FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

DOCTORAL THESIS

Matěj Ryston

Theory of Relativity – How to Develop its Understanding at a Secondary School Level

Department of Physics Education

Supervisor of the doctoral thesis: doc. RNDr. Leoš Dvořák, CSc.
Study programme: Physics
Study branch: Physics Education and General Problems of Physics

Prague 2022

I declare that I carried out this doctoral thesis independently, and only with the cited sources, literature and other professional sources. It has not been used to obtain another or the same degree.

I understand that my work relates to the rights and obligations under the Act No. 121/2000 Sb., the Copyright Act, as amended, in particular the fact that the Charles University has the right to conclude a license agreement on the use of this work as a school work pursuant to Section 60 subsection 1 of the Copyright Act.

In .... date ....

Author's signature

I thank my supervisor, doc. RNDr. Leoš Dvořák, CSc., for giving me the opportunity to work on a topic that combines the two great passions of my professional life, education and theory of relativity, for his inspiring comments and for always finding the time to go through the many pages of my writing. I also thank my consultant, doc. RNDr. Mgr. Vojtěch Žák, Ph.D., for helpful discussions. I am grateful to both gentlemen for their patience with me and the lengthy process of my work.

My thanks belongs also to RNDr. Otakar Svítek, Ph.D., Mgr. Martin Malachov and my gymnasium students for reviewing and commenting on the study text on relativity that was created as a part of this work.

Most of all, I am thankful to my awesome wife, Verča, for her loving support, patience and being always there when I needed. Baba Ganoush, my love...

Title: Theory of Relativity – How to Develop its Understanding at a Secondary School Level

Author: Matěj Ryston

Department: Department of Physics Education

Supervisor: doc. RNDr. Leoš Dvořák, CSc., Department of Physics Education

Abstract: The goal of this doctoral thesis is to find and implement possible ways to facilitate secondary school students' understanding of relativity. The thesis starts with an overview of existing literature as well as ongoing studies concerned with teaching relativity to secondary students, especially in connection with possible misconceptions and other learning difficulties. Furthermore, it maps available book and internet sources on relativity in both Czech and English that students might use outside of school. A research of Czech curricular documents for upper secondary education and