. Such a theory, however, is not known from first principles and the situation is further complicated by the presence of the mass gap that separates the octet of pseudoscalar mesons from the rest of the hadronic spectrum.

Restricting ourselves to the energy region below the mass of the first hadronic resonance, i.e. the $\rho(770)$ meson, the strong interaction can then be described by the means of an effective theory called the Chiral perturbation theory (ChPT)

[59, 60, 61]. The ChPT is built on the account of the spontaneous chiral symmetry breaking

$$G \equiv \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to H \equiv \mathrm{SU}(N_f)_V \,, \tag{1.30}$$

of the QCD Lagrangian with N_f massless quarks, due to which $N_f^2 - 1$ Goldstone bosons arise in the coset space G/H as the effective degrees of freedom of the theory. These are then associated with the pseudoscalar mesons as the lowest states of the hadronic spectrum. In the case of the two-flavour ChPT, i.e. for $N_f = 2$, only the triplet of pions is taken into account. However, for $N_f = 3$, one needs to include also the kaons and the eta meson.

Chiral operators. Following the formalism of building effective Lagrangians with spontaneous symmetry breaking proposed in [62, 63], one may utilize a suitable way of parametrization of the Goldstone bosons provided therein.

Let us denote ϕ as the Goldstone bosons in the coset space G/H and a corresponding coset representative as $u_{L,R}(\phi)$. The transformation under a chiral transformation $g = (g_\mathrm{L}, g_\mathrm{R}) \in G$ is given by

$$u_L(\phi) \xrightarrow{G} g_L u_L(\phi) h^\dagger(g,\phi) \,, \tag{1.31a}$$

$$u_R(\phi) \xrightarrow{G} g_R u_R(\phi) h^\dagger(g,\phi) \,, \tag{1.31b}$$

where $h(g,\phi) \in H$ is the compensator field. In fact, one may take the choice of the coset representative such that

$$u_R(\phi) = u_L^\dagger(\phi) \equiv u \,. \tag{1.32}$$

A suitable form of the above-mentioned parametrization is the exponential one, in which the Goldstone bosons are incorporated as

$$u = \exp\left(\frac{i}{\sqrt{2}F}\phi\right) , \tag{1.33}$$

where the low-energy parameter F is the pion decay constant and, in the case of $N_f = 3$,

$$\phi = \sqrt{2}\phi^a T^a = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\dfrac{2\eta_8}{\sqrt{6}} \end{pmatrix} . \tag{1.34}$$

Mesonic chiral Lagrangians can be constructed by taking traces of products of chiral operators X that either transform as [59, 60, 61, 62, 63]

$$X \xrightarrow{G} h(g,\phi) X h^\dagger(g,\phi) \tag{1.35}$$

or remain invariant under chiral transformations. It can be shown that the following operators fulfill such a requirement and, in fact, form a complete set. They are as follows:

$$u_\mu = i\left[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right] , \tag{1.36a}$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,, \tag{1.36b}$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u \,. \tag{1.36c}$$

In the operators above, we have made a use of the following quantities: ℓ^μ and r^μ are the left and right external sources,

$$\chi = 2B_0(s + ip)\,, \tag{1.37}$$

s and p are the scalar and pseudoscalar sources, B_0 is a constant related to the quark condensate and

$$F_L^{\mu\nu} = \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i\,[\ell^\mu, \ell^\nu]\,, \tag{1.38a}$$
$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i\,[r^\mu, r^\nu]\,, \tag{1.38b}$$

are the left and right nonabelian field strength tensors. It is also convenient to introduce the operator

$$h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu \tag{1.39}$$

in which the covariant derivative is defined by

$$\nabla_\mu \bullet = \partial_\mu \bullet + [\Gamma_\mu, \bullet]\,, \tag{1.40}$$

and the chiral connection reads

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right]. \tag{1.41}$$

Chiral Lagrangian. Similarly to any other effective field theory, a Lagrangian is organized according to its power counting. Here, the ChPT Lagrangian can be written down schematically as a series of terms with an increasing number of derivatives and quark masses, ordered by powers of a generic momentum p. As of the day of completion of this work, the chiral Lagrangian is known up to $\mathcal{O}(p^8)$:

$$\mathcal{L}_{\text{ChPT}} = \underbrace{\mathcal{L}^{(2)}_{\text{ChPT}}}_{\text{LO}} + \underbrace{\mathcal{L}^{(4)}_{\text{ChPT}} + \mathcal{L}^{(4)}_{\text{WZW}}}_{\text{NLO}} + \underbrace{\mathcal{L}^{(6)}_{\text{ChPT}} + \mathcal{L}^{(6)}_{\text{WZW}}}_{\text{NNLO}} + \underbrace{\mathcal{L}^{(8)}_{\text{ChPT}}}_{\text{NNNLO}} + \ldots\,, \tag{1.42}$$

where we have denoted which Lagrangians belong to the leading order (LO), next-to-leading order (NLO) etc.

Instead of writing down all the Lagrangians in detail, which would not be either desirable or well-arranged, we rather provide a brief commentary for each of them and refer the reader to the original papers where the necessary details can be found comfortably.[18] Also, the Lagrangians relevant specifically for our purposes have been discussed concisely in ref. [2], see section 2.1 therein.

a) The ChPT Lagrangian of the lowest order, $\mathcal{L}^{(2)}_{\text{ChPT}}$, was introduced in [60, 61] and depends only on two parameters: the pion decay constant F and the parameter B_0. Due to its remarkable simplicity, stemming from basic properties of current algebra, we make an exception and present its form:

$$\mathcal{L}^{(2)}_{\text{ChPT}} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle\,. \tag{1.43}$$

[18] We strictly follow the widely accepted notation so as the reader can compare it with the literature without confusion.

b) The NLO Lagrangian $\mathcal{L}^{(4)}_{\text{ChPT}}$ was derived in [60, 61] also and, contrary to the previous one, is already a bit complicated. Usually, it is presented in a form consisted of ten terms with the couplings $L_1, \ldots, L_{10}$. Sometimes, however, another terms are included — these are the terms proportional to constants L_{11} and L_{12} that vanish when the equations of motion are employed, and the terms proportional to H_1 and H_2 that are needed only for the renormalization.

c) The another NLO ChPT Lagrangian $\mathcal{L}^{(4)}_{\text{WZW}}$ is actually the leading-order contribution of the pure Goldstone-boson part of the odd-intrinsic parity sector and its parameters are set entirely by the chiral anomaly. Such a Lagrangian was introduced in [64] upon the previous work presented in ref. [65] — due to which it is usually called as the Wess–Zumino–Witten Lagrangian.

d) The NNLO Lagrangian $\mathcal{L}^{(6)}_{\text{ChPT}}$ was studied in [66, 67] and, in SU(3), it consists of 94 terms.

e) A classification of a minimal set of independent terms of the anomalous NNLO ChPT Lagrangian $\mathcal{L}^{(6)}_{\text{WZW}}$ was initiated in [66, 68] and, according to [69, 70], there are 23 terms in SU(3) in total.

f) Finally, the present state of the art is concluded by the recently-obtained NNNLO Lagrangian $\mathcal{L}^{(8)}_{\text{ChPT}}$, derived in ref. [71]. In SU(3), it contains 1254 terms in total.

1.4 Resonance chiral theory

Considering the large-N_c limit, an effective theory of QCD for an intermediate energy region, that also satisfies all symmetries of the underlying theory, can be constructed. Such a theory is the Resonance chiral theory (RChT) and is relevant for energies $M_\rho \leq E \leq 2\,\text{GeV}$ [72].

RChT increases the number of degrees of freedom of ChPT by including massive U(3) multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances, denoted generically as a nonet field R that can be decomposed into singlet R^0 and octet R^a [72, 73]:

$$R = \frac{R_0}{\sqrt{3}} + \sqrt{2} R_a T^a \,. \tag{1.44}$$

A procedure of constructing the RChT Lagrangian was presented in detail in refs. [72, 73] and summarized also in [74]. We thus refrain ourselves from going into unnecessary details here and provide only a brief summary. To this end, a construction of the RChT Lagrangian is based upon the single resonance approximation, in which only the lightest multiplets of resonances are accounted for. In such an approach, the Goldstone bosons are coupled to massive U(3) multiplets. Then, the construction follows the path of building operators that transform similarly to (1.35) — in our case, we desire such tensors to transform as

$$X \xrightarrow{G'} h(g,\phi) X h^\dagger(g,\phi) \,, \tag{1.45}$$

with

$$G' \equiv \mathrm{U}(3)_L \times \mathrm{U}(3)_R \,. \tag{1.46}$$

Within the large-N_c approach, there is no limit to the number of resonances that can be included in the effective Lagrangian. We can thus construct the RChT Lagrangian as an expansion in the number of resonance fields, i.e. [72]

$$\mathcal{L}_{\mathrm{RChT}} = \mathcal{L}_{\mathrm{ChPT}} + \mathcal{L}^{(\mathrm{kin})}_{R_1 R_1} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1,\, R_2} \mathcal{L}_{R_1 R_2} + \sum_{R_1,\, R_2,\, R_3} \mathcal{L}_{R_1 R_2 R_3} + \ldots \,, \tag{1.47}$$

with the kinetic term given as

$$\mathcal{L}^{(\mathrm{kin})}_{R_1 R_1} = -\frac{1}{2}\langle \nabla^\mu R_{\mu\nu} \nabla_\alpha R^{\alpha\nu} \rangle + \frac{1}{4} M_R^2 \langle R_{\mu\nu} R^{\mu\nu} \rangle + \frac{1}{2}\langle \nabla^\alpha R' \nabla_\alpha R' \rangle - \frac{1}{2} M_{R'}^2 \langle R' R' \rangle \,, \tag{1.48}$$

in which we have denoted $R = V, A$ and $R' = S, P$ for clarity.

The individual terms of the Lagrangian (1.47) can also be classified by the chiral order, for which they contribute after integrating the resonances out. Then, the resonance Lagrangian can be schematically written down as

$$\mathcal{L}_{\mathrm{RChT}} = \mathcal{L}^{(4)}_{\mathrm{RChT}} + \mathcal{L}^{(6)}_{\mathrm{RChT}} + \ldots \,. \tag{1.49}$$

We alert the reader that the vector and axial-vector resonances are spin-one particles so there is a freedom of choice which formalism one uses to describe them.[19] As examples, we mention the Proca (vector) formalism [75] or the first-order formalism [76, 77]. In our previous work [2], however, we have employed the antisymmetric tensor formalism and in what follows, we briefly present the respective Lagrangians.

a) The couplings of the lowest massive U(3) multiplets with the pseudoscalar fields and external sources in the leading order of $1/N_c$ are given by the linear part of the interaction resonance Lagrangian (1.49), i.e. [72]

$$\begin{aligned}\mathcal{L}^{(4)}_{\mathrm{RChT}} = \; & \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle + \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle \\ & + c_d \langle S u^\mu u_\mu \rangle + c_m \langle S \chi_+ \rangle + i d_m \langle P \chi_- \rangle + \frac{i d_{m0}}{N_f} \langle P \rangle \langle \chi_- \rangle \,. \end{aligned} \tag{1.50}$$

b) The $\mathcal{O}(p^6)$ Lagrangian, relevant in the odd-intrinsic parity sector, was classified for the first time in ref. [78]. Its form consists of 67 terms in total, parameterized as[20]

$$\mathcal{L}^{(6,\,\mathrm{odd})}_{\mathrm{RChT}} = -\varepsilon^{\mu\nu\alpha\beta} \sum_{(i,\, X)} \kappa_i^X (\widehat{\mathcal{O}}_i^X)_{\mu\nu\alpha\beta} \,, \tag{1.51}$$

[19] Needless to say, the solution of any calculation should be independent of used formalism. However, since one works with perturbation theories, the calculations are performed only up to a given order, which makes different formalisms nonequivalent with respect to each other.

[20] A note regarding the convention is in order here. Ref. [78] uses an opposite convention for the Levi-Civita tensor with respect to the convention used in this thesis and in the papers [1, 2]. So as the meaning of the corresponding coupling constants remains the same, we have modified the respective Lagrangian (1.51) by adding the overall minus sign.

where κ_i^X are the coupling constants, i being a serial number of such an operator according to tables 1-7 presented in ref. [78] and X denotes the relevant combination of resonance fields. In detail, the Lagrangian (1.51) takes into account operators with the single (V, A, S, P), double (VV, AA, SA, SV, VA, PA, PV) and triple (VVP, VAS, AAP) combinations of resonances. As we have suggested, the individual operators $(\widehat{\mathcal{O}}_i^X)_{\mu\nu\alpha\beta}$ can be found well-arranged in ref. [78], see tables 1-7 at pages 10-11 therein.[21]

[21] For a curious reader, we list here how many operators with the respective allowed combination X of resonances there actually exist. In a compact and intuitive notation of the form (X, $i_{\max}$), it is as follows: (V, 18), (A, 16), (S, 2), (P, 5), (VV, 4), (AA, 4), (SA, 2), (SV, 2), (VA, 6), (PA, 2), (PV, 3), (VVP, 1), (VAS, 1) and (AAP, 1).

2. Framework of Green functions

In this second introductory chapter, we will summarize the basic properties of the Green functions made of the chiral currents.[22] After their formal definition, we shall briefly discuss the relevant Ward identities that allow us to find out which Green functions are indeed physical and which are not allowed under the certain symmetries. Then, we will pay an attention to the decomposition of the Green functions and a general structure of contributions of resonance multiplets.

2.1 Green functions of chiral currents

As much as the concept of Green functions is trivial and their formal definition intuitive, they undoubtedly belong among the most important objects of quantum field theory due to their both theoretical and phenomenological significance. A canonical example may be demonstrated on the fact that the amplitudes of physical processes can be calculated using the Lehmann–Symanzik–Zimmermann (LSZ) reduction formula from the Green functions.

The Green functions are the vacuum expectation values of the time-ordered products of the quantum fields. One may thus denote the n-point Green function symbolically as

$$\Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}(x_1,\ldots,x_n) \equiv \langle 0|\mathrm{T}\,\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)|0\rangle\,, \tag{2.1}$$

where the operators $\mathcal{O}$ stand for any of the currents (1.25a), (1.25b) or the densities (1.28a), (1.28b), and where the symbol $|0\rangle$ stands for the nonperturbative QCD vacuum — which we will devote a special attention to in the next chapter.[23]

The Green functions of colourless sources are gauge-invariant objects and must be then invariant also with respect to translation. Without loss of generality, the translation invariance allows us to perform a shift by the n-th coordinate and thus, eventually, eliminate it by setting it to zero:[24]

$$\begin{aligned}\Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}&(x_1,\ldots,x_{n-1},x_n)\\ &= \Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}(x_1-x_n,\ldots,x_{n-1}-x_n,0)\\ &\overset{(x_n=0)}{\equiv} \Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}(x_1,\ldots,x_{n-1})\,.\end{aligned} \tag{2.2}$$

Such a property may not be, however, apparent immediately in practical calculations and one may resort to using a computer brute force to verify it.

The Green function, as introduced above, is quantified in the coordinate representation. Its conversion to the momentum representation can be made by

[22] A Green function can be also called a "correlator". Throughout this thesis, we will use both terms alternately, with emphasis on a more appropriate aesthetic use in a given context.

[23] We will desist ourselves from writing down the vacuum state from now on, i.e. $\langle 0|\bullet|0\rangle \equiv \langle\bullet\rangle$.

[24] Such a shift of a coordinate is usually a very standard operation. On the other hand, as ref. [79] argues, one needs to be careful when the correlator is made of derivative currents. Then, one would have not been eligible to perform such a shift and set the coordinate of the respective current into the origin from the beginning. As an example of such a situation are the calculations of the quark loops in the external gluon field.

applying the Fourier transform, i.e.

$$\begin{aligned}\Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}&(p_1,\ldots,p_{n-1};p_n) \\ &= \int \mathrm{d}^4x_1\cdots\mathrm{d}^4x_{n-1}\, e^{-i(p_1\cdot x_1+\ldots+p_{n-1}\cdot x_{n-1})}\Pi_{\mathcal{O}_1\ldots\mathcal{O}_n}(x_1,\ldots,x_{n-1})\,,\end{aligned} \tag{2.3}$$

where the above-mentioned translation invariance amounts to the conservation of the four-momenta in the form

$$p_1+\ldots+p_{n-1}+p_n=0\,. \tag{2.4}$$

To simplify the designation of the Green functions in the text a bit, we shall use a shortened notation of $\langle\mathcal{O}_1\cdots\mathcal{O}_n\rangle$ with no regard as what the respective representation of such a correlator is — which will be obvious from the context either way. Also, we will restrict ourselves only to $n=2$ and $n=3$ throughout this thesis, i.e. only the two- and three-point Green functions will be of relevance here.

2.2 Ward identities

The Ward identities reflect the symmetry properties of a given theory on the quantum level. Considering as an example the three-point Green functions in the coordinate representation, their Ward identities correspond to the divergences thereof expressed as linear combinations of the two-point correlators. In the momentum representation, the derivative is then changed for a multiplication with the respective four-momentum.

Fur our purpose, let us write down the formulae for the Ward identities explicitly for the two- and three-point Green functions. Assuming that $\mathcal{O}_1(x)$ stands for any of the Noether currents, which we will thus denote specifically together with the Lorentz index as $\mathcal{O}_1^\mu(x)$, we have[25]

$$\begin{aligned}\partial_\mu^x\langle\mathrm{T}\,\mathcal{O}_1^\mu(x)\mathcal{O}_2(y)\rangle &= \langle\mathrm{T}\,\partial_\mu^x\mathcal{O}_1^\mu(x)\mathcal{O}_2(y)\rangle \\ &\qquad + \delta(x^0-y^0)\langle\mathrm{T}\,[\mathcal{O}_1^0(x),\mathcal{O}_2(y)]\rangle\,, \end{aligned} \tag{2.5}$$

$$\begin{aligned}\partial_\mu^x\langle\mathrm{T}\,\mathcal{O}_1^\mu(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle &= \text{anomaly} + \langle\mathrm{T}\,\partial_\mu^x\mathcal{O}_1^\mu(x)\mathcal{O}_2(y)\mathcal{O}_3(z)\rangle \\ &\qquad + \delta(x^0-y^0)\langle\mathrm{T}\,[\mathcal{O}_1^0(x),\mathcal{O}_2(y)]\mathcal{O}_3(z)\rangle \\ &\qquad + \delta(x^0-z^0)\langle\mathrm{T}\,\mathcal{O}_2(y)[\mathcal{O}_1^0(x),\mathcal{O}_3(z)]\rangle\,. \end{aligned} \tag{2.6}$$

Now, two quantities are left to be determined: the divergences of currents and the equal-time commutators. The latter can be obtained easily after some algebraical manipulations. In detail, the mutual commutators of the vector and axial-vector currents read[26]

$$[V_0^a(t,\boldsymbol{x}),V_\mu^b(t,\boldsymbol{y})] = [A_0^a(t,\boldsymbol{x}),A_\mu^b(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})f^{abc}V_\mu^c(t,\boldsymbol{x})\,, \tag{2.7a}$$

$$[V_0^a(t,\boldsymbol{x}),A_\mu^b(t,\boldsymbol{y})] = [A_0^a(t,\boldsymbol{x}),V_\mu^b(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})f^{abc}A_\mu^c(t,\boldsymbol{x})\,, \tag{2.7b}$$

[25] We note that $\partial_\mu^x\equiv\frac{\partial}{\partial x^\mu}$.

[26] This can be left to the reader as a simple exercise — one may, however, find a hint in a useful book [50], see sections 2.3 and 2.4 therein.

whilst for the commutators of these currents with the scalar and pseudoscalar densities we have

$$[V_0^a(t,\boldsymbol{x}), S^b(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) f^{abc} S^c(t,\boldsymbol{x})\,, \tag{2.8a}$$
$$[A_0^a(t,\boldsymbol{x}), P^b(t,\boldsymbol{y})] = -i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) d^{abc} S^c(t,\boldsymbol{x})\,, \tag{2.8b}$$
$$[V_0^a(t,\boldsymbol{x}), P^b(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) f^{abc} P^c(t,\boldsymbol{x})\,, \tag{2.8c}$$
$$[A_0^a(t,\boldsymbol{x}), S^b(t,\boldsymbol{y})] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) d^{abc} P^c(t,\boldsymbol{x})\,. \tag{2.8d}$$

Then, also after some algebra, the divergences of the Noether currents (1.25a) and (1.25b) can be expressed as

$$\partial^\mu V_\mu^a(x) = i\overline{q}(x)\,[\mathcal{M}, T^a]\,q(x)\,, \tag{2.9a}$$
$$\partial^\mu A_\mu^a(x) = i\overline{q}(x)\gamma_5\,\{\mathcal{M}, T^a\}\,q(x)\,, \tag{2.9b}$$

where the mass matrix $\mathcal{M}$, as given by (1.10), can be rewritten as a linear combination of the Gell-Mann matrices,

$$\mathcal{M} = (m_u - m_d)T^3 + \frac{m_u + m_d - 2m_s}{\sqrt{3}}T^8 + \frac{m_u + m_d + m_s}{3}\,, \tag{2.10}$$

which eventually leads to

$$[\mathcal{M}, T^a] = i(m_u - m_d) f^{3ab} T^b + \frac{i(m_u + m_d - 2m_s)}{\sqrt{3}} f^{8ab} T^b\,, \tag{2.11a}$$
$$\begin{aligned}\{\mathcal{M}, T^a\} = (m_u - m_d)\left(\frac{1}{3}\delta^{3a} + d^{3ab}T^b\right) + \frac{m_u + m_d - 2m_s}{\sqrt{3}}\left(\frac{1}{3}\delta^{8a} + d^{8ab}T^b\right)\\ + \frac{2(m_u + m_d + m_s)}{3}T^a\,,\end{aligned} \tag{2.11b}$$

from which one easily sees the emergence of the scalar and pseudoscalar densities in (2.9a) and (2.9b).

To find explicit results of the Ward identities for all the relevant Green functions, see subsections 2.2.1 and 2.2.2 in our paper [1], where we have derived them in detail, i.e. for all relevant channels.

2.3 Classification

Having established the Ward identities, we may now embark on identifying the relevant Green functions of chiral currents and densities. The total number of all possible n-point Green functions is given as a number of all n-combinations with repetitions from a set of k currents and densities.[27] Based on the introductory paragraphs in the previous chapter, we have two currents (vector V and axial-vector A) and two densities (pseudoscalar P and scalar S) at disposal.

[27] Naturally, such a number of combinations with repetitions is determined by the binomial coefficient

$$\binom{n+k-1}{n}\,.$$

Being interested only in the total number of their possible combinations at this stage, we have ten two-point and twenty three-point Green functions. Needless to say, aforementioned numbers include also such Green functions, whose existence is forbidden on the grounds of parity invariance etc. To present all the Green functions in a clear manner, we introduce the table below.

n	Green functions				
2	$\langle VV\rangle$	$\boldsymbol{\langle VA\rangle}$	$\boldsymbol{\langle VS\rangle}$	$\boldsymbol{\langle VP\rangle}$	$\langle AA\rangle$
	$\boldsymbol{\langle AS\rangle}$	$\langle AP\rangle$	$\langle SS\rangle$	$\boldsymbol{\langle SP\rangle}$	$\langle PP\rangle$
3	$\langle ASP\rangle$	$\langle VSS\rangle$	$\langle VPP\rangle$	$\langle VVA\rangle$	$\langle AAA\rangle$
	$\langle AAV\rangle$	$\langle VVV\rangle$	$\langle SSS\rangle$	$\langle SPP\rangle$	$\langle VVP\rangle$
	$\langle AAP\rangle$	$\langle VAS\rangle$	$\langle VVS\rangle$	$\langle AAS\rangle$	$\langle VAP\rangle$
	$\boldsymbol{\langle SSP\rangle}$	$\boldsymbol{\langle PPP\rangle}$	$\boldsymbol{\langle VSP\rangle}$	$\boldsymbol{\langle ASS\rangle}$	$\boldsymbol{\langle APP\rangle}$

Table 2.1: A complete list of all the two- and three-point Green functions made of the V and A currents and P and S densities. The correlators forbidden in QCD are written down in bold.

To make the set of all the Green functions a bit well-arranged, we proposed an organization scheme in ref. [1]. Therein, we have divided the correlators into two groups, based on what contributions they acquire in the chiral limit. The scheme is as follows.

- Set 1: The correlators with the perturbative contribution in the chiral limit, i.e. those with a nonzero contribution of a Feynman graph that is obtained by contracting together all the respective currents in such a way so a loop is formed. The following Green functions satisfy such a definition:
  - $\langle VV\rangle$, $\langle AA\rangle$, $\langle SS\rangle$, $\langle PP\rangle$.
  - $\langle ASP\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle VVV\rangle$, $\langle AAV\rangle$.
- Set 2: The order parameters of the chiral symmetry breaking in the chiral limit. This constitutes such Green functions, whose perturbative contribution vanishes in the chiral limit and their OPE starts with the contribution of the quark condensate (see section 3.2 for details). The respective Green functions are as follows:
  - $\langle AP\rangle$.
  - $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$.

2.4 Tensor decomposition

Knowledge of the tensor decomposition of the Green functions is of fundamental importance since it is the very essence of what such a correlator looks like. On top of that, such an information allows us to write down the respective contributions to the Green functions in a comprehensible fashion — usually only in a form of the invariant scalar functions. In practice, however, it is sometimes a bit complicated identifying such a decomposition since the relevant symmetries and respective Ward identities must be taken into account.

To this end, we take the liberty to refer the reader to our paper [1], where an extensive overview of the tensor decompositions of all the relevant three-point Green functions can be found in subsections 2.3.1 and 2.3.2. Furthermore, in the appendices E and F therein, we have also provided a thorough discussion on the explicit derivation of the decompositions of the $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle AAV\rangle$ and $\langle VVV\rangle$ Green functions.

In contrast with the above-mentioned correlators, some of them actually do have a remarkably simple structure. As an example, we present the tensor decompositions of the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions — not only because of the fact that we will pay a special attention to these functions in the following chapters. Then, due to their transversality and the Lorentz and parity invariance, the decompositions of these correlators can be written down as

$$\big[\Pi_{VVP}(p,q;r)\big]^{abc}_{\mu\nu} = \mathcal{F}_{VVP}(p^2,q^2,r^2)d^{abc}\varepsilon^{\mu\nu(p)(q)}\,, \tag{2.12a}$$

$$\big[\Pi_{VAS}(p,q;r)\big]^{abc}_{\mu\nu} = \mathcal{F}_{VAS}(p^2,q^2,r^2)f^{abc}\varepsilon^{\mu\nu(p)(q)}\,, \tag{2.12b}$$

$$\big[\Pi_{AAP}(p,q;r)\big]^{abc}_{\mu\nu} = \mathcal{F}_{AAP}(p^2,q^2,r^2)d^{abc}\varepsilon^{\mu\nu(p)(q)}\,, \tag{2.12c}$$

where the invariant functions $\mathcal{F}_{VVP}$ and $\mathcal{F}_{AAP}$ are symmetrical in the first two arguments because of the Bose symmetry.

3. Vacuum structure of QCD

In this chapter, we shall discuss the properties of the vacuum structure of QCD and thus formally conclude the introductory parts of this thesis. To this end, we point out that apart from references cited throughout the text below, this section is partially based also on ref. [1], specifically on subsections 3.1-3.4 therein.

3.1 QCD condensates

Unlike in the case of the quantum electrodynamics, the QCD vacuum is nonperturbative and its structure nontrivial due to which there exist fluctuations of quarks and gluons that are represented by nonvanishing QCD condensates. These condensates are vacuum expectation values of the local gauge-invariant composite operators made of the quark and gluon fields and, eventually, the derivatives thereof.

In our previous work [1], only a few of the QCD condensates have been taken into account in our analysis of their contributions to the OPE of Green functions in the chiral limit (see below). In detail, we have restricted ourselves only to the condensates with canonical dimension $D \leq 6$. Then, the investigation consisted of the contributions of the quark ($D = 3$), gluon ($D = 4$), quark-gluon ($D = 5$) and four-quark ($D = 6$) condensates.[28]

Since the QCD condensates are nonperturbative parameters, their numerical values can not be calculated directly and need to be obtained either experimentally or using, for example, lattice QCD. In what follows, we briefly summarize the main properties of the above-mentioned condensates and introduce their present values, extracted by using various methods.

Quark condensate. The lowest nontrivial QCD condensate is the quark condensate, $\langle \bar{q}q \rangle$. It actually possesses a particular significance since its nonzero value represents a sufficient condition for the spontaneous breakdown of the chiral symmetry in QCD — then, the quark condensate is also called as the order parameter of the chiral symmetry breaking.[29] On top of that, it represents a significant contribution to the QCD sum rules [82].

The numerical value of the quark condensate is a quantity that depends on the renormalization scale μ — in most cases, the scale of $\mu = 1\,\text{GeV}$ is considered, for which one has[30]

$$\langle \bar{q}q \rangle = -(1.38 \pm 0.17) \cdot 10^{-2}\,\text{GeV}^3\,. \tag{3.1}$$

Needless to say, the value presented above is known already from the late 60's, namely from the Gell-Mann Oakes Renner relation [83]. In spite of this, it is still a largely accepted valued, although newer ascertainments based on the lattice

[28] In ref. [1], we have tacitly omitted the contribution of another QCD condensate with $D = 6$, that is the three-gluon condensate. The reason is that it is argued that such a contribution vanishes in the chiral limit for any two-point Green function [80, 81].

[29] We point out that it is indeed a sufficient, and not a necessary condition.

[30] The logarithmic dependence on the renormalization scale occurs if the respective condensate has a nonzero anomalous dimension.

QCD give a slightly different values — see for example [84, 85, 86, 87, 88, 89, 90] for details.

Gluon condensate. The gluon condensate is the vacuum expectation value of a product of two gluon field strength tensors, $\langle G^a_{\mu\nu} G^{a\,\mu\nu}\rangle$. To be able to manage the indices economically, we will employ a short-hand notation and denote it simply as $\langle G^2\rangle$ from now on.

Its existence was proposed in the seminal papers [82, 91] and the authors obtained the value from the charmonium sum rules, for which the gluon condensate is the leading nonperturbative correction. It reads[31]

$$\frac{\alpha_s}{\pi}\langle G^2\rangle = (1.2 \pm 0.4)\cdot 10^{-2}\,\mathrm{GeV}^4 \tag{3.2}$$

and does not depend on the renormalization scale due to its zero anomalous dimension.

Since then, many other estimated have been proposed by different authors. For a comprehensive overview of the obtained values, see [92] and references therein.

Quark-gluon condensate. A vacuum expectation value of two quark fields and one gluon field strength tensor is the quark-gluon condensate, $\langle \overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle$. Once again, to save indices as much as possible, we shall denote it as $\langle \overline{q}\sigma\cdot Gq\rangle$.

In the literature, the quark-gluon condensate is usually parameterized proportionately to the quark condensate, i.e.

$$g_s\langle \overline{q}\sigma\cdot Gq\rangle = m_0^2\langle \overline{q}q\rangle\,, \tag{3.3}$$

with

$$m_0^2 = (0.8\pm 0.2)\,\mathrm{GeV}^2 \tag{3.4}$$

at $\mu = 1\,\mathrm{GeV}$ being a parameter obtained either from the sum rules for baryonic resonances [93] or from an analysis of B-mesons by QCD sum rules [94]. Another estimates using various models can be also found in refs. [95, 96, 97, 98, 99, 100, 101, 102, 103].

Four-quark condensate. Finally, the four-quark condensate is the vacuum expectation value of four quark fields that can be accompanied by an additional spinor or flavour structure, $\langle \overline{q}Xq\,\overline{q}Xq\rangle$. Here, the symbol X thus stands for such a combination of $(1,\gamma_\mu,\gamma_5,\gamma_\mu\gamma_5,\sigma_{\mu\nu})$ and $(1,T^a)$ matrices that preserves the Lorentz invariance.

As we have argued in ref. [1], evaluation of the contribution of such a condensate to the Green functions is by far the most complicated of all the condensates discussed so far. On top of that, not very much is actually known about the four-quark condensates — both theoretically and experimentally.

On the other hand, one simplifying assumption is available — it is believed that intermediate vacuum states dominate at large-N_c limit, which allows us to

[31]Here, we have chosen to add the numerical factor of $\frac{\alpha_s}{\pi}$, as it is commonly considered in the literature.

express the four-quark condensate in a form proportional to the quark condensate squared, i.e. schematically [91, 104][32]

$$\langle \overline{q}Xq\,\overline{q}Xq\rangle = \langle \overline{q}Xq \left(\sum_{n=0}^{\infty} |n\rangle\langle n|\right) \overline{q}Xq\rangle \sim \langle \overline{q}q\rangle^2 \,. \tag{3.5}$$

For an illustration of the applicability of the formula (3.5), we point out that we have shown in ref. [1] that the following formula holds:

$$\langle \overline{q}\gamma_\mu T^a q\,\overline{q}\gamma^\mu T^a q\rangle = -\frac{4}{27}\langle \overline{q}q\rangle^2 \,. \tag{3.6}$$

3.2 Operator product expansion

The framework of the Operator product expansion (OPE) was developed by K. G. Wilson in his pioneering work [105].[33] It constitutes a method which allow us to study short-distance behaviour of the Green functions or, equivalently in QCD, their high-energy behaviour.

The idea behind the OPE is as follows. Similarly to the right-hand side of (2.3), let us consider a vacuum expectation value of a time-ordered product of several local gauge-invariant composite operators, made of the field variables of the theory. If the coordinates of these operators are in a close proximity to each other, such an object can be rewritten as a series of the so-called Wilson coefficients (denoted generally as $C_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}$), accompanied by the vacuum averages of the local gauge-invariant operators, i.e. the QCD condensates. Then, the Wilson coefficients carry all the informations about the short-distance physics and are calculable in perturbative QCD with the help of the standard technique of Feynman graphs.

In the paragraph above, we have assumed the special case of OPE for simplicity, in which all the coordinates of the respective operators are close to each other. Such a choice is then equivalent to the situation, where all the momenta are simultaneously large, i.e. the scaling of the momenta goes as $(p,q,r) \to (\lambda p, \lambda q, \lambda r)$ for $\lambda \to \infty$. This is exactly the case that was studied in our previous paper [1], where we have investigated the OPE of all the existing three-point Green functions of chiral currents in the chiral limit. Taking into account only the contributions of the QCD condensates studied in ref. [1], the OPE applied to the three-point Green function in the momentum representation gives

$$\begin{aligned}\Pi_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(\lambda p, \lambda q; \lambda r) &\xrightarrow{\lambda\to\infty} \lambda\, C^{1}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r) + \frac{1}{\lambda^2} C^{\langle\overline{q}q\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle\overline{q}q\rangle \\ &+ \frac{1}{\lambda^3} C^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle G^a_{\mu\nu}G^{a\,\mu\nu}\rangle + \frac{1}{\lambda^4} C^{\langle\overline{q}\sigma\cdot Gq\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle \\ &+ \frac{1}{\lambda^5} C^{\langle\overline{q}q\rangle^2}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle\overline{q}q\rangle^2 + \ldots \,.\end{aligned} \tag{3.7}$$

[32]It is assumed that an accuracy of such a factorization hypothesis is $\sim 1/N_c^2$, i.e. for $N_c = 3$ it is roughly 10%.

[33]We point out that the acronym "OPE" is sometimes used in literature also for the term "one-pion exchange". Needless to say, such an expression is not of any importance nor relevance throughout this thesis and we reassure the reader that there can not occur any ambiguities.

The explicit results of the OPE for all the relevant three-point Green functions are quite complicated. Therefore, instead of reproducing all of these expressions here unnecessarily, we refer the reader to ref. [1], especially to sections 4 to 8. Nevertheless, we present the results of the OPE for the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions at the end of section 3.5 as illustrative examples that will be of a further use in chapter 4.

3.3 Fock–Schwinger gauge

The purpose of this section is to summarize a pedagogical introduction to the basics of the Fock–Schwinger gauge, as we have already presented it in ref. [1]. Needless to say, the basics of the Fock–Schwinger gauge has been covered by many authors already and a curious reader may find suitable sources of informations elsewhere — these are often, however, of a brief nature.[34] On top of that, said gauge has been called by various alternative names in the literature so the reader should be aware of possible ambiguities.[35]

The Fock–Schwinger gauge has been originally introduced with respect to quantum electrodynamics in order to simplify calculations involving the photon fields in refs. [106, 107] and later applied in an equivalent way to quantum chromodynamics for the gluon fields as well [79, 108, 109]. Within the context of this thesis, the Fock–Schwinger gauge corresponds to the condition

$$x^\mu \mathcal{A}^a_\mu(x) = 0\,, \tag{3.8}$$

with $\mathcal{A}^a_\mu(x)$ being the gluon field. As we will see shortly, an advantage of the Fock–Schwinger gauge (3.8) stems from the fact that it allows us to express the gluon field in terms of the gluon field strength tensor and the covariant derivatives thereof.

On the other hand, such a gauge violates the translation invariance. Nevertheless, as we have already pointed out in section 2.1, the Green functions of colourless sources are gauge-invariant — and since any gauge-invariant quantity must be invariant also with respect to translation, one must thus make sure that all the necessary Feynman diagrams for a given contribution, with the condition (3.8) taken into consideration, are actually accounted for.[36]

We will show how to express the gluon field in terms of the gluon field strength tensor within the framework of the Fock–Schwinger gauge now. Although such

[34]The discussion below is based, apart from appendix B in [1], on refs. [47, 79, 110].

[35]It might be interesting to list some of the denominations used in the literature. As ref. [111] mentions, the name "Fock–Schwinger gauge" is used by the authors of [79, 112, 113, 114, 115, 116, 117], while the following names are sometimes used as well: the "Fock gauge" [118], the "Schwinger gauge" [119, 120, 121], the "complete Lorentz invariant gauge" [108, 122, 123], the "coordinate gauge" [117, 123, 124, 125], the "fixed–point gauge" [80], the "Cronström–Dubikov–Smilga gauge" [123, 126, 127], the "Poincare gauge" [118, 128, 129, 130], the "homogenous gauge" [131] or the "multipolar gauge" [132, 133, 134].

[36]In fact, a general form of the gauge condition (3.8) is such that the gluon field therein is multiplied not by the four-vector x but $x - x_0$, with an arbitrary coordinate x_0 being the gauge parameter. Based on what we have stated above, any gauge-invariant quantity must be independent of the parameter x_0 — cancellation of x_0-dependent terms thus represents a validity check of the evaluation procedure. However, the special case (3.8) of $x_0 = 0$ is often considered so as the algebraical difficulties are avoided as much as possible.

a property has important consequences, the derivation itself is trivial. In detail, the gluon field can be easily rewritten as

$$\mathcal{A}^a_\mu(x) = \partial_\mu(x^\nu \mathcal{A}^a_\nu(x)) - x^\nu \partial_\mu \mathcal{A}^a_\nu(x) = -x^\nu \partial_\mu \mathcal{A}^a_\nu(x)\,, \tag{3.9}$$

where the assumption of the gauge condition (3.8) has been utilized in the last step. Having the gluon field given by an expression made of a product of a coordinate and a derivative of the gluon field, we shall consider to evaluate a product of such a coordinate and the gluon field strength tensor — a quantity that contains derivatives. Once again, assuming the validity of (3.8), one obtains

$$x^\nu G^a_{\mu\nu}(x) = -x^\nu(\partial_\nu \mathcal{A}^a_\mu(x) - \partial_\mu \mathcal{A}^a_\nu(x))\,, \tag{3.10}$$

since the commutator, present in the gluon field strength tensor, vanishes upon the above-mentioned condition. Then, substituting (3.9) into (3.10), one gets

$$x^\nu G^a_{\mu\nu}(x) = -\mathcal{A}^a_\mu(x) - x^\nu \partial_\nu \mathcal{A}^a_\mu(x)\,. \tag{3.11}$$

A crucial modification of (3.11) now lies in rescaling the coordinate. In fact, let us consider $x = \alpha y$, with α being a real number. Then, the relation (3.11) modificates to

$$\alpha y^\nu G^a_{\mu\nu}(\alpha y) = -\mathcal{A}^a_\mu(\alpha y) - y^\nu \partial_\nu \mathcal{A}^a_\mu(\alpha y) = -\frac{\mathrm{d}}{\mathrm{d}\alpha}(\alpha \mathcal{A}^a_\mu(\alpha y))\,, \tag{3.12}$$

where we have rewritten the second equality as the total derivative. Performing an integration of both sides of (3.12) with respect to α over the interval $[0, 1]$, the right-hand side becomes free of the parameter α, and one gets

$$\mathcal{A}^a_\mu(x) = -x^\nu \int_0^1 \alpha\, G^a_{\mu\nu}(\alpha x)\, \mathrm{d}\alpha\,, \tag{3.13}$$

where we have changed conventionally the coordinate y to x. Finally, we perform the Taylor expansion of the gluon field strength tensor around the origin, which takes the form

$$G^a_{\mu\nu}(\alpha x) = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} x^{\rho_1} \cdots x^{\rho_n}\, \partial_{\rho_1} \cdots \partial_{\rho_n} G^a_{\mu\nu}(x)\big|_{x=0}\,. \tag{3.14}$$

One may now employ an important property of the Fock–Schwinger gauge, which allow us to interchange the partial derivatives in (3.14) for the covariant ones in the adjoint representation. Indeed, it is obvious to notice that the relation

$$x^\mu(\partial_\mu G^a_{\alpha\beta}(x)) = x^\mu (D_\mu G_{\alpha\beta}(x))^a \tag{3.15}$$

holds when (3.8) is considered — and an extension of such a relation to a product of multiple coordinates and respective derivatives is then a mere trivial task. Inserting (3.14) into (3.13), together with the above property, gives us the result

$$\mathcal{A}^a_\mu(x) = -x^\nu \sum_{n=0}^{\infty} \frac{1}{n!\,(n+2)} x^{\rho_1} \cdots x^{\rho_n} (D_{\rho_1} \cdots D_{\rho_n} G_{\mu\nu}(x))^a\big|_{x=0}\,, \tag{3.16}$$

i.e.

$$\mathcal{A}^a_\mu(x) = -\frac{1}{2}x^\nu G^a_{\mu\nu}(0) - \frac{1}{3}x^\nu x^\rho (D_\rho G_{\mu\nu}(x))^a\big|_{x=0} + \dots\,, \tag{3.17}$$

where we have written down explicitly only the terms that are relevant for calculations presented in this thesis.

Contrary to the derivation of the formula (3.16) above, a similar one can be obtained also for the expansion of the quark field — but in much easier way. In fact, it is only a minor modification of the respective Taylor series, in which the partial derivatives of the quark field are interchanged for the covariant ones in the fundamental representation — once again, due to the gauge condition (3.8). Then, such an expansion can be written down as

$$q(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^{\rho_1} \cdots x^{\rho_n} (\nabla_{\rho_1} \cdots \nabla_{\rho_n} q(x))\big|_{x=0}\,, \tag{3.18}$$

where we have suppressed all the relevant indices attached to the quark field, as introduced in section 1.2, for simplicity. As an example, the first two terms of the expansion above read

$$q(x) = q(0) + x^\mu (\nabla_\mu q(x))\big|_{x=0} + \frac{1}{2} x^\mu x^\nu (\nabla_\mu \nabla_\nu q(x))\big|_{x=0} + \dots\,. \tag{3.19}$$

A similar formula to (3.18) can be found also for the Dirac-conjugated quark field $\overline{q}(x)$. There, however, the quark field and the derivatives switch places, with derivatives acting then to the left.

3.4 Quark propagator in external gluon field

As we have emphasized in section 3.3, an advantage of the Fock–Schwinger gauge is that it allows us to express the gluon field in terms of the gluon field strength tensor and the covariant derivatives thereof. On top of that, one can employ such a gauge in a straightforward manner to evaluate the propagation of a quark in the external gluon field — the respective propagator is then a helpful object that simplifies significantly calculations of the gluonic condensates.

The quark propagator in an external gluon field can be made a use of in two representations, either the coordinate or the momentum one, mutually equivalent to each other under the application of the Fourier transform.[37] We present expressions of said propagator in both representations since we will utilize each of them in different instances — firstly in the chiral limit and secondly in the mass case. Nevertheless, we will make do with the propagator expanded up to terms with two gluon field strength tensors with no derivatives since our interest is limited only to one relevant gluonic condensate, that is the gluon condensate.

Coordinate representation. An explicit form of the massless quark propagator in the external gluon field, expanded up to the term with two gluon field strength tensors coupled to the quark line, has been presented in ref. [79] and reads

$$S(x,y) = S_0(x,y) + S_1^{\alpha\beta}(x,y) G_{\alpha\beta}(0) + S_2(x,y) G_{\alpha\beta}(0) G^{\alpha\beta}(0) + \dots\,, \tag{3.20}$$

[37] For a convention regarding the Fourier transform, see appendix A.

with the coordinate dependence summarized in the following coefficients:

$$S_0(x,y) = \frac{i}{2\pi^2}\frac{\not{x}-\not{y}}{(x-y)^4}\,, \tag{3.21a}$$

$$S_1^{\alpha\beta}(x,y) = \frac{g_s}{4\pi^2}\left(\frac{\not{x}-\not{y}}{(x-y)^4}x^\alpha y^\beta - \frac{i(x-y)_\mu}{4(x-y)^2}\gamma_\nu\gamma_5\varepsilon^{\mu\nu\alpha\beta}\right), \tag{3.21b}$$

$$S_2(x,y) = \frac{ig_s^2}{192\pi^2}\frac{\not{x}-\not{y}}{(x-y)^4}\left((x\cdot y)^2 - x^2y^2\right). \tag{3.21c}$$

Obviously, the propagator (3.20) is not translation-invariant and thus can not be written down in the form of a function of the difference of the coordinates, i.e. $S(x,y) \neq S(x-y,0)$. Obviously, such a property is the consequence of the gauge condition (3.8) — which is, on an algebraical level, manifested by the first term in the bracket of (3.21b) and a whole term (3.21c). However, the propagator simplifies considerably when either one of the coordinates is set to the origin. Indeed, let us choose $y=0$, which then leads to a simple expression[38]

$$S(x,0) = \frac{i\not{x}}{2\pi^2x^4} - \frac{ig_sx_\mu}{16\pi^2x^2}\gamma_\nu\gamma_5\varepsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}(0) + \ldots\,, \tag{3.22}$$

from which the contribution of the two gluon field strength tensors vanished.

An advantage of the propagator (3.20) is that it acts generally between two coordinates and thus can be used for any n-point Green function. On the other hand, its limitation is that it can be used only in the chiral limit — unless an infinite series of terms proportional to the powers of quark mass is added to it. To this end, one may schematically denote an extension for the correction linear in quark mass as[39]

$$S(x,y) \longrightarrow S(x,y;m) = S(x,y) - \frac{m}{4\pi^2(x-y)^2} + \mathcal{O}(m^2)\,. \tag{3.23}$$

Momentum representation. In the special case, when the propagator acts between a nonzero coordinate and the origin, one can make a use of the result obtained in ref. [109], where the quark propagator in the momentum representation is presented — on top of that, even the nonzero mass of such a quark is considered therein.[40] The propagator reads

$$S(p,m) = S_0(p,m) + S_1^{\alpha\beta}(p,m)G_{\alpha\beta}(0) + S_2^{\alpha\beta\gamma\delta}(p,m)G_{\alpha\beta}(0)G_{\gamma\delta}(0) + \ldots\,, \tag{3.24}$$

with the explicit form of the contributing parts given as

$$S_0(p,m) = \frac{i(\not{p}+m)}{p^2-m^2}\,, \tag{3.25a}$$

$$S_1^{\alpha\beta}(p,m) = -\frac{ig_s}{4}\frac{\{\sigma^{\alpha\beta},\not{p}+m\}}{(p^2-m^2)^2}\,, \tag{3.25b}$$

$$S_2^{\alpha\beta\gamma\delta}(p,m) = -\frac{ig_s^2}{4}\frac{f^{\alpha\beta\gamma\delta}(p,m)+f^{\alpha\gamma\beta\delta}(p,m)+f^{\alpha\gamma\delta\beta}(p,m)}{(p^2-m^2)^5}\,, \tag{3.25c}$$

[38] It even holds that $S(x,0) = -S(-x,0) = -S(0,x)$.

[39] The terms proportional to higher powers of m get quite complicated since the mass terms arise also in the respective logarithms — see section 4.1 in ref. [135].

[40] As we will show in appendix B, this can be used to obtain the contribution of the gluon condensate to the two-point Green functions in a very straightforward way.

where we have denoted[41]

$$f^{\alpha\beta\gamma\delta}(p,m) = (\not{p}+m)\gamma^\alpha(\not{p}+m)\gamma^\beta(\not{p}+m)\gamma^\gamma(\not{p}+m)\gamma^\delta(\not{p}+m)\,. \tag{3.26}$$

Obviously, the expressions (3.25a)-(3.25b) are analogous to (3.21a)-(3.21b), when both the mass and one of the coordinates are set to zero — then, the first two terms of the propagator (3.24) should be equivalent to (3.22). To convince ourselves, one may apply the Fourier transform (A.1) on (3.22) and make a use of the formulae presented in appendix A in ref. [1].[42]

The remaining question then relates to the presence of the term with two gluon field strength tensors in (3.24). As we have mentioned in the paragraph above, such a term should vanish in the chiral limit, as it disappeared in (3.22). As it turns out, the key to answering this issue is projecting out the Lorentz structure of these tensors. To this end, the formula below is of the importance:[43]

$$G_{\alpha\beta}(0)G_{\gamma\delta}(0) = \frac{1}{72}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})G^2\,, \tag{3.27}$$

where $G^2 \equiv G^a_{\mu\nu}G^{a\,\mu\nu}$ is the local operator that, upon being placed between the vacuum states, gives arise to the gluon condensate.

For the given purpose, one may neglect the numerical factors and pay attention only to the Lorentz structure of the quantities in question. In detail, one shall realize that upon substituting (3.27) into the last term of (3.24), the following structure emerges:

$$(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})(f^{\alpha\beta\gamma\delta}(p,m) + f^{\alpha\gamma\beta\delta}(p,m) + f^{\alpha\gamma\delta\beta}(p,m))\,. \tag{3.28}$$

The algebraical adjustments are then nothing but a trivial exercise and one finds out that (3.28) is, in fact, equal to the difference of the two functions (3.26). Specifically, it is equal to

$$f^{\alpha\alpha\beta\beta}(p,m) - f^{\alpha\beta\beta\alpha}(p,m) = -12m(p^2-m^2)(p^2+m\not{p})\,, \tag{3.29}$$

which vanishes for $m=0$, as expected.

3.5 Propagation formulae

As we have depicted above, the QCD condensates are, among other, local operators. In reality, the condensates that one obtains when evaluating the contributions of respective Feynman diagrams are nonlocal since the quark and/or gluon fields between the vacuum states come from currents with different space-time arguments. In order to obtain the genuine QCD condensates, one is then required

[41] We point out to the reader that although we use a different notation for the function (3.26) in comparison with the ref. [109], their version of the propagator (3.25c) mistakenly contains one redundant term $(\not{p}+m)$ on the far-most right-hand side of eq. (3.45c) at page no. 27 therein.

[42] Only a minor inconvenience arises when one is required to rewrite the commutator of the sigma-tensor and the momentum-slash in a form proportional to the Levi-Civita tensor. To this end, the following expressions prove to be useful: $\sigma^{\mu\nu}\gamma^\rho = i(g^{\nu\rho}\gamma^\mu - g^{\mu\rho}\gamma^\nu) - \varepsilon^{\mu\nu\rho\alpha}\gamma_\alpha\gamma_5$ and $\gamma^\rho\sigma^{\mu\nu} = i(g^{\mu\rho}\gamma^\nu - g^{\nu\rho}\gamma^\mu) - \varepsilon^{\mu\nu\rho\alpha}\gamma_\alpha\gamma_5$. Then, one has $\{\sigma^{\mu\nu},\not{p}\} = -2\varepsilon^{(p)\mu\nu\alpha}\gamma_\alpha\gamma_5$.

[43] Such a formula follows directly from the relation $G^a_{\alpha\beta}(0)G^b_{\gamma\delta}(0) = \frac{1}{96}\delta^{ab}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})G^2$ upon multiplying it with T^aT^b and using the fact that $T^aT^a = \frac{4}{3}$.

to employ the propagation formulae that express such nonlocal condensates in terms of the local ones.

These formulae are one of the main results of our paper [1], where we have formulated and derived them in full length. Due to their importance, we present them here once again. In detail, the following vacuum expectation values and their expansion were of the essence in order to obtain the contributions of the local quark, gluon, quark-gluon and four-quark condensates in the chiral limit:

$$\begin{aligned}
\langle \bar{q}^A_{i,\alpha}(x) q^B_{k,\beta}(y)\rangle &= \Bigg(\frac{\langle \bar{q}q\rangle}{2^2\cdot 3^2}\delta_{ik} - \frac{g_s\langle \bar{q}\sigma\cdot Gq\rangle}{2^5\cdot 3^2}[F^{\langle \bar{q}q\rangle}(x,y)]_{ki} \\
&\quad + \frac{i\pi\alpha_s\langle \bar{q}q\rangle^2}{2^3\cdot 3^7}[G^{\langle \bar{q}q\rangle}(x,y)]_{ki} + \dots \Bigg)\delta_{\alpha\beta}\delta^{AB}\,, && (3.30a)\\
\alpha_s\langle \mathcal{A}^a_\mu(x)\mathcal{A}^b_\nu(y)\rangle &= \frac{\alpha_s\langle G^2\rangle}{2^7\cdot 3}H^{\langle G^2\rangle}_{\mu\nu}(x,y)\delta^{ab} + \dots\,, && (3.30b)\\
g_s\langle \bar{q}^A_{i,\alpha}(x)\mathcal{A}^a_\mu(u) q^B_{k,\beta}(y)\rangle &= \Bigg(\frac{g_s\langle \bar{q}\sigma\cdot Gq\rangle}{2^7\cdot 3^2}[F^{\langle \bar{q}\mathcal{A}q\rangle}_\mu(x,u,y)]_{ki} \\
&\quad + \frac{\pi\alpha_s\langle \bar{q}q\rangle^2}{2^3\cdot 3^5}[G^{\langle \bar{q}\mathcal{A}q\rangle}_\mu(x,u,y)]_{ki} + \dots \Bigg)(T^a)_{\beta\alpha}\delta^{AB}\,, && (3.30c)
\end{aligned}$$

where the coordinate dependence is summarized in the following functions:

$$\begin{aligned}
F^{\langle \bar{q}q\rangle}(x,y) &= \frac{1}{2}(x-y)^2 + \frac{i}{3}\sigma^{(x)(y)}\,, && (3.31a)\\
G^{\langle \bar{q}q\rangle}(x,y) &= 4(x\cdot y)(\not{x}-\not{y}) - (x^2-y^2)(\not{x}+\not{y})\,, && (3.31b)\\
H^{\langle G^2\rangle}_{\mu\nu}(x,y) &= (x\cdot y)g_{\mu\nu} - y_\mu x_\nu\,, && (3.31c)\\
F^{\langle \bar{q}\mathcal{A}q\rangle}_\mu(x,u,y) &= \sigma^{(u)\mu}\,, && (3.31d)\\
G^{\langle \bar{q}\mathcal{A}q\rangle}_\mu(x,u,y) &= \frac{1}{6}\gamma^\mu\big(3u\cdot(x+y) - 4u^2\big) + \frac{1}{6}\not{u}\big(4u^\mu - 3(x+y)^\mu\big) \\
&\quad - \frac{i}{2}\varepsilon^{\mu(x-y)(u)\alpha}\gamma_\alpha\gamma_5\,. && (3.31e)
\end{aligned}$$

In the formulae above, one substitutes for the quark and gluon fields according to (3.16) and (3.18), respectively. Only then one needs to perform various manipulations such as the integration by parts or employing the equations of motion for the quark and gluon fields so as the local QCD condensates are extracted.

We will not discuss these formulae here in detail — rather, we refer the reader to our article [1], especially to sections 3.4 and 3.5. We also take the liberty to point reader's attention to appendix C therein, where the derivation of the above-mentioned formulae is explained rigorously step by step as *exempli gratia.*

Having the aforementioned propagation formulae at disposal, one can indeed start performing calculations of corresponding Feynman diagrams in order to obtain the contributions of individual QCD condensates to the Green functions. Discussing such a procedure here in detail is, however, beyond the scope of this thesis — we thus kindly refer the reader once again to the ref. [1], specifically to sections 5, 6, 7 and 8, where the respective calculations of the contributions of the quark, gluon, quark-gluon and four-quark condensates, respectively, are discussed thoroughly, together with the final results.

Only so as we can comfortably utilize some of the results obtained therein in the next chapter, we reproduce here the contributions of the quark and quark-gluon condensates to the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions — see eqs. (5.5)-(5.7) and (7.12)-(7.14), respectively, in ref. [1]. The results read

$$\begin{aligned}\mathcal{F}^{\mathrm{OPE}}_{VVP}\big((\lambda p)^2,(\lambda q)^2,(\lambda r)^2\big) &= \frac{\langle\overline{q}q\rangle}{6\lambda^4}\frac{p^2+q^2+r^2}{p^2q^2r^2}\\ &- \frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{72\lambda^6}\frac{r^2(p^4+q^4)+3(p^2-q^2)^2(p^2+q^2)+4r^6}{p^4q^4r^4}+\mathcal{O}\left(\frac{1}{\lambda^8}\right),\end{aligned} \tag{3.32a}$$

$$\begin{aligned}\mathcal{F}^{\mathrm{OPE}}_{VAS}\big((\lambda p)^2,(\lambda q)^2,(\lambda r)^2\big) &= \frac{\langle\overline{q}q\rangle}{6\lambda^4}\frac{p^2-q^2-r^2}{p^2q^2r^2}\\ &- \frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{72\lambda^6}\frac{r^2(p^4-q^4)+3(p^2-q^2)(p^4+q^4)-4r^6}{p^4q^4r^4}+\mathcal{O}\left(\frac{1}{\lambda^8}\right),\end{aligned} \tag{3.32b}$$

$$\begin{aligned}\mathcal{F}^{\mathrm{OPE}}_{AAP}\big((\lambda p)^2,(\lambda q)^2,(\lambda r)^2\big) &= \frac{\langle\overline{q}q\rangle}{6\lambda^4}\frac{p^2+q^2-r^2}{p^2q^2r^2}\\ &- \frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{72\lambda^6}\frac{r^2(p^4+q^4)+3(p^2-q^2)^2(p^2+q^2)-4r^6}{p^4q^4r^4}+\mathcal{O}\left(\frac{1}{\lambda^8}\right)\end{aligned} \tag{3.32c}$$

for $\lambda\to\infty$.

4. Contribution of resonances within RChT

In the last chapter of this thesis, we will be interested in the contributions of the resonance multiplets to the Green functions of chiral currents and densities. To this end, based on our previous work [2], we will restrict ourselves only to the odd-intrinsic parity sector of QCD. In other words, we will discuss only the resonance contributions to the $\langle VVP \rangle$, $\langle VAS \rangle$ and $\langle AAP \rangle$ correlators.

In a way, this chapter serves as an extension of the paper [2] that is currently submitted for publication. On top of that, we fulfill our promise given in such paper by showing here the explicit results of the RChT–OPE matching and the resonance saturation.

4.1 Duplication of resonance multiplets

At this stage, we have the individual contributions of the quark and quark-gluon condensates in the form of the respective Operator product expansions (3.32a)-(3.32c). One may thus utilize such a knowledge and compare them with the respective contributions obtained within the framework of RChT. Such resonance contributions are given by the expressions (4.2), (4.7) and (4.12) in ref. [2].[44] However, such a matching between the respective contributions fails due to algebraical reasons.

General parametrization. As we have found in ref. [2], a remedy to such an ache — at least in the minimal case — is to duplicate the resonance multiplets, for which a detailed discussion and explicit derivation can be found in appendix A therein. To this end, one can introduce the invariant scalar functions of the $\langle VVP \rangle$, $\langle VAS \rangle$ and $\langle AAP \rangle$ Green functions as follows:

$$\begin{aligned}
\mathcal{F}_{VVP}(p^2,q^2,r^2) &= \frac{\mathcal{R}_{VVP}(p^2,q^2,r^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)} \\
&\quad\times \frac{B_0F^2}{r^2(r^2-M_{P_1}^2)(r^2-M_{P_2}^2)}\,, \qquad (4.1a)\\
\mathcal{F}_{VAS}(p^2,q^2,r^2) &= \frac{\mathcal{R}_{VAS}(p^2,q^2,r^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{A_1}^2)(q^2-M_{A_2}^2)} \\
&\quad\times \frac{B_0F^2}{(r^2-M_{S_1}^2)(r^2-M_{S_2}^2)}\,, \qquad (4.1b)\\
\mathcal{F}_{AAP}(p^2,q^2,r^2) &= \frac{\mathcal{R}_{AAP}(p^2,q^2,r^2)}{(p^2-M_{A_1}^2)(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)(q^2-M_{A_2}^2)} \\
&\quad\times \frac{B_0F^2}{r^2(r^2-M_{P_1}^2)(r^2-M_{P_2}^2)}\,, \qquad (4.1c)
\end{aligned}$$

[44] The results (4.2) and (4.7) in ref. [2] have been calculated already in ref. [78] — we, however, refer here to our work [2] for unanimity of convention.

with the respective polynomials $\mathcal{R}(p^2, q^2, r^2)$ given by eqs. (A.11) and (A.31) presented in [2].

The parametrization of (4.1a) and the subsequent expressions is adopted from ref. [136]. Nevertheless, it might be appropriate to explain the reasoning behind it. To do so, let us restrict ourselves to $\langle VVP \rangle$ correlator and a simpler case of its OPE given only by the contribution of the quark condensate. As can be read off from (3.32a), the mass dimension of the invariant function $\mathcal{F}_{VVP}$ is -1 (this goes also for $\mathcal{F}_{VAS}$ and $\mathcal{F}_{AAP}$). Due to the incorporation of the Goldstone bosons and resonances, the denominator of (4.1a) is thus of mass dimension 14 and consists of respective propagators. Consequently, the numerator is then of mass dimension 13. However, since (4.1a) is supposed to be matched onto the expression proportional to the quark condensate, which is momentum-independent quantity of mass dimension 3, we can simply adopt the quark condensate into the parametrization of (4.1a) and consider only the remaining dimension of 10 given by the momentum-dependent polynomial $\mathcal{R}_{VVP}$.[45] In the case of the $\langle VAS \rangle$ correlator, there is no contribution from the Goldstone bosons, which lowers the dimension of the polynomial $\mathcal{R}_{VAS}$ to 8.

Reduction of the coefficients. As can be seen from the above-mentioned expressions (A.11) and (A.31) in refs. [2], the polynomials $\mathcal{R}_{VVP}$ and $\mathcal{R}_{AAP}$ consists of 34 terms, while the $\mathcal{R}_{VAS}$ is made of 35 parts due to the lack of the Bose symmetry. Obviously, one may try to minimalize the number of unknown coefficients of these polynomials by introducing several well-supported requirements. In this case, the following conditions were considered.

1) The ansatz (4.1a) is assumed to satisfy the following essential requirements.

   - The OPE (3.32a) with all momenta simultaneously large.
   - The OPE with only two momenta large [137]:

$$\mathcal{F}_{VVP}^{\text{OPE}}\left((\lambda p)^2, (q-\lambda p)^2, q^2\right) = \frac{\langle \bar{q}q \rangle}{3\lambda^2}\frac{1}{p^2 q^2} + \mathcal{O}\left(\frac{1}{\lambda^3}\right) \tag{4.2}$$

   for $\lambda \to \infty$.

   - For the pion transition form factor, defined as

$$\mathcal{F}_{\gamma^*\gamma^*\pi^0}(p^2, q^2) = \frac{2}{3B_0 F}\lim_{r^2\to 0} r^2 \mathcal{F}_{VVP}(p^2, q^2, r^2)\,, \tag{4.3}$$

   the Brodsky–Lepage behaviour [138, 139]

$$\lim_{Q^2\to\infty} Q^2 \mathcal{F}_{\gamma^*\gamma^*\pi^0}(0, -Q^2) = 2F \tag{4.4}$$

   is assumed to be fulfilled.

   - The chiral anomaly at the photon point [64, 65]:

$$\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,0) = \frac{N_c}{12\pi^2 F}\,. \tag{4.5}$$

[45] Needless to say, the relation $\langle \bar{q}q \rangle = -3B_0 F^2$ is taken into account when swapping between $\langle \bar{q}q \rangle$ and $B_0 F^2$.

2) The situation with the $\langle VAS \rangle$ Green function is quite dismal — apart from a brief discussion regarding its OPE in refs. [1, 78, 140], there is probably no further knowledge of a respective form factor that could have been connected to phenomenology. Therefore, the only condition we require from (4.1b) to fulfill is the OPE (3.32b).

3) Similarly to the previous case, only a little is known about the $\langle AAP \rangle$ Green function. With respect to the current state of the art, we require (4.1c) to satisfy the two following conditions.

   – The OPE (3.32c) with all momenta simultaneously large.

   – The OPE with only two momenta large [137]:

$$\mathcal{F}_{AAP}^{\mathrm{OPE}}\left((\lambda p)^2, (q-\lambda p)^2, q^2\right) = \frac{\langle \bar{q}q \rangle}{3\lambda^2} \frac{1}{p^2 q^2} + \mathcal{O}\left(\frac{1}{\lambda^3}\right) \tag{4.6}$$

   for $\lambda \to \infty$.

The results of demanding the above-mentioned conditions to come into effect can be found in appendix A in ref. [2]. To help the reader a bit, we mention that the results for the $\langle VVP \rangle$ are the expressions (A.12), (A.13), (A.15), (A.18), (A.21) and (A.23). For $\langle VAS \rangle$, the results are eqs. (A.32) and (A.33). Finally, the results for $\langle AAP \rangle$ are eqs. (A.38), (A.39) and (A.41). Specifically, the parametrizations (4.1a)-(4.1c) thus have 8, 10 and 12 free parameters, respectively.

Extension of Lagrangians. So far, we have established the need of duplicating the resonance multiplets, in order to satisfy the OPE contributions (3.32a)-(3.32c), on the general parametrizations (4.1a)-(4.1c). Therefore, let us now advance directly to extending the resonance Lagrangians and briefly comment on such a procedure.

The required duplication of the resonance multiplets at the Lagrangian level is quite easy. In fact, it is rather trivial for single-resonance operators — one simply takes into account the same operators with the original resonance fields exchanged for the ones from the nearest higher-mass multiplet of the same kind. However, for the two- and three-resonance operators, one must be careful since all the independent combinations need to be taken into account and some algebraical manipulations must be performed to make sure that all the linearly independent terms are accounted for.

For simplicity, we will not present the extended Lagrangians here and only refer the reader to our paper [2]. Therein, the Lagrangian (3.17) and the tables 4-15 are of the importance.

Lastly, we present a short remark regarding the notation. When having to duplicate the contribution of a respective operator by replacing the original resonance fields for the duplicated ones, we accompany this newly-arised operator with a coupling constant that acquires its form according to the following rule:

$$\begin{aligned} \kappa_i^{X'} &\longrightarrow \lambda_i^{X'}, \\ \kappa_i^{X'Y'} &\longrightarrow \lambda_{i1}^{X'Y'}, \lambda_{i2}^{X'Y'}, \dots, \\ \kappa^{X'Y'Z'} &\longrightarrow \lambda_1^{X'Y'Z'}, \lambda_2^{X'Y'Z'}, \dots, \end{aligned} \tag{4.7}$$

where X', Y' and Z' stand either for V, A, S or P.[46] Such a choice of unambiguous notation in (4.7) indeed allows us to avoid confusion when having to duplicate the operators that did not play any role in paper [2] whatsoever but might be needed in the same context in the future. Also, the lower indices "$i1$" etc. represent a decimal notation and the ellipses are due to the need to take into account every necessary combinations of the resonances inside the individual two- or three-resonance operators. The additional operators are then accompanied by the couplings of the type $\mu_i^{X'}$ with the labelling chosen accordingly.

4.2 Two-multiplet resonance contribution

In this section, we simply reproduce the ChPT and RChT contributions that we have obtained in ref. [2] in order to utilize them properly in sections 4.3 and 4.4.

ChPT contribution. In the $N_c \to \infty$ limit, the ChPT contributions to the $\langle VVP \rangle$, $\langle VAS \rangle$ and $\langle AAP \rangle$ Green functions up to $\mathcal{O}(p^6)$ are given by eqs. (4.1), (4.6) and (4.11) in ref. [2], i.e.

$$\mathcal{F}_{VVP}^{\text{ChPT}}(p^2,q^2,r^2) = \frac{B_0 N_c}{8\pi^2 r^2} - 32 B_0 C_7^W + 8 B_0 C_{22}^W \frac{p^2+q^2}{r^2}\,, \tag{4.8a}$$

$$\mathcal{F}_{VAS}^{\text{ChPT}}(p^2,q^2,r^2) = 32 B_0 C_{11}^W\,, \tag{4.8b}$$

$$\mathcal{F}_{AAP}^{\text{ChPT}}(p^2,q^2,r^2) = \frac{B_0 N_c}{24\pi^2 r^2} - 32 B_0 C_9^W + 8 B_0 C_{23}^W \frac{p^2+q^2}{r^2}\,, \tag{4.8c}$$

RChT contribution. Unlike the aforementioned expressions, the RChT contributions to the $\langle VVP \rangle$, $\langle VAS \rangle$ and $\langle AAP \rangle$ Green functions up to $\mathcal{O}(p^6)$ are quite lengthy. They are given by eqs. (4.5), (4.10) and (4.15) in [2], i.e.

$$\begin{aligned}
\mathcal{F}_{VVP}^{\text{RChT}}(p^2,q^2,r^2) &= \frac{B_0 N_c}{8\pi^2 r^2} \\
&+ \frac{2\sqrt{2} B_0 F_{V_1}}{(p^2-M_{V_1}^2) r^2}\big[(2\kappa_{12}^V+\kappa_{16}^V)(p^2-q^2-r^2)+2\kappa_{17}^V q^2-8\kappa_{14}^V r^2\big] \\
&+ \frac{2\sqrt{2} B_0 F_{V_1}}{(q^2-M_{V_1}^2) r^2}\big[(2\kappa_{12}^V+\kappa_{16}^V)(q^2-p^2-r^2)+2\kappa_{17}^V p^2-8\kappa_{14}^V r^2\big] \\
&+ \frac{64 B_0 d_m^{(1)}\kappa_5^P}{r^2-M_{P_1}^2} - \frac{4 B_0 F_{V_1}^2(8\kappa_2^{VV}-\kappa_3^{VV})}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)} - \frac{4 B_0 F_{V_1}^2 \kappa_3^{VV}(p^2+q^2)}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2) r^2} \\
&+ \frac{16\sqrt{2} B_0 F_{V_1} d_m^{(1)}\kappa_3^{PV}}{(p^2-M_{V_1}^2)(r^2-M_{P_1}^2)} + \frac{16\sqrt{2} B_0 F_{V_1} d_m^{(1)}\kappa_3^{PV}}{(q^2-M_{V_1}^2)(r^2-M_{P_1}^2)} \\
&+ \frac{16 B_0 F_{V_1}^2 d_m^{(1)}\kappa^{VVP}}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)(r^2-M_{P_1}^2)} \\
&+ \frac{2\sqrt{2} B_0 F_{V_2}}{(p^2-M_{V_2}^2) r^2}\big[(2\lambda_{12}^V+\lambda_{16}^V)(p^2-q^2-r^2)+2\lambda_{17}^V q^2-8\lambda_{14}^V r^2\big]
\end{aligned} \tag{4.9a}$$

[46] To avoid any clash in the notation, we have added the apostrophes to clearly differentiate from the symbol X that has been used in (1.51) to denote various combinations of the resonance fields.

$$
\begin{aligned}
&+\frac{2\sqrt{2}B_0F_{V_2}}{(q^2-M_{V_2}^2)r^2}\big[(2\lambda_{12}^V+\lambda_{16}^V)(q^2-p^2-r^2)+2\lambda_{17}^V p^2-8\lambda_{14}^V r^2\big]\\
&+\frac{64B_0d_m^{(2)}\lambda_5^P}{r^2-M_{P_2}^2}-\frac{16B_0F_{V_1}F_{V_2}\lambda_{21}^{VV}}{(p^2-M_{V_1}^2)(q^2-M_{V_2}^2)}-\frac{16B_0F_{V_1}F_{V_2}\lambda_{21}^{VV}}{(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)}\\
&-\frac{4B_0F_{V_2}^2(8\lambda_{22}^{VV}-\lambda_{34}^{VV})}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)}-\frac{4B_0F_{V_2}^2\lambda_{34}^{VV}(p^2+q^2)}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)r^2}\\
&-\frac{2B_0F_{V_1}F_{V_2}}{(p^2-M_{V_1}^2)(q^2-M_{V_2}^2)r^2}\big[(p^2+q^2-r^2)(\lambda_{31}^{VV}+2\lambda_{32}^{VV}+\lambda_{33}^{VV})-4\lambda_{32}^{VV}p^2\big]\\
&-\frac{2B_0F_{V_1}F_{V_2}}{(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)r^2}\big[(p^2+q^2-r^2)(\lambda_{31}^{VV}+2\lambda_{32}^{VV}+\lambda_{33}^{VV})-4\lambda_{32}^{VV}q^2\big]\\
&+\frac{16\sqrt{2}B_0F_{V_2}d_m^{(1)}\lambda_{31}^{PV}}{(q^2-M_{V_2}^2)(r^2-M_{P_1}^2)}+\frac{16\sqrt{2}B_0F_{V_2}d_m^{(1)}\lambda_{31}^{PV}}{(p^2-M_{V_2}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16\sqrt{2}B_0F_{V_1}d_m^{(2)}\lambda_{32}^{PV}}{(q^2-M_{V_1}^2)(r^2-M_{P_2}^2)}+\frac{16\sqrt{2}B_0F_{V_1}d_m^{(2)}\lambda_{32}^{PV}}{(p^2-M_{V_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16\sqrt{2}B_0F_{V_2}d_m^{(2)}\lambda_{33}^{PV}}{(q^2-M_{V_2}^2)(r^2-M_{P_2}^2)}+\frac{16\sqrt{2}B_0F_{V_2}d_m^{(2)}\lambda_{33}^{PV}}{(p^2-M_{V_2}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{(p^2-M_{V_1}^2)(q^2-M_{V_2}^2)(r^2-M_{P_1}^2)}+\frac{16B_0F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16B_0F_{V_1}^2d_m^{(2)}\lambda_2^{VVP}}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)(r^2-M_{P_2}^2)}+\frac{16B_0F_{V_2}^2d_m^{(1)}\lambda_3^{VVP}}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16B_0F_{V_1}F_{V_2}d_m^{(2)}\lambda_4^{VVP}}{(p^2-M_{V_1}^2)(q^2-M_{V_2}^2)(r^2-M_{P_2}^2)}+\frac{16B_0F_{V_1}F_{V_2}d_m^{(2)}\lambda_4^{VVP}}{(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{V_2}^2d_m^{(2)}\lambda_5^{VVP}}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)(r^2-M_{P_2}^2)}+\frac{32B_0F_{V_1}^2\mu_2^{VV}r^2}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)}\\
&+\frac{16B_0F_{V_1}F_{V_2}\mu_{21}^{VV}r^2}{(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)}+\frac{16B_0F_{V_1}F_{V_2}\mu_{21}^{VV}r^2}{(p^2-M_{V_1}^2)(q^2-M_{V_2}^2)}+\frac{32B_0F_{V_2}^2\mu_{22}^{VV}r^2}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)}\\
&+\frac{16B_0F_{V_1}^2d_m^{(1)}\mu_1^{VVP}(p^4+q^4)}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)(r^2-M_{P_1}^2)}+\frac{16B_0F_{V_1}^2d_m^{(2)}\mu_2^{VVP}(p^4+q^4)}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{V_2}^2d_m^{(1)}\mu_3^{VVP}(p^4+q^4)}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)(r^2-M_{P_1}^2)}+\frac{16B_0F_{V_2}^2d_m^{(2)}\mu_4^{VVP}(p^4+q^4)}{(p^2-M_{V_2}^2)(q^2-M_{V_2}^2)(r^2-M_{P_2}^2)}\,,
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{F}_{VAS}^{\mathrm{RChT}}(p^2,q^2,r^2)=-\frac{8\sqrt{2}B_0F_{V_1}(\kappa_4^V-2\kappa_{15}^V)}{p^2-M_{V_1}^2}-\frac{16\sqrt{2}B_0F_{A_1}\kappa_{14}^A}{q^2-M_{A_1}^2} \qquad (4.9\mathrm{b})\\
&-\frac{32B_0c_m^{(1)}\kappa_2^S}{r^2-M_{S_1}^2}-\frac{16\sqrt{2}B_0F_{A_1}c_m^{(1)}\kappa_1^{SA}}{(q^2-M_{A_1}^2)(r^2-M_{S_1}^2)}+\frac{8\sqrt{2}B_0F_{V_1}c_m^{(1)}(2\kappa_1^{SV}+\kappa_2^{SV})}{(p^2-M_{V_1}^2)(r^2-M_{S_1}^2)}\\
&+\frac{16B_0F_{V_1}F_{A_1}\kappa_6^{VA}}{(p^2-M_{V_1}^2)(q^2-M_{A_1}^2)}-\frac{16B_0F_{V_1}F_{A_1}c_m^{(1)}\kappa^{VAS}}{(p^2-M_{V_1}^2)(q^2-M_{A_1}^2)(r^2-M_{S_1}^2)}\\
&-\frac{8\sqrt{2}B_0F_{V_2}(\lambda_4^V-2\lambda_{15}^V)}{p^2-M_{V_2}^2}-\frac{16\sqrt{2}B_0F_{A_2}\lambda_{14}^A}{q^2-M_{A_2}^2}-\frac{32B_0c_m^{(2)}\lambda_2^S}{r^2-M_{S_2}^2}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{16\sqrt{2}B_0 F_{A_2} c_m^{(1)} \lambda_{11}^{SA}}{(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} - \frac{16\sqrt{2}B_0 F_{A_1} c_m^{(2)} \lambda_{12}^{SA}}{(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16\sqrt{2}B_0 F_{A_2} c_m^{(2)} \lambda_{13}^{SA}}{(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} + \frac{8\sqrt{2}B_0 F_{V_2} c_m^{(1)} (2\lambda_{11}^{SV} + \lambda_{21}^{SV})}{(p^2 - M_{V_2}^2)(r^2 - M_{S_1}^2)} \\
&+ \frac{8\sqrt{2}B_0 F_{V_1} c_m^{(2)} (2\lambda_{12}^{SV} + \lambda_{22}^{SV})}{(p^2 - M_{V_1}^2)(r^2 - M_{S_2}^2)} + \frac{8\sqrt{2}B_0 F_{V_2} c_m^{(2)} (2\lambda_{13}^{SV} + \lambda_{23}^{SV})}{(p^2 - M_{V_2}^2)(r^2 - M_{S_2}^2)} \\
&+ \frac{16 B_0 F_{V_2} F_{A_1} \lambda_{61}^{VA}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)} + \frac{16 B_0 F_{V_1} F_{A_2} \lambda_{62}^{VA}}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)} \\
&+ \frac{16 B_0 F_{V_2} F_{A_2} \lambda_{63}^{VA}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)} - \frac{16 B_0 F_{V_2} F_{A_1} c_m^{(1)} \lambda_{1}^{VAS}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_1}^2)} \\
&- \frac{16 B_0 F_{V_1} F_{A_2} c_m^{(1)} \lambda_{2}^{VAS}}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} - \frac{16 B_0 F_{V_1} F_{A_1} c_m^{(2)} \lambda_{3}^{VAS}}{(p^2 - M_{V_1}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 F_{V_2} F_{A_2} c_m^{(1)} \lambda_{4}^{VAS}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} - \frac{16 B_0 F_{V_2} F_{A_1} c_m^{(2)} \lambda_{5}^{VAS}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 F_{V_1} F_{A_2} c_m^{(2)} \lambda_{6}^{VAS}}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} - \frac{16 B_0 F_{V_2} F_{A_2} c_m^{(2)} \lambda_{7}^{VAS}}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 c_m^{(1)} F_{V_2} F_{A_1} \mu_{11}^{VAS} p^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_1}^2)} - \frac{16 B_0 c_m^{(1)} F_{V_2} F_{A_2} \mu_{12}^{VAS} p^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} \\
&- \frac{16 B_0 c_m^{(2)} F_{V_2} F_{A_1} \mu_{13}^{VAS} p^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} - \frac{16 B_0 c_m^{(2)} F_{V_2} F_{A_2} \mu_{14}^{VAS} p^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 c_m^{(1)} F_{V_1} F_{A_2} \mu_{21}^{VAS} q^4}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} - \frac{16 B_0 c_m^{(1)} F_{V_2} F_{A_2} \mu_{22}^{VAS} q^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_1}^2)} \\
&- \frac{16 B_0 c_m^{(2)} F_{V_1} F_{A_2} \mu_{23}^{VAS} q^4}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} - \frac{16 B_0 c_m^{(2)} F_{V_2} F_{A_2} \mu_{24}^{VAS} q^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 c_m^{(2)} F_{V_1} F_{A_1} \mu_{31}^{VAS} r^4}{(p^2 - M_{V_1}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} - \frac{16 B_0 c_m^{(2)} F_{V_2} F_{A_1} \mu_{32}^{VAS} r^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_1}^2)(r^2 - M_{S_2}^2)} \\
&- \frac{16 B_0 c_m^{(2)} F_{V_1} F_{A_2} \mu_{33}^{VAS} r^4}{(p^2 - M_{V_1}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)} - \frac{16 B_0 c_m^{(2)} F_{V_2} F_{A_2} \mu_{34}^{VAS} r^4}{(p^2 - M_{V_2}^2)(q^2 - M_{A_2}^2)(r^2 - M_{S_2}^2)},
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{F}_{AAP}^{\mathrm{RChT}}(p^2, q^2, r^2) = \frac{B_0 N_c}{24\pi^2 r^2} \qquad (4.9c) \\
&+ \frac{2\sqrt{2}B_0 F_{A_1}}{(q^2 - M_{A_1}^2) r^2} \left[(q^2 - p^2 - r^2)(\kappa_3^A + 2\kappa_8^A + \kappa_{15}^A) + 2p^2 \kappa_{16}^A \right] \\
&+ \frac{2\sqrt{2}B_0 F_{A_1}}{(p^2 - M_{A_1}^2) r^2} \left[(p^2 - q^2 - r^2)(\kappa_3^A + 2\kappa_8^A + \kappa_{15}^A) + 2q^2 \kappa_{16}^A \right] \\
&- \frac{8\sqrt{2}B_0 F_{A_1}(2\kappa_{11}^A + \kappa_{12}^A)}{q^2 - M_{A_1}^2} - \frac{8\sqrt{2}B_0 F_{A_1}(2\kappa_{11}^A + \kappa_{12}^A)}{p^2 - M_{A_1}^2} + \frac{64 B_0 d_m^{(1)} \kappa_1^P}{r^2 - M_{P_1}^2} \\
&- \frac{32 B_0 F_{A_1}^2 \kappa_2^{AA}}{(p^2 - M_{A_1}^2)(q^2 - M_{A_1}^2)} - \frac{4 B_0 F_{A_1}^2 \kappa_3^{AA}(p^2 + q^2 - r^2)}{(p^2 - M_{A_1}^2)(q^2 - M_{A_1}^2) r^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{8\sqrt{2}B_0F_{A_1}d_m^{(1)}(2\kappa_1^{PA}+\kappa_2^{PA})}{(p^2-M_{A_1}^2)(r^2-M_{P_1}^2)}+\frac{8\sqrt{2}B_0F_{A_1}d_m^{(1)}(2\kappa_1^{PA}+\kappa_2^{PA})}{(q^2-M_{A_1}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16B_0F_{A_1}^2d_m^{(1)}\kappa^{AAP}}{(p^2-M_{A_1}^2)(q^2-M_{A_1}^2)(r^2-M_{P_1}^2)}\\
&+\frac{2\sqrt{2}B_0F_{A_2}}{(q^2-M_{A_2}^2)r^2}\big[(q^2-p^2-r^2)(\lambda_3^A+2\lambda_8^A+\lambda_{15}^A)+2p^2\lambda_{16}^A\big]\\
&+\frac{2\sqrt{2}B_0F_{A_2}}{(p^2-M_{A_2}^2)r^2}\big[(p^2-q^2-r^2)(\lambda_3^A+2\lambda_8^A+\lambda_{15}^A)+2q^2\lambda_{16}^A\big]\\
&-\frac{8\sqrt{2}B_0F_{A_2}(2\lambda_{11}^A+\lambda_{12}^A)}{q^2-M_{A_2}^2}-\frac{8\sqrt{2}B_0F_{A_2}(2\lambda_{11}^A+\lambda_{12}^A)}{p^2-M_{A_2}^2}+\frac{64B_0d_m^{(2)}\lambda_1^P}{r^2-M_{P_2}^2}\\
&-\frac{16B_0F_{A_1}F_{A_2}\lambda_{21}^{AA}}{(p^2-M_{A_1}^2)(q^2-M_{A_2}^2)}-\frac{16B_0F_{A_1}F_{A_2}\lambda_{21}^{AA}}{(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)}\\
&-\frac{4B_0F_{A_2}^2(8\lambda_{22}^{AA}-\lambda_{34}^{AA})}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)}-\frac{4B_0F_{A_2}^2\lambda_{34}^{AA}(p^2+q^2)}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)r^2}\\
&-\frac{2B_0F_{A_1}F_{A_2}}{(p^2-M_{A_1}^2)(q^2-M_{A_2}^2)r^2}\big[(p^2+q^2-r^2)(\lambda_{31}^{AA}+2\lambda_{32}^{AA}+\lambda_{33}^{AA})-4\lambda_{32}^{AA}p^2\big]\\
&-\frac{2B_0F_{A_1}F_{A_2}}{(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)r^2}\big[(p^2+q^2-r^2)(\lambda_{31}^{AA}+2\lambda_{32}^{AA}+\lambda_{33}^{AA})-4\lambda_{32}^{AA}q^2\big]\\
&+\frac{8\sqrt{2}B_0F_{A_2}d_m^{(1)}(2\lambda_{11}^{PA}+\lambda_{21}^{PA})}{(p^2-M_{A_2}^2)(r^2-M_{P_1}^2)}+\frac{8\sqrt{2}B_0F_{A_2}d_m^{(1)}(2\lambda_{11}^{PA}+\lambda_{21}^{PA})}{(q^2-M_{A_2}^2)(r^2-M_{P_1}^2)}\\
&+\frac{8\sqrt{2}B_0F_{A_1}d_m^{(2)}(2\lambda_{12}^{PA}+\lambda_{22}^{PA})}{(p^2-M_{A_1}^2)(r^2-M_{P_2}^2)}+\frac{8\sqrt{2}B_0F_{A_1}d_m^{(2)}(2\lambda_{12}^{PA}+\lambda_{22}^{PA})}{(q^2-M_{A_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{8\sqrt{2}B_0F_{A_2}d_m^{(2)}(2\lambda_{13}^{PA}+\lambda_{23}^{PA})}{(p^2-M_{A_2}^2)(r^2-M_{P_2}^2)}+\frac{8\sqrt{2}B_0F_{A_2}d_m^{(2)}(2\lambda_{13}^{PA}+\lambda_{23}^{PA})}{(q^2-M_{A_2}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{A_1}F_{A_2}d_m^{(1)}\lambda_1^{AAP}}{(p^2-M_{A_1}^2)(q^2-M_{A_2}^2)(r^2-M_{P_1}^2)}+\frac{16B_0F_{A_1}F_{A_2}d_m^{(1)}\lambda_1^{AAP}}{(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16B_0F_{A_1}^2d_m^{(2)}\lambda_2^{AAP}}{(p^2-M_{A_1}^2)(q^2-M_{A_1}^2)(r^2-M_{P_2}^2)}+\frac{16B_0F_{A_2}^2d_m^{(1)}\lambda_3^{AAP}}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)(r^2-M_{P_1}^2)}\\
&+\frac{16B_0F_{A_1}F_{A_2}d_m^{(2)}\lambda_4^{AAP}}{(p^2-M_{A_1}^2)(q^2-M_{A_2}^2)(r^2-M_{P_2}^2)}+\frac{16B_0F_{A_1}F_{A_2}d_m^{(2)}\lambda_4^{AAP}}{(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{A_2}^2d_m^{(2)}\lambda_5^{AAP}}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)(r^2-M_{P_2}^2)}+\frac{32B_0F_{A_1}^2\mu_2^{AA}r^2}{(p^2-M_{A_1}^2)(q^2-M_{A_1}^2)}\\
&+\frac{16B_0F_{A_1}F_{A_2}\mu_{21}^{AA}r^2}{(p^2-M_{A_2}^2)(q^2-M_{A_1}^2)}+\frac{16B_0F_{A_1}F_{A_2}\mu_{21}^{AA}r^2}{(p^2-M_{A_1}^2)(q^2-M_{A_2}^2)}+\frac{32B_0F_{A_2}^2\mu_{22}^{AA}r^2}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)}\\
&+\frac{16B_0F_{A_1}^2d_m^{(1)}\mu_1^{AAP}(p^4+q^4)}{(p^2-M_{A_1}^2)(q^2-M_{A_1}^2)(r^2-M_{P_1}^2)}+\frac{16B_0F_{A_1}^2d_m^{(2)}\mu_2^{AAP}(p^4+q^4)}{(p^2-M_{A_1}^2)(q^2-M_{A_1}^2)(r^2-M_{P_2}^2)}\\
&+\frac{16B_0F_{A_2}^2d_m^{(1)}\mu_3^{AAP}(p^4+q^4)}{(p^2-M_{A_2}^2)(q^2-M_{A_2}^2)(r^2-M_{P_1}^2)}\,.
\end{aligned}
$$

4.3 Matching RChT–OPE

In this section, we perform the matching between the RChT and OPE contributions, i.e. between (3.32a), (3.32b), (3.32c) and (4.9a), (4.9b), (4.9c), respectively. In detail, such a matching is performed by scaling all the momenta of the RChT contributions simultaneously as $(p, q, r) \to (\lambda p, \lambda q, \lambda r)$ for $\lambda \to \infty$ and subsequently expanding such expressions in terms of λ. A comparison with the respective OPE contributions is then made.

In order to present the results of the matching as economically as possible, we provide them in the form of an auxiliary file — the MATHEMATICA notebook. Therein, one can find what some of the couplings of the extended resonance Lagrangian are supposed to be equal to in order to satisfy the desired behaviour of the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions.

Further, in the said file, we provide expressions for the coefficients of the polynomials in (4.1a)-(4.1c) that help us to present the final results of the resonance contributions of these correlators in a compact form. To verify the validity of such results, some of these expressions can be then compared to their forms obtained at the algebraical level — see eqs. (A.12), (A.13), (A.15), (A.18), (A.21) and (A.23) for $\langle VVP\rangle$, eqs. (A.32) and (A.33) for $\langle VAS\rangle$ and, finally, eqs. (A.38), (A.39) and (A.41) for $\langle AAP\rangle$, referring once again to the article [2].

4.4 Resonance saturation

Finally, let us turn our attention to the ChPT contributions (4.8a), (4.8b) and (4.8c) that are given in terms of the respective low-energy constants C_i^W. In RChT, the contribution of these constants is accounted for in terms of the resonance multiplets. A natural task is then to perform the matching of both contributions and obtain the expressions for the low-energy constants in terms of the couplings of the resonance Lagrangian.

The matching itself is carried out as follows. Firstly, we take the resonance contributions (4.9a), (4.9b), (4.9c) and perform the Taylor expansion in terms of the kinematical variables p^2, q^2 and r^2 around zero. Then, the relevant polynomial structures of such series are compared to the tree-level ChPT results (4.8a), (4.8b) and (4.8c), respectively.

The results of the above-mentioned matching are quite lengthy. They are as follows:[47]

$$
\begin{aligned}
C_7^W = &-\boldsymbol{\frac{F_{V_1}\kappa_{12}^V}{2\sqrt{2}M_{V_1}^2}} - \boldsymbol{\frac{\sqrt{2}F_{V_1}\kappa_{14}^V}{M_{V_1}^2}} - \boldsymbol{\frac{F_{V_1}\kappa_{16}^V}{4\sqrt{2}M_{V_1}^2}} + \boldsymbol{\frac{2d_m^{(1)}\kappa_5^P}{M_{P_1}^2}} + \boldsymbol{\frac{F_{V_1}^2\kappa_2^{VV}}{M_{V_1}^4}} \\
&- \boldsymbol{\frac{F_{V_1}^2\kappa_3^{VV}}{8M_{V_1}^4}} - \boldsymbol{\frac{\sqrt{2}F_{V_1}d_m^{(1)}\kappa_3^{PV}}{M_{V_1}^2M_{P_1}^2}} + \boldsymbol{\frac{F_{V_1}^2d_m^{(1)}\kappa^{VVP}}{2M_{V_1}^4M_{P_1}^2}} - \frac{F_{V_2}\lambda_{12}^V}{2\sqrt{2}M_{V_2}^2} \\
&- \frac{\sqrt{2}F_{V_2}\lambda_{14}^V}{M_{V_2}^2} - \frac{F_{V_2}\lambda_{16}^V}{4\sqrt{2}M_{V_2}^2} + \frac{2d_m^{(2)}\lambda_5^P}{M_{P_2}^2} + \frac{F_{V_1}F_{V_2}\lambda_{21}^{VV}}{M_{V_1}^2M_{V_2}^2} + \frac{F_{V_2}^2\lambda_{22}^{VV}}{M_{V_2}^4}
\end{aligned}
$$

[47]In the expressions below, we have highlighted in bold such contributions that one would have obtained if only the lowest resonance multiplets would have been taken into account. These can be compared also with eqs. (39) and (87) in ref. [78] and with eq. (B.3) in ref. [2].

$$
\begin{aligned}
&- \frac{F_{V_1}F_{V_2}\lambda_{31}^{VV}}{8M_{V_1}^2M_{V_2}^2} - \frac{F_{V_1}F_{V_2}\lambda_{32}^{VV}}{4M_{V_1}^2M_{V_2}^2} - \frac{F_{V_1}F_{V_2}\lambda_{33}^{VV}}{8M_{V_1}^2M_{V_2}^2} - \frac{F_{V_2}^2\lambda_{34}^{VV}}{8M_{V_2}^4} \\
&- \frac{\sqrt{2}F_{V_2}d_m^{(1)}\lambda_{31}^{PV}}{M_{V_2}^2M_{P_1}^2} - \frac{\sqrt{2}F_{V_1}d_m^{(2)}\lambda_{32}^{PV}}{M_{V_1}^2M_{P_2}^2} - \frac{\sqrt{2}F_{V_2}d_m^{(2)}\lambda_{33}^{PV}}{M_{V_2}^2M_{P_2}^2} \\
&+ \frac{F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{M_{V_1}^2M_{V_2}^2M_{P_1}^2} + \frac{F_{V_1}^2d_m^{(2)}\lambda_2^{VVP}}{2M_{V_1}^4M_{P_2}^2} + \frac{F_{V_2}^2d_m^{(1)}\lambda_3^{VVP}}{2M_{V_2}^4M_{P_1}^2} \\
&+ \frac{F_{V_1}F_{V_2}d_m^{(2)}\lambda_4^{VVP}}{M_{V_1}^2M_{V_2}^2M_{P_2}^2} + \frac{F_{V_2}^2d_m^{(2)}\lambda_5^{VVP}}{2M_{V_2}^4M_{P_2}^2}\,, \qquad (4.10a)
\end{aligned}
$$

$$
\begin{aligned}
C_9^W =& -\boldsymbol{\frac{F_{A_1}\kappa_3^A}{4\sqrt{2}M_{A_1}^2}} - \boldsymbol{\frac{F_{A_1}\kappa_8^A}{2\sqrt{2}M_{A_1}^2}} - \boldsymbol{\frac{\sqrt{2}F_{A_1}\kappa_{11}^A}{M_{A_1}^2}} - \boldsymbol{\frac{F_{A_1}\kappa_{12}^A}{\sqrt{2}M_{A_1}^2}} - \boldsymbol{\frac{F_{A_1}\kappa_{15}^A}{4\sqrt{2}M_{A_1}^2}} \\
&+ \boldsymbol{\frac{2d_m^{(1)}\kappa_1^P}{M_{P_1}^2}} + \boldsymbol{\frac{F_{A_1}^2\kappa_2^{AA}}{M_{A_1}^4}} - \boldsymbol{\frac{F_{A_1}^2\kappa_3^{AA}}{8M_{A_1}^4}} - \boldsymbol{\frac{\sqrt{2}F_{A_1}d_m^{(1)}\kappa_1^{PA}}{M_{A_1}^2M_{P_1}^2}} - \boldsymbol{\frac{F_{A_1}d_m^{(1)}\kappa_2^{PA}}{\sqrt{2}M_{A_1}^2M_{P_1}^2}} \\
&+ \boldsymbol{\frac{F_{A_1}^2d_m^{(1)}\kappa^{AAP}}{2M_{A_1}^4M_{P_1}^2}} - \frac{F_{A_2}\lambda_3^A}{4\sqrt{2}M_{A_2}^2} - \frac{F_{A_2}\lambda_8^A}{2\sqrt{2}M_{A_2}^2} - \frac{\sqrt{2}F_{A_2}\lambda_{11}^A}{M_{A_2}^2} - \frac{F_{A_2}\lambda_{12}^A}{\sqrt{2}M_{A_2}^2} \\
&- \frac{F_{A_2}\lambda_{15}^A}{4\sqrt{2}M_{A_2}^2} + \frac{2d_m^{(2)}\lambda_1^P}{M_{P_2}^2} + \frac{F_{A_1}F_{A_2}\lambda_{21}^{AA}}{M_{A_1}^2M_{A_2}^2} + \frac{F_{A_2}^2\lambda_{22}^{AA}}{M_{A_2}^4} - \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{8M_{A_1}^2M_{A_2}^2} \\
&- \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{4M_{A_1}^2M_{A_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{8M_{A_1}^2M_{A_2}^2} - \frac{F_{A_2}^2\lambda_{34}^{AA}}{8M_{A_2}^4} - \frac{\sqrt{2}F_{A_2}d_m^{(1)}\lambda_{11}^{PA}}{M_{A_2}^2M_{P_1}^2} \\
&- \frac{\sqrt{2}F_{A_1}d_m^{(2)}\lambda_{12}^{PA}}{M_{A_1}^2M_{P_2}^2} - \frac{\sqrt{2}F_{A_2}d_m^{(2)}\lambda_{13}^{PA}}{M_{A_2}^2M_{P_2}^2} - \frac{F_{A_2}d_m^{(1)}\lambda_{21}^{PA}}{\sqrt{2}M_{A_2}^2M_{P_1}^2} - \frac{F_{A_1}d_m^{(2)}\lambda_{22}^{PA}}{\sqrt{2}M_{A_1}^2M_{P_2}^2} \\
&- \frac{F_{A_2}d_m^{(2)}\lambda_{23}^{PA}}{\sqrt{2}M_{A_2}^2M_{P_2}^2} + \frac{F_{A_1}d_m^{(1)}F_{A_2}\lambda_1^{AAP}}{M_{A_1}^2M_{A_2}^2M_{P_1}^2} + \frac{F_{A_1}^2d_m^{(2)}\lambda_2^{AAP}}{2M_{A_1}^4M_{P_2}^2} + \frac{F_{A_2}^2d_m^{(1)}\lambda_3^{AAP}}{2M_{A_2}^4M_{P_1}^2} \\
&+ \frac{F_{A_1}F_{A_2}d_m^{(2)}\lambda_4^{AAP}}{M_{A_1}^2M_{A_2}^2M_{P_2}^2} + \frac{F_{A_2}^2d_m^{(2)}\lambda_5^{AAP}}{2M_{A_2}^4M_{P_2}^2}\,, \qquad (4.10b)
\end{aligned}
$$

$$
\begin{aligned}
C_{11}^W =& \ \boldsymbol{\frac{F_{V_1}\kappa_4^V}{2\sqrt{2}M_{V_1}^2}} - \boldsymbol{\frac{F_{V_1}\kappa_{15}^V}{\sqrt{2}M_{V_1}^2}} + \boldsymbol{\frac{F_{A_1}\kappa_{14}^A}{\sqrt{2}M_{A_1}^2}} + \boldsymbol{\frac{c_m^{(1)}\kappa_2^S}{M_{S_1}^2}} - \boldsymbol{\frac{F_{A_1}c_m^{(1)}\kappa_1^{SA}}{\sqrt{2}M_{A_1}^2M_{S_1}^2}} \\
&+ \boldsymbol{\frac{F_{V_1}c_m^{(1)}\kappa_1^{SV}}{\sqrt{2}M_{V_1}^2M_{S_1}^2}} + \boldsymbol{\frac{F_{V_1}c_m^{(1)}\kappa_2^{SV}}{2\sqrt{2}M_{V_1}^2M_{S_1}^2}} + \boldsymbol{\frac{F_{V_1}F_{A_1}\kappa_6^{VA}}{2M_{V_1}^2M_{A_1}^2}} + \boldsymbol{\frac{F_{V_1}F_{A_1}c_m^{(1)}\kappa^{VAS}}{2M_{V_1}^2M_{A_1}^2M_{S_1}^2}} \\
&+ \frac{F_{V_2}\lambda_4^V}{2\sqrt{2}M_{V_2}^2} - \frac{F_{V_2}\lambda_{15}^V}{\sqrt{2}M_{V_2}^2} + \frac{F_{A_2}\lambda_{14}^A}{\sqrt{2}M_{A_2}^2} + \frac{c_m^{(2)}\lambda_2^S}{M_{S_2}^2} - \frac{F_{A_2}c_m^{(1)}\lambda_{11}^{SA}}{\sqrt{2}M_{A_2}^2M_{S_1}^2} \\
&- \frac{F_{A_1}c_m^{(2)}\lambda_{12}^{SA}}{\sqrt{2}M_{A_1}^2M_{S_2}^2} - \frac{F_{A_2}c_m^{(2)}\lambda_{13}^{SA}}{\sqrt{2}M_{A_2}^2M_{S_2}^2} + \frac{F_{V_2}c_m^{(1)}\lambda_{11}^{SV}}{\sqrt{2}M_{V_2}^2M_{S_1}^2} + \frac{F_{V_1}c_m^{(2)}\lambda_{12}^{SV}}{\sqrt{2}M_{V_1}^2M_{S_2}^2} \\
&+ \frac{F_{V_2}c_m^{(2)}\lambda_{13}^{SV}}{\sqrt{2}M_{V_2}^2M_{S_2}^2} + \frac{F_{V_2}c_m^{(1)}\lambda_{21}^{SV}}{2\sqrt{2}M_{V_2}^2M_{S_1}^2} + \frac{F_{V_1}c_m^{(2)}\lambda_{22}^{SV}}{2\sqrt{2}M_{V_1}^2M_{S_2}^2} + \frac{F_{V_2}c_m^{(2)}\lambda_{23}^{SV}}{2\sqrt{2}M_{V_2}^2M_{S_2}^2} \\
&+ \frac{F_{V_2}F_{A_1}\lambda_{61}^{VA}}{2M_{V_2}^2M_{A_1}^2} + \frac{F_{V_1}F_{A_2}\lambda_{62}^{VA}}{2M_{V_1}^2M_{A_2}^2} + \frac{F_{V_2}F_{A_2}\lambda_{63}^{VA}}{2M_{V_2}^2M_{A_2}^2} + \frac{F_{V_2}F_{A_1}c_m^{(1)}\lambda_1^{VAS}}{2M_{V_2}^2M_{A_1}^2M_{S_1}^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{F_{V_1}F_{A_2}c_m^{(1)}\lambda_2^{VAS}}{2M_{V_1}^2M_{A_2}^2M_{S_1}^2}+\frac{F_{V_1}F_{A_1}c_m^{(2)}\lambda_3^{VAS}}{2M_{V_1}^2M_{A_1}^2M_{S_2}^2}+\frac{F_{V_2}F_{A_2}c_m^{(1)}\lambda_4^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_1}^2}\\
&+\frac{F_{V_2}F_{A_1}c_m^{(2)}\lambda_5^{VAS}}{2M_{V_2}^2M_{A_1}^2M_{S_2}^2}+\frac{F_{V_1}F_{A_2}c_m^{(2)}\lambda_6^{VAS}}{2M_{V_1}^2M_{A_2}^2M_{S_2}^2}+\frac{F_{V_2}F_{A_2}c_m^{(2)}\lambda_7^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}\,,
\end{aligned}
\tag{4.10c}
$$

$$
\begin{aligned}
C_{22}^W=&-\boldsymbol{\frac{F_{V_1}\kappa_{17}^V}{\sqrt{2}M_{V_1}^2}}-\boldsymbol{\frac{F_{V_1}^2\kappa_3^{VV}}{2M_{V_1}^4}}-\frac{F_{V_2}\lambda_{17}^V}{\sqrt{2}M_{V_2}^2}-\frac{F_{V_1}F_{V_2}\lambda_{31}^{VV}}{2M_{V_1}^2M_{V_2}^2}\\
&-\frac{F_{V_1}F_{V_2}\lambda_{33}^{VV}}{2M_{V_1}^2M_{V_2}^2}-\frac{F_{V_2}^2\lambda_{34}^{VV}}{2M_{V_2}^4}\,,
\end{aligned}
\tag{4.10d}
$$

$$
\begin{aligned}
C_{23}^W=&-\boldsymbol{\frac{F_{A_1}\kappa_{16}^A}{\sqrt{2}M_{A_1}^2}}-\boldsymbol{\frac{F_{A_1}^2\kappa_3^{AA}}{2M_{A_1}^4}}-\frac{F_{A_2}\lambda_{16}^A}{\sqrt{2}M_{A_2}^2}-\frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{2M_{A_1}^2M_{A_2}^2}\\
&-\frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{2M_{A_1}^2M_{A_2}^2}-\frac{F_{A_2}^2\lambda_{34}^{AA}}{2M_{A_2}^4}\,.
\end{aligned}
\tag{4.10e}
$$

Upon applying the constraints obtained from the RChT–OPE matching, the expressions above can be rewritten as follows:

$$
\begin{aligned}
C_7^W=\;&\frac{1}{M_{V_1}^2+M_{V_2}^2}\Bigg(\frac{F_{V_1}^2\kappa_3^{VV}M_{V_1}^4}{4M_{V_2}^4M_{P_2}^2}+\frac{F_{V_1}F_{V_2}\lambda_{32}^{VV}M_{V_1}^2}{4M_{V_2}^4}+\frac{F_{V_2}^2d_m^{(1)}\lambda_3^{VVP}M_{V_1}^2}{2M_{V_2}^4M_{P_1}^2}\\
&+\frac{\sqrt{2}F_{V_1}d_m^{(1)}\kappa_3^{PV}M_{V_1}^2}{M_{V_2}^2M_{P_1}^2}-\frac{\sqrt{2}F_{V_1}d_m^{(1)}\kappa_3^{PV}M_{V_1}^2}{M_{V_2}^2M_{P_2}^2}-\frac{F_{V_1}^2\kappa_3^{VV}M_{V_1}^2}{4M_{V_2}^2M_{P_2}^2}\\
&-\frac{3F^2M_{V_1}^2}{16M_{V_2}^2M_{P_2}^2}-\frac{F_{V_1}F_{V_2}\lambda_{21}^{VV}M_{V_1}^2}{2M_{V_2}^4}-\frac{F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}M_{V_1}^2}{M_{V_2}^4M_{P_2}^2}\\
&-\frac{F_{V_1}F_{V_2}d_m^{(2)}\lambda_4^{VVP}M_{V_1}^2}{M_{V_2}^4M_{P_2}^2}-\frac{F_{V_1}^2d_m^{(1)}\kappa^{VVP}M_{V_1}^2}{2M_{V_2}^4M_{P_2}^2}-\frac{F_{V_1}^2d_m^{(2)}\lambda_2^{VVP}M_{V_1}^2}{2M_{V_2}^4M_{P_2}^2}\\
&-\frac{F_{V_2}^2d_m^{(1)}\lambda_3^{VVP}M_{V_1}^2}{2M_{V_2}^4M_{P_2}^2}+\frac{N_cM_{V_1}^2}{256\pi^2M_{P_2}^2}-\frac{\langle\overline{q}q\rangle M_{V_1}^2}{192B_0M_{V_2}^4}-\frac{\langle\overline{q}q\rangle M_{V_1}^2}{48B_0M_{V_2}^2M_{P_2}^2}\\
&+\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle M_{V_1}^2}{192B_0M_{V_2}^2M_{P_1}^2M_{P_2}^2}-\frac{\langle\overline{q}q\rangle}{32B_0M_{P_2}^2}-\frac{\langle\overline{q}q\rangle}{64B_0M_{V_2}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_1}^2}\\
&+\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{192B_0M_{P_1}^2M_{P_2}^2}-\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{1152B_0M_{V_2}^2M_{P_2}^2}-\frac{\langle\overline{q}q\rangle M_{V_2}^2}{96B_0M_{V_1}^2M_{P_2}^2}\\
&-\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{1152B_0M_{V_1}^2M_{P_2}^2}-\frac{\langle\overline{q}q\rangle M_{V_2}^2}{192B_0M_{V_1}^4}+\frac{F_{V_1}^2M_{V_2}^2\kappa_3^{VV}}{4M_{V_1}^2M_{P_2}^2}+\frac{F_{V_1}^2\kappa_3^{VV}}{2M_{V_1}^2}\\
&+\frac{F_{V_1}^2d_m^{(1)}M_{V_2}^2\kappa^{VVP}}{2M_{V_1}^4M_{P_1}^2}+\frac{F_{V_1}^2d_m^{(1)}\kappa^{VVP}}{2M_{V_1}^2M_{P_1}^2}+\frac{F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{M_{V_1}^2M_{P_1}^2}\\
&+\frac{F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{M_{V_2}^2M_{P_1}^2}+\frac{F_{V_1}F_{V_2}\lambda_{21}^{VV}}{2M_{V_2}^2}+\frac{F_{V_1}^2d_m^{(2)}M_{V_2}^2\lambda_2^{VVP}}{2M_{V_1}^4M_{P_2}^2}+\frac{N_c}{256\pi^2}\\
&+\frac{F_{V_1}F_{V_2}\lambda_{21}^{VV}}{2M_{V_1}^2}+\frac{F_{V_1}^2d_m^{(2)}\lambda_2^{VVP}}{2M_{V_1}^2M_{P_2}^2}+\frac{F_{V_1}F_{V_2}\lambda_{32}^{VV}}{4M_{V_1}^2}+\frac{F_{V_2}^2d_m^{(1)}\lambda_3^{VVP}}{2M_{V_2}^2M_{P_1}^2}\\
&+\frac{F_{V_1}F_{V_2}d_m^{(2)}\lambda_4^{VVP}}{M_{V_1}^2M_{P_2}^2}+\frac{F_{V_1}d_m^{(1)}M_{V_2}^2\kappa_3^{PV}\sqrt{2}}{M_{V_1}^2M_{P_2}^2}+\frac{3F^2}{32M_{V_1}^2}-\frac{N_cM_{V_2}^2}{256\pi^2M_{V_1}^2}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{\sqrt{2}F_{V_1} d_m^{(1)} M_{V_2}^2 \kappa_3^{PV}}{M_{V_1}^2 M_{P_1}^2} - \frac{F_{V_1}F_{V_2}M_{V_2}^2\lambda_{21}^{VV}}{2M_{V_1}^4} - \frac{F_{V_1}^2 M_{V_2}^2\kappa_3^{VV}}{4M_{V_1}^4} - \frac{F_{V_1}^2\kappa_3^{VV}}{4M_{P_2}^2} \\
&- \frac{F_{V_1}F_{V_2}\lambda_{32}^{VV}}{2M_{V_2}^2} - \frac{F_{V_1}^2\kappa_3^{VV}}{4M_{V_2}^2} - \frac{3F^2}{32M_{V_2}^2} + \frac{N_c M_{V_2}^2}{256\pi^2 M_{P_2}^2} - \frac{F_{V_1}F_{V_2}d_m^{(1)}\lambda_1^{VVP}}{M_{V_2}^2 M_{P_2}^2} \\
&- \frac{3F^2}{16M_{P_2}^2} - \frac{F_{V_1}^2 d_m^{(1)}\kappa^{VVP}}{2M_{P_2}^2 M_{V_2}^2} - \frac{F_{V_1}^2 d_m^{(2)}\lambda_2^{VVP}}{2M_{V_2}^2 M_{P_2}^2} - \frac{F_{V_2}^2 d_m^{(1)}\lambda_3^{VVP}}{2M_{V_2}^2 M_{P_2}^2}\Bigg), \qquad (4.11a)
\end{aligned}
$$

$$
\begin{aligned}
C_9^W = \;& \frac{g_s M_{A_1}^2\langle\bar{q}\sigma\cdot Gq\rangle}{384B_0 M_{A_2}^4 M_{P_1}^2 M_{P_2}^2} + \frac{g_s\langle\bar{q}\sigma\cdot Gq\rangle}{128B_0 M_{A_2}^2 M_{P_1}^2 M_{P_2}^2} - \frac{g_s\langle\bar{q}\sigma\cdot Gq\rangle}{192B_0 M_{A_2}^2 M_{P_1}^4} \\
&- \frac{g_s\langle\bar{q}\sigma\cdot Gq\rangle}{1152B_0 M_{A_1}^2 M_{A_2}^2 M_{P_2}^2} - \frac{\langle\bar{q}q\rangle}{96B_0 M_{A_2}^2 M_{P_2}^2} + \frac{\langle\bar{q}q\rangle}{192B_0 M_{A_2}^4} \\
&- \frac{\langle\bar{q}q\rangle}{96B_0 M_{A_1}^2 M_{P_2}^2} + \frac{\langle\bar{q}q\rangle}{192B_0 M_{A_1}^2 M_{A_2}^2} + \frac{\langle\bar{q}q\rangle}{192B_0 M_{A_1}^4} - \frac{N_c}{768\pi^2 M_{P_2}^2} \\
&+ \frac{d_m^{(1)}F_{A_1}^2\kappa^{AAP}}{2M_{A_1}^4 M_{P_1}^2} - \frac{d_m^{(1)}F_{A_1}^2\kappa^{AAP}}{2M_{A_2}^4 M_{P_2}^2} + \frac{d_m^{(1)}F_{A_1}F_{A_2}\lambda_1^{AAP}}{M_{A_1}^2 M_{A_2}^2 M_{P_1}^2} - \frac{d_m^{(1)}F_{A_1}F_{A_2}\lambda_1^{AAP}}{M_{A_2}^4 M_{P_2}^2} \\
&- \frac{\sqrt{2}d_m^{(1)}F_{A_1}\kappa_1^{PA}}{M_{A_1}^2 M_{P_1}^2} - \frac{d_m^{(1)}F_{A_1}\kappa_2^{PA}}{\sqrt{2}M_{A_1}^2 M_{P_1}^2} + \frac{\sqrt{2}d_m^{(1)}F_{A_1}\kappa_1^{PA}}{M_{A_1}^2 M_{P_2}^2} + \frac{d_m^{(1)}F_{A_1}\kappa_2^{PA}}{\sqrt{2}M_{A_1}^2 M_{P_2}^2} \\
&+ \frac{\sqrt{2}d_m^{(1)}F_{A_1}\kappa_1^{PA}}{M_{A_2}^2 M_{P_1}^2} + \frac{d_m^{(1)}F_{A_1}\kappa_2^{PA}}{\sqrt{2}M_{A_2}^2 M_{P_1}^2} - \frac{\sqrt{2}d_m^{(1)}F_{A_1}\kappa_1^{PA}}{M_{A_2}^2 M_{P_2}^2} - \frac{d_m^{(1)}F_{A_1}\kappa_2^{PA}}{\sqrt{2}M_{A_2}^2 M_{P_2}^2} \\
&+ \frac{d_m^{(1)}F_{A_2}^2\lambda_3^{AAP}}{2M_{A_2}^4 M_{P_1}^2} - \frac{d_m^{(1)}F_{A_2}^2\lambda_3^{AAP}}{2M_{A_2}^4 M_{P_2}^2} + \frac{d_m^{(2)}F_{A_1}^2\lambda_2^{AAP}}{2M_{A_1}^4 M_{P_2}^2} - \frac{d_m^{(2)}F_{A_1}^2\lambda_2^{AAP}}{2M_{A_2}^4 M_{P_2}^2} \\
&+ \frac{d_m^{(2)}F_{A_1}F_{A_2}\lambda_4^{AAP}}{M_{A_1}^2 M_{A_2}^2 M_{P_2}^2} - \frac{d_m^{(2)}F_{A_1}F_{A_2}\lambda_4^{AAP}}{M_{A_2}^4 M_{P_2}^2} + \frac{F_{A_1}^2\kappa_3^{AA}M_{A_1}^2}{4M_{A_2}^4 M_{P_2}^2} - \frac{F_{A_1}^2\kappa_3^{AA}}{4M_{A_1}^2 M_{P_2}^2} \\
&- \frac{F_{A_1}F_{A_2}\lambda_{21}^{AA}}{2M_{A_1}^4} + \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{16M_{A_1}^4} + \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{8M_{A_1}^4} + \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{16M_{A_1}^4} \\
&+ \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}M_{A_1}^2}{8M_{A_2}^4 M_{P_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}M_{A_1}^2}{4M_{A_2}^4 M_{P_2}^2} + \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}M_{A_1}^2}{8M_{A_2}^4 M_{P_2}^2} \\
&+ \frac{F_{A_1}F_{A_2}\lambda_{21}^{AA}}{M_{A_1}^2 M_{A_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{8M_{A_1}^2 M_{A_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{4M_{A_1}^2 M_{A_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{8M_{A_1}^2 M_{A_2}^2} \\
&- \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{8M_{A_1}^2 M_{P_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{4M_{A_1}^2 M_{P_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{8M_{A_1}^2 M_{P_2}^2} - \frac{F_{A_1}F_{A_2}\lambda_{21}^{AA}}{2M_{A_2}^4} \\
&+ \frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{16M_{A_2}^4} + \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{8M_{A_2}^4} + \frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{16M_{A_2}^4} + \frac{F_{A_1}F_{A_2}\lambda_{32}^{AA}}{2M_{A_2}^2 M_{P_2}^2}, \qquad (4.11b)
\end{aligned}
$$

$$
\begin{aligned}
C_{11}^W = &- \frac{F_{A_1}c_m^{(1)}\kappa_1^{SA}}{\sqrt{2}M_{A_1}^2 M_{S_1}^2} + \frac{F_{A_1}c_m^{(1)}\kappa_1^{SA}}{\sqrt{2}M_{A_1}^2 M_{S_2}^2} + \frac{F_{A_1}c_m^{(1)}\kappa_1^{SA}}{\sqrt{2}M_{A_2}^2 M_{S_1}^2} - \frac{F_{A_1}c_m^{(1)}\kappa_1^{SA}}{\sqrt{2}M_{A_2}^2 M_{S_2}^2} \\
&+ \frac{F_{V_1}c_m^{(1)}\kappa_1^{SV}}{\sqrt{2}M_{V_1}^2 M_{S_1}^2} - \frac{F_{V_1}c_m^{(1)}\kappa_1^{SV}}{\sqrt{2}M_{V_1}^2 M_{S_2}^2} + \frac{F_{V_1}c_m^{(1)}\kappa_1^{SV}}{\sqrt{2}M_{V_2}^2 M_{S_2}^2} - \frac{F_{V_1}c_m^{(1)}\kappa_1^{SV}}{\sqrt{2}M_{V_2}^2 M_{S_1}^2} \\
&+ \frac{F_{V_1}c_m^{(1)}\kappa_2^{SV}}{2\sqrt{2}M_{V_1}^2 M_{S_1}^2} - \frac{F_{V_1}c_m^{(1)}\kappa_2^{SV}}{2\sqrt{2}M_{V_2}^2 M_{S_1}^2} - \frac{F_{V_1}c_m^{(1)}\kappa_2^{SV}}{2\sqrt{2}M_{V_1}^2 M_{S_2}^2} + \frac{F_{V_1}c_m^{(1)}\kappa_2^{SV}}{2\sqrt{2}M_{V_2}^2 M_{S_2}^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{F_{V_1}F_{A_1}\kappa_6^{VA}}{2M_{V_1}^2M_{A_1}^2}-\frac{F_{V_1}F_{A_1}\kappa_6^{VA}}{2M_{V_2}^2M_{A_1}^2}-\frac{F_{V_1}F_{A_1}\kappa_6^{VA}}{2M_{V_1}^2M_{A_2}^2}+\frac{F_{V_1}F_{A_1}\kappa_6^{VA}}{2M_{V_2}^2M_{A_2}^2}\\
&+\frac{F_{V_1}F_{A_1}c_m^{(1)}\kappa^{VAS}}{2M_{V_1}^2M_{A_1}^2M_{S_1}^2}-\frac{F_{V_1}F_{A_1}c_m^{(1)}\kappa^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}+\frac{F_{V_2}F_{A_1}c_m^{(1)}\lambda_1^{VAS}}{2M_{V_2}^2M_{A_1}^2M_{S_1}^2}\\
&-\frac{F_{V_2}F_{A_1}c_m^{(1)}\lambda_1^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}+\frac{F_{V_1}F_{A_2}c_m^{(1)}\lambda_2^{VAS}}{2M_{V_1}^2M_{A_2}^2M_{S_1}^2}-\frac{F_{V_1}F_{A_2}c_m^{(1)}\lambda_2^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}\\
&+\frac{F_{V_1}F_{A_1}c_m^{(2)}\lambda_3^{VAS}}{2M_{V_1}^2M_{A_1}^2M_{S_2}^2}-\frac{F_{V_1}F_{A_1}c_m^{(2)}\lambda_3^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}+\frac{F_{V_2}F_{A_2}c_m^{(1)}\lambda_4^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_1}^2}\\
&-\frac{F_{V_2}F_{A_2}c_m^{(1)}\lambda_4^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}+\frac{F_{V_2}F_{A_1}c_m^{(2)}\lambda_5^{VAS}}{2M_{V_2}^2M_{A_1}^2M_{S_2}^2}-\frac{F_{V_2}F_{A_1}c_m^{(2)}\lambda_5^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}\\
&+\frac{F_{V_1}F_{A_2}c_m^{(2)}\lambda_6^{VAS}}{2M_{V_1}^2M_{A_2}^2M_{S_2}^2}-\frac{F_{V_1}F_{A_2}c_m^{(2)}\lambda_6^{VAS}}{2M_{V_2}^2M_{A_2}^2M_{S_2}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_1}^2M_{A_2}^2}\\
&-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_1}^2M_{S_2}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_2}^2M_{S_1}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_2}^2M_{A_2}^2}\\
&+\frac{\langle\overline{q}q\rangle}{192B_0M_{A_2}^2M_{S_2}^2}+\frac{\langle\overline{q}q\rangle}{192B_0M_{A_2}^2M_{S_1}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{A_1}^2M_{V_2}^2}\\
&+\frac{\langle\overline{q}q\rangle}{192B_0M_{A_1}^2M_{S_2}^2}-\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{2304B_0M_{V_1}^2M_{V_2}^2M_{S_2}^2}+\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{768B_0M_{V_2}^2M_{S_1}^2M_{S_2}^2}\\
&-\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{768B_0M_{A_2}^2M_{S_1}^2M_{S_2}^2}+\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{2304B_0M_{A_1}^2M_{A_2}^2M_{S_2}^2}-\frac{\langle\overline{q}q\rangle}{192B_0M_{V_2}^2M_{S_2}^2}\,, \qquad (4.11c)
\end{aligned}
$$

$$
C_{22}^W=-\frac{3F^2}{8M_{V_1}^2M_{V_2}^2}+\frac{N_c}{64\pi^2M_{V_1}^2}+\frac{N_c}{64\pi^2M_{V_2}^2}\,, \qquad (4.11d)
$$

$$
\begin{aligned}
C_{23}^W=&-\frac{F_{A_1}\kappa_3^A}{2\sqrt{2}M_{A_1}^2}+\frac{F_{A_1}\kappa_3^A}{\sqrt{2}M_{A_2}^2}-\frac{F_{A_1}\kappa_3^AM_{A_1}^2}{2\sqrt{2}M_{A_2}^4}-\frac{F_{A_1}\kappa_8^A}{\sqrt{2}M_{A_1}^2}+\frac{\sqrt{2}F_{A_1}\kappa_8^A}{M_{A_2}^2}\\
&-\frac{F_{A_1}\kappa_8^AM_{A_1}^2}{\sqrt{2}M_{A_2}^4}-\frac{F_{A_1}\kappa_{15}^A}{2\sqrt{2}M_{A_1}^2}+\frac{F_{A_1}\kappa_{15}^A}{\sqrt{2}M_{A_2}^2}-\frac{F_{A_1}\kappa_{15}^AM_{A_1}^2}{2\sqrt{2}M_{A_2}^4}-\frac{F_{A_1}^2\kappa_3^{AA}}{2M_{A_1}^4}\\
&+\frac{F_{A_1}^2\kappa_3^{AA}}{2M_{A_2}^4}-\frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{2M_{A_1}^2M_{A_2}^2}+\frac{F_{A_1}F_{A_2}\lambda_{31}^{AA}}{2M_{A_2}^4}-\frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{2M_{A_1}^2M_{A_2}^2}+\frac{F_{A_1}F_{A_2}\lambda_{33}^{AA}}{2M_{A_2}^4}\\
&+\frac{N_c}{192\pi^2M_{A_2}^2}+\frac{\langle\overline{q}q\rangle}{48B_0M_{A_2}^4}-\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{192B_0M_{A_2}^4M_{P_1}^2}+\frac{g_s\langle\overline{q}\sigma\cdot Gq\rangle}{192B_0M_{A_1}^2M_{A_2}^2M_{P_1}^2}\,.
\end{aligned}
\qquad (4.11e)
$$

Interestingly enough, we see that we have obtained a simple relation (4.11d) for C_{22}^W, in which all the parameters are known. This thus allows us to extract its numerical value:

$$
C_{22}^W=7.68\cdot10^{-3}\,\mathrm{GeV}^{-2} \qquad (4.12)
$$

4.5 Three-multiplet resonance contribution

As we have shown in ref. [2], the pion transition form factor obtained from the $\langle VVP\rangle$ Green functions with two vector and two pseudoscalar resonance multiplets reproduces the one of the THS — cf. eqs. (A.24) and (A.25) in [2] and eq. (17) in [136]. Naturally, one might try to increase the number of respective multiplets in accordance with the previous strategy. Then, instead of (4.1a), one is required to consider the ansatz

$$\begin{aligned}\mathcal{F}_{VVP}(p^2,q^2,r^2) &= \frac{B_0F^2}{r^2(r^2-M_{P_1}^2)(r^2-M_{P_2}^2)(r^2-M_{P_3}^2)} \\ &\times \frac{\mathcal{S}_{VVP}(p^2,q^2,r^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(p^2-M_{V_3}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)(q^2-M_{V_3}^2)}\,,\end{aligned} \tag{4.13}$$

where $\mathcal{S}_{VVP}(p^2,q^2,r^2)$ is a polynomial of mass dimension 16 and, after taking into account the requirements (3.32a), (4.2), (4.4) and (4.5), encompass of 40 unknown coefficients — we shall denote them, for example, as α_i.[48]

The pion transition form factor, corresponding to the ansatz (4.13), reads

$$\begin{aligned}\mathcal{F}_{\gamma^*\gamma^*\pi^0}(p^2,q^2) &= \frac{N_c}{12\pi^2F} \\ &\times \frac{M_{V_1}^4M_{V_2}^4M_{V_3}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(p^2-M_{V_3}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)(q^2-M_{V_3}^2)} \\ &\times \Bigg[1+\frac{4\pi^2F^2}{N_cM_{V_1}^4M_{V_2}^4M_{V_3}^4}\Big(6M_{V_1}^2M_{V_2}^2M_{V_3}^2(p^4+q^4)-p^4q^4(p^2+q^2)\Big) \\ &- \frac{8\pi^2F^2}{N_cM_{V_1}^4M_{V_2}^4M_{V_3}^4M_{P_1}^2M_{P_2}^2M_{P_3}^2}\Big(\alpha_2(p^2+q^2)+\alpha_6p^2q^2+\alpha_{10}p^2q^2(p^2+q^2) \\ &+ \alpha_{16}p^2q^2(p^4+q^4)+\alpha_{20}p^4q^4+\alpha_{25}p^2q^2(p^2-q^2)^2(p^2+q^2)+\alpha_{37}p^2q^2(p^2-q^2)^4\Big)\Bigg]\end{aligned} \tag{4.14}$$

and depends on seven coefficients. Naturally, it is desirable to determine them.

We have thus introduced three additional requirements that the parametrization (4.13) should satisfy. These are as follows.

- The “subleading” Brodsky–Lepage behaviour [104, 141]:

$$\frac{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(-Q^2,-Q^2)}{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,0)} = \frac{8\pi^2F^2}{3}\left[\frac{1}{Q^2}-\frac{8\delta^2}{9Q^4}+\mathcal{O}\left(\frac{1}{Q^6}\right)\right], \tag{4.15}$$

  with the parameter $\delta^2 = (0.20\pm0.02)\,\mathrm{GeV}^2$.

- The asymptotic behaviour of the form factor $\langle V_1|V|\pi^0\rangle$ for $q^2\to\infty$:

$$\lim_{\substack{p^2\to M_{V_1}^2\\ r^2\to 0}}(p^2-M_{V_1}^2)r^2\mathcal{F}_{VVP}(p^2,q^2,r^2) \sim \frac{1}{q^2}\,. \tag{4.16}$$

[48] Originally, before one applies the above-mentioned requirements, the polynomial consists of 95 Bose-symmetric terms and the same number of corresponding coefficients. Although it would not make sense to list the full polynomial, the reader might be interested which coefficient belongs to a specific monomial. To this end, we describe the organization of such a polynomial as follows. Let us consider a general Bose-symmetric monomial of a dimension d in a form $(p^{2a}+q^{2a})p^{2b}q^{2b}r^{2c}$, where a, b and c are natural numbers and the factors of two guarantee that there can not occur odd powers of any of the momenta p, q or r. Obviously, the relation $2(a+2b+c)=d$ holds. Then, starting with a constant term, we sort the individual monomials in such an order so as the size of its dimension is saturated primarily by the number a, only then by b and, lastly, by c.

- The semi on-shell Brodsky–Lepage behaviour for the pseudoscalar resonances for $Q^2 \to \infty$ [142]:

$$\lim_{r^2 \to M_{P_i}^2} (r^2 - M_{P_i}^2)\mathcal{F}_{VVP}(0, -Q^2, r^2) \sim \frac{1}{Q^2}\,, \tag{4.17}$$

with factorization in all pseudoscalar channels, i.e. $i = 1, 2, 3$.

Although the results that one gets by utilizing these conditions are shown in ref. [2], let us reproduce them here in more detail. The reasoning behind this is the fact that, as it turns out, one actually obtains constraints also for some coefficients that are not present in the form factor (4.14).

After substituting the pion transition form factor (4.14) into (4.15), one gets

$$\begin{aligned}\frac{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(-Q^2,-Q^2)}{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,0)} &= \frac{8\pi^2F^2}{3} \\ &\times \left[\frac{1}{Q^2} - \frac{1}{Q^4}\left(\frac{2\alpha_{16}+\alpha_{20}}{M_{P_1}^2M_{P_2}^2M_{P_3}^2} + 2(M_{V_1}^2+M_{V_2}^2+M_{V_3}^2)\right) + \mathcal{O}\left(\frac{1}{Q^6}\right)\right],\end{aligned} \tag{4.18}$$

i.e. so as to fulfill the “subleading” Brodsky–Lepage behaviour one has

$$\alpha_{20} = -2\alpha_{16} - 2M_{P_1}^2M_{P_2}^2M_{P_3}^2\left(M_{V_1}^2+M_{V_2}^2+M_{V_3}^2-\frac{4}{9}\delta^2\right). \tag{4.19}$$

The condition (4.16) gives us the series

$$\begin{aligned}-&\Big[q^4\alpha_{37} + q^2\left(\alpha_{25} - \alpha_{37}(3M_{V_1}^2 - M_{V_2}^2 - M_{V_3}^2)\right) + \frac{1}{2}M_{V_1}^2M_{P_1}^2M_{P_2}^2M_{P_3}^2 + \alpha_{16} \\ &+ \alpha_{25}(M_{V_2}^2+M_{V_3}^2) + \alpha_{37}\left(3M_{V_1}^2(M_{V_1}^2-M_{V_2}^2-M_{V_3}^2) + M_{V_2}^4 + M_{V_3}^4 + M_{V_2}^2M_{V_3}^2\right)\Big] \\ &\times \frac{B_0F^2M_{V_1}^2}{(M_{V_1}^2-M_{V_2}^2)(M_{V_1}^2-M_{V_3}^2)M_{P_1}^2M_{P_2}^2M_{P_3}^2} + \mathcal{O}\left(\frac{1}{q^2}\right),\end{aligned} \tag{4.20}$$

from which, if one enforces its behaviour according to (4.16), it is obvious that the polynomial structure in the brackets must vanish. This leads to

$$\alpha_{16} = -\frac{1}{2}M_{V_1}^2M_{P_1}^2M_{P_2}^2M_{P_3}^2\,, \tag{4.21a}$$

$$\alpha_{25} = \alpha_{37} = 0\,. \tag{4.21b}$$

Finally, the behaviour (4.17) gives us three sets of conditions. Writing down explicitly the expansion in terms of Q^2 for the factorization of the first pseudoscalar resonance multiplet, we get

$$\begin{aligned}-&\frac{B_0F^2}{M_{V_1}^2M_{V_2}^2M_{V_3}^2(M_{P_1}^2-M_{P_2}^2)(M_{P_1}^2-M_{P_3}^2)} \\ &\times \Big[\alpha_{36}Q^4 - \left(\alpha_{24} + \alpha_{36}(M_{V_1}^2+M_{V_2}^2+M_{V_3}^2) + \beta_{62}M_{P_1}^2\right)Q^2 + \alpha_{15} \\ &+ \alpha_{26}M_{P_1}^2 + \alpha_{36}(M_{V_1}^4 + M_{V_1}^2M_{V_3}^2 + M_{V_2}^2M_{V_3}^2 + M_{V_1}^2M_{V_2}^2 + M_{V_2}^4 + M_{V_3}^4) \\ &+ (\alpha_{24} + \alpha_{38}M_{P_1}^2)(M_{V_1}^2+M_{V_2}^2+M_{V_3}^2) + \alpha_{40}M_{P_1}^4\Big] + \mathcal{O}\left(\frac{1}{Q^2}\right),\end{aligned} \tag{4.22}$$

with the equivalent expressions for the second and the third multiplets obtained easily by replacing M_{P_1} for M_{P_2} and M_{P_3}, respectively. Since the elimination of the terms in the bracket is straightforward, we readily get the following constrains:

$$\alpha_{15} = \alpha_{24} = \alpha_{26} = \alpha_{36} = \alpha_{38} = \alpha_{40} = 0\,. \tag{4.23}$$

Taking into account the results obtained above and substituting them back into the form factor (4.14), only three coefficients remain undetermined: α_2, α_6 and α_{10}.

A procedure of identifying the values of these parameters is, however, a bit complicated. First of all, they are dimensionful, so scaling them accordingly is a useful start. Then, it is obvious to notice that once one of the momenta is set to zero, the form factor is then a function of only α_2. This can be fitted onto the experimental data obtained by the BABAR [143], BELLE [144] and CLEO [145] collaborations. On top of that, the parameter α_2 can be fixed also from the decays $\rho^+ \to \pi^+\gamma$ and $\omega \to \pi^0\gamma$. In ref. [2], we have investigated these approaches and acquired values that are in a mutual agreement — see section 5.2 therein.

The determination of the parameters α_6 and α_{10} is far more complicated. A detailed discussion can be found also in ref. [2], see section 5.3 therein. We will not go into details here, let us only mention that we have employed the decays $\omega \to \pi^0 e^+e^-$ and $\omega \to \pi^0\mu^+\mu^-$, from which the obtained values are burdened with a large error due to the corresponding unreliable χ^2 function. Nevertheless, the obtained central values were subsequently used, together with the decay $\pi^0 \to e^+e^-$ and the effective parameter $\chi^{(\mathrm{r})}$, for a prediction of the pion-pole contribution to the muon anomalous magnetic moment. The result, having a large error as well, reads

$$a_\mu^{\mathrm{LbyL},\,\pi^0} = 65 \pm 45 \cdot 10^{-11}\,. \tag{4.24}$$

Conclusion

In this thesis and the respective attached journal articles, we have studied all the relevant two- and three-point Green functions of the chiral currents and densities using the ChPT/RChT and OPE approaches.

We have presented the leading-order contributions of the QCD condensates up to dimension six to these correlators, obtained within the framework of the Operator product expansion. In detail, such an effort consisted of recalculating the well-known perturbative contributions and then by evaluating the respective contributions of the QCD condensates — both in the chiral limit. We have accomplished to obtain the following results:

- The contributions of the gluon and four-quark condensates to the $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle VVV\rangle$, $\langle ASP\rangle$, $\langle AAV\rangle$, $\langle VSS\rangle$ and $\langle VPP\rangle$ Green functions.
- The contributions of the quark and quark-gluon condensates to the $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$ and $\langle VAP\rangle$ Green functions.

In the second part of the work, we have then limited ourselves to the order parameters of the chiral symmetry breaking in the chiral limit, i.e. to the $\langle VVP\rangle$, $\langle VAS\rangle$ and $\langle AAP\rangle$ Green functions. These were subjected to an investigation within the Chiral perturbation theory and Resonance chiral theory. In order to obtain some constraints on the parameters of the effective Lagrangians, we have required their high-energy behaviour to match OPE, for which the contributions of the quark and quark-gluon condensates were taken into account. The result of such an approach was a realization that the duplication of all the lowest vector, axial-vector, scalar and pseudoscalar resonance multiplets in the corresponding Lagrangians is inevitable. On top that, taking into account several derivative operators of a higher dimension was proved to be necessary. Then, the respective resonance contributions were successfully matched onto OPE, from which a series of constraints for the respective coupling constants of the resonance Lagrangians were extracted in the form of "sum rules".

As a special case, we have also studied the $\langle VVP\rangle$ Green function with three vector and three pseudoscalar resonance multiplets taken into account. Needless to say, this investigation was performed only on an algebraic level since the corresponding extension of the resonance Lagrangian would be extremely complicated. To this end, we were able to construct the relevant double off-shell pion transition form factor $\mathcal{F}_{\gamma^*\gamma^*\pi^0}(p^2,q^2)$ as a function of three parameters only. Their values were eventually identified with the help of various phenomenological inputs, such as using the fit on the experimental data for the quantity $Q^2\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,-Q^2)$ and the decays $\rho^+\to\pi^+\gamma$, $\omega\to\pi^0\gamma$, $\omega\to\pi^0e^+e^-$ and $\omega\to\pi^0\mu^+\mu^-$. Finally, a study of the correlation of the pion-pole contribution to the muon $g-2$ factor and the effective parameter $\chi^{(\mathrm{r})}$, related to the $\pi^0\to e^+e^-$ decay, was performed.

Future prospects. In the second part of our work, we have knowingly ignored the anomalous three-point Green functions, i.e. the $\langle VVA\rangle$ and $\langle AAA\rangle$ correlators. The reason for such a simplification is that their OPE starts, even in the chiral limit, with the perturbative contribution. A presence of the respective logarithmic terms makes the matching on their resonance contributions difficult since these terms can not be treated within RChT with a finite number of resonances. This, however, represents a challenging task that would be interesting to study further. Also, as another future prospect, it would be desirable to perform the presented study completely beyond the chiral limit.

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List of Tables

List of Abbreviations

DESY	Deutsches Elektronen Synchrotron
GB	Goldstone boson
ChPT	Chiral perturbation theory
LSZ	Lehmann–Symanzik–Zimmermann
OPE	Operator product expansion
PETRA	Positron-Elektron Tandem Ring Anlage
QCD	Quantum chromodynamics
RChT	Resonance chiral theory
TASSO	Two Arm Spectrometer Solenoid
THS	Two-hadron saturation

List of Publications

In this section, the author's publications — both journal articles and conference proceedings — are presented in a chronological order.

Journal articles. During the author's doctoral studies, two journal articles have been accomplished. The article [1] was published in the Journal of High Energy Physics, while the article [2] is currently submitted therein and is awaiting the reviewing process. As what regards the authorship, the author of this thesis is the main author of these articles. In detail, the author has carried out all the respective calculations, that were continuously discussed with the supervisors, and prepared the first versions of the respective texts and figures, that were consequently subjected to various adjustments proposed by the supervisors as well.

[1] *OPE of Green functions of chiral currents*

T. Kadavý, K. Kampf and J. Novotný

Published in JHEP **10** (2020) 142

[2] *On the three-point order parameters of chiral symmetry breaking*

T. Kadavý, K. Kampf and J. Novotný

Submitted to JHEP

Conference proceedings. In what follows, we present a detailed list of articles published in the proceedings of some of the conferences that the author of the dissertation has attended during the whole course of his studies and where he presented topics closely related to the theme of this thesis, either in the form of an oral presentation (seven times) or as a poster (five times). We also point out that the proceedings [3]-[6] were prepared during bachelor and master studies, so only the articles [7]-[14] correspond to the author's doctoral studies.[49]

[3] *Resonances in the odd-intrinsic sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

MesonNet 2013 International Workshop (Prague, Czech Republic; June 17-19, 2013)

[4] *On hadronic light-by-light contribution to the muon $g-2$ factor*

T. Husek, T. Kadavý, K. Kampf and J. Novotný

MesonNet 2014 International Workshop (Frascati, Italy; September 29 - October 1, 2014)

[49] It must be, however, pointed out that the presented list of conferences attended is far from complete since not every opportunity to present the work this thesis is based on was accompanied with a possibility to contribute to the conference proceedings.

[5] *Green functions of currents in the odd-intrinsic parity sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

11th Conference on Quark Confinement and the Hadron Spectrum (St. Petersburg, Russia; September 8-12, 2014)

[6] *Green functions of currents in the odd-intrinsic parity sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

18th High-Energy Physics International Conference in Quantum Chromodynamics (Montpellier, France; June 29 - July 3, 2015)

[7] *Three-point Green functions in the odd sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

14th International Workshop on Meson Production, Properties and Interaction (Kraków, Poland; June 2-7, 2016)

[8] *OPE for the Three-point Green Functions of Currents in the Odd-intrinsic Parity Sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

Week of Doctoral Students 2016 (Prague, Czech Republic; June 7-9, 2016)

[9] *OPE of Green functions in the odd sector of QCD*

T. Kadavý, K. Kampf and J. Novotný

12th Conference on Quark Confinement and the Hadron Spectrum (Thessaloniki, Greece; August 28 - September 4, 2016)

[10] *Contribution of QCD condensates to the OPE of Green functions of chiral currents*

T. Kadavý, K. Kampf and J. Novotný

15th International Workshop on Meson Physics (Kraków, Poland; June 7-12, 2018)

[11] *Contribution of QCD Condensates to the OPE of Green Functions of Chiral Currents*

T. Kadavý, K. Kampf and J. Novotný

39th International Conference on High Energy Physics (Seoul, South Korea; July 4-11, 2018)

[12] *Contribution of quantum chromodynamics condensates to the operator product expansion of Green functions of chiral currents*

T. Kadavý, K. Kampf and J. Novotný

8th International Workshop on Astronomy and Relativistic Astrophysics (Ollantaytambo, Peru; September 8-15, 2018)

[13] *OPE of Green Functions of Chiral Currents*

T. Kadavý

22th High-Energy Physics International Conference in Quantum Chromodynamics (Montpellier, France; July 2-5, 2019)

[14] *Green Functions of Chiral Currents within OPE*

T. Kadavý

40th International Conference on High Energy Physics (Prague, Czech Republic; July 28 - August 6, 2020)

Study text. Finally, we present an additional material that the author carried out during the pedagogical part that accompanied his doctoral studies. Specifically, it is a study text full of solved exercises devoted to the tutorials to the course "Jaderná a částicová fyzika" that bachelor students are required to pass in the third year of their studies. It is, however, written in Czech in order to be accessible and comprehensible to the students as much as possible. Also, the text is not publicly available — rather, it is distributed directly between the students by the author or upon request.

[15] *Jaderná a částicová fyzika. Sbírka detailně řešených příkladů ke cvičení*

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A. Fourier transform

In ref. [1], it was desirable to convert the results of contributions of the QCD condensates to the Green functions in the coordinate representation into the momentum one — the reason being that, eventually, one would need to compare them with the corresponding ChPT/RChT contributions, which are obtained within the momentum representation straight away.

The Fourier transform is the tool that does such a conversion. A reader might find different conventions regarding the sign in the exponential in various literature — the one we have used in ref. [1] corresponds to a convention in which all the momenta of a Green function are taken as ingoing. For simplicity, let us restrict ourselves into one variable only. Then, the Fourier transform reads

$$\widetilde{F}(p) = \int \mathrm{d}^4x\, e^{-ip\cdot x} F(x) \quad \longleftrightarrow \quad F(x) = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\, e^{ip\cdot x} \widetilde{F}(p)\,, \tag{A.1}$$

from which it can be noticed that the following relations hold:[50]

$$\begin{aligned} \int \frac{\mathrm{d}^4p}{(2\pi)^4}\, e^{ip\cdot x} p_\mu \widetilde{F}(p) &= -i\partial^x_\mu \int \frac{\mathrm{d}^4p}{(2\pi)^4}\, e^{ip\cdot x} \widetilde{F}(p)\,, \\ \int \mathrm{d}^4x\, e^{-ip\cdot x} x_\mu F(x) &= i\partial^p_\mu \int \mathrm{d}^4x\, e^{-ip\cdot x} F(x)\,. \end{aligned} \tag{A.2}$$

For the purposes of ref. [1], one thus needs only the Fourier transforms of $1/x^2$, $1/x^4$ and $\log(-\mu^2 x^2)$, with μ being an arbitrary scale. Then, any other expressions needed can be obtained using the formulae (A.2) repeatedly. For the first two relations one has ($n \in \mathbb{N}$) [79, 109]

$$\begin{aligned} \int \mathrm{d}^4x\, \frac{e^{-ip\cdot x}}{x^2} &= -\frac{4i\pi^2}{p^2}\,, \\ \int \mathrm{d}^4x\, \frac{e^{-ip\cdot x}}{x^{2n}} &= i(-1)^n \frac{2^{4-2n}\pi^2}{\Gamma(n-1)\Gamma(n)} p^{2(n-2)} \ln\left(-\frac{p^2}{\mu^2}\right) + P_{n-2}(p^2)\,, \end{aligned} \tag{A.3}$$

where $n > 1$ and $P_{n-2}(p^2)$ is a polynomial of power $n-2$ in p^2 and $\Gamma(n) = (n-1)!$ is a gamma function. The latter integral is generally UV-divergent because the coefficients of such polynomial are ill-defined.[51] Nevertheless, the polynomial does not depend of p^2 for $n = 2$ and its derivation with respect to p_μ vanishes, which allows us to comfortly express the results presented in appendix A in ref. [1].

Finally, the Fourier transform of the logarithmic term can be found in ref. [135] and it reads[52]

$$\int \mathrm{d}^4x\, e^{-ip\cdot x} \log(-\mu^2 x^2) = \frac{16i\pi^2}{p^4}\,. \tag{A.4}$$

[50] Since there is indeed a must to distinct between the derivatives with respect to coordinate x and momentum p, we add the upper index that does the job.

[51] As ref. [79] remarks, a presence of such ill-defined polynomials without singularities in p^2 is typical for a study of sum rules, where they disappear after borelization.

[52] A ref. [135] presents a large number of useful Fourier transforms, including details of their calculation — see appendices A and C therein.

B. OPE beyond chiral limit

To be fair with the reader, we will not discuss here the OPE beyond chiral limit and its individual contributions in detail. Rather, we will fulfill our obligation and discuss one particular example — the contribution of the the gluon condensate to the two-point Green functions in the case of nonzero quark mass.

As we have argued in section 3.4, the quark propagator in the external gluon field in the momentum representation (3.24) simplifies significantly, among other procedures, the calculation of the contribution of the gluon condensate to the two-point Green functions. Before we prove such a statement, we remind the reader the procedure of a standard calculation performed in the coordinate representation, followed by the application of the Fourier transform, as we have presented it in ref. [1] — see appendix D.3 therein.

Let us now turn our attention to the calculation itself. To obtain the contribution of the gluon condensate to the two-point Green functions, one is required to evaluate the contributions of the figs. 13(a)-13(c) at page no. 61 in ref. [1]. This is a straightforward calculation similar to an evaluation of the perturbative contribution with the only difference being that one uses the relevant parts of the propagator (3.24). After some algebraical manipulations, it is indeed trivial to obtain the contributions of the respective Feynman graphs in the following form:

$$
\begin{aligned}
\big[\Pi^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b}(p)\big]_{(a)} ={}& \frac{\pi\alpha_s\langle G^2\rangle}{96}\delta^{ab}\times \\
&\times\int\frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\mathrm{Tr}\left[\Gamma_1\frac{\{\sigma_{\alpha\beta},\not{\ell}+m\}}{(\ell^2-m^2)^2}\Gamma_2\frac{\{\sigma^{\alpha\beta},\not{\ell}+\not{p}+m\}}{\big((\ell+p)^2-m^2\big)^2}\right], \qquad \text{(B.1a)}\\
\big[\Pi^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b}(p)\big]_{(b)} ={}& -\frac{\pi\alpha_s\langle G^2\rangle}{24}\delta^{ab}\times \\
&\times\int\frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\mathrm{Tr}\left[\Gamma_1\frac{f^{\alpha\alpha\beta\beta}(\ell,m)-f^{\alpha\beta\beta\alpha}(\ell,m)}{(\ell^2-m^2)^5}\Gamma_2\frac{\not{\ell}+\not{p}+m}{(\ell+p)^2-m^2}\right], \qquad \text{(B.1b)}\\
\big[\Pi^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b}(p)\big]_{(c)} ={}& -\frac{\pi\alpha_s\langle G^2\rangle}{24}\delta^{ab}\times \\
&\times\int\frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\mathrm{Tr}\left[\Gamma_1\frac{\not{\ell}+m}{\ell^2-m^2}\Gamma_2\frac{f^{\alpha\alpha\beta\beta}(\ell+p,m)-f^{\alpha\beta\beta\alpha}(\ell+p,m)}{\big((\ell+p)^2-m^2\big)^5}\right], \\
&\qquad \text{(B.1c)}
\end{aligned}
$$

with the only necessary ingredients being (3.27) and the knowledge of the fact that (3.28) reduces to (3.29).[53]

Then, using the relation (3.29), the expressions (B.1b) and (B.1c) can be simplified a bit:

$$
\begin{aligned}
\big[\Pi^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b}(p)\big]_{(b)} ={}& \frac{\pi\alpha_s\langle G^2\rangle}{2}\delta^{ab}\times \\
&\times m\int\frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\mathrm{Tr}\left[\Gamma_1\frac{\ell^2+m\not{\ell}}{(\ell^2-m^2)^4}\Gamma_2\frac{\not{\ell}+\not{p}+m}{(\ell+p)^2-m^2}\right], \qquad \text{(B.2a)}
\end{aligned}
$$

[53] To be thorough, let us point out that (3.27) is utilized since one needs to employ the relation $\mathrm{Tr}\,[G_{\alpha\beta}(0)G_{\gamma\delta}(0)] = \frac{1}{24}(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma})G^2$ in a mid-stage of the calculation.

$$\left[\Pi^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b}(p)\right]_{(c)} = \frac{\pi\alpha_s\langle G^2\rangle}{2}\delta^{ab}\times$$
$$\times\, m\int\frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\mathrm{Tr}\left[\Gamma_1\frac{\not\ell+m}{\ell^2-m^2}\Gamma_2\frac{(\ell+p)^2+m(\not\ell+\not p)}{\big((\ell+p)^2-m^2\big)^4}\right], \tag{B.2b}$$

from which it is obvious that the contributions of the graphs 13(b) and 13(c) vanish in the chiral limit, as expected — cf. the discussion above the relation (D.10) in ref. [1]. For clarity, let us mention that performing the integration in (B.1a) within the chiral limit is an elementary task and one indeed recovers the already obtained results — cf. eqs. (B.6) and (B.7) in [1].

Let us now, however, advance and finalize the integration of the expressions (B.1a), (B.2a) and (B.2b) for $m \neq 0$. As a special example, let us choose the $\langle VV\rangle$ Green function — after substituting for the respective Dirac matrices, performing the integrations and summing up the contributions of the three Feynman graphs, the final result of the contribution of the gluon condensate beyond the chiral limit reads[54]

$$\Pi^{\langle G^2\rangle}_{VV}(p^2) = \frac{i\alpha_s\langle G^2\rangle}{24\pi p^4} \tag{B.3}$$
$$\times\,\frac{24m^6\big(\Lambda(p^2;m,m)+2\big)-4m^4p^2\big(3\Lambda(p^2;m,m)+7\big)+8m^2p^4-p^6}{(p^2-4m^2)^3}\,.$$

In (B.3), we have denoted a dimensionless function of momenta and masses Λ, which is the part of the Passarino–Veltman B_0 function containing the s-plane branch cut. Its form reads[55]

$$\begin{aligned}\Lambda(s;m_1,m_2) &= \frac{\lambda_{\mathrm{K}}(s,m_1^2,m_2^2)}{2s}\lim_{\varepsilon\to0^+}\int_0^1\frac{\mathrm{d}z}{sz^2-(s+m_1^2-m_2^2)z+m_1^2-i\varepsilon}\\ &= \frac{\sqrt{\lambda_{\mathrm{K}}(s,m_1^2,m_2^2)}}{s}\log\left(\frac{\sqrt{\lambda_{\mathrm{K}}(s,m_1^2,m_2^2)}+m_1^2+m_2^2-s}{2m_1m_2}\right),\end{aligned} \tag{B.4}$$

where[56]

$$\lambda_{\mathrm{K}}(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz \tag{B.5}$$

is the Källén's function.[57] Finally, we point out that the function (B.4) simplifies a bit in the special case of both masses being equal to each other, as in (B.3):

$$\Lambda(s;m,m) = \frac{\sqrt{s(s-4m^2)}}{s}\log\left(\frac{\sqrt{s(s-4m^2)}+2m^2-s}{2m^2}\right). \tag{B.6}$$

[54]We remind the reader that we present only the Lorentz-invariant scalar coefficient, i.e. the coefficient in front of the structure $(p^2g_{\mu\nu}-p_\mu p_\nu)\delta^{ab}$.

[55]We note that (B.4) has a branch cut discontinuity in the complex s-plane running from $s=(m_1+m_2)^2$ to infinity, and a non-Landaunian pole at $s=0$.

[56]The special cases of $\lambda_{\mathrm{K}}(x,y,y)=x(x-4y)$ and $\lambda_{\mathrm{K}}(x,y,0)=(x-y)^2$ are useful to remember.

[57]Since Källén was a Swedish, it might be interesting for a Czech reader to find out what the correct pronunciation of his name actually is. To this end, we point out that the syllable "kä" should be read as the Czech syllable "še".

C. Journal article [1]

The article *OPE of Green functions of chiral currents*, published in JHEP **10** (2020) 142, is attached to the thesis. Its first version can be found at

- https://arxiv.org/abs/2006.13006,

while the published version is available at

- https://link.springer.com/article/10.1007/JHEP10(2020)142.

D. Journal article [2]

The article *On the three-point order parameters of chiral symmetry breaking*, submitted to JHEP, is attached to the thesis. It can be found at

- https://arxiv.org/abs/2206.02579.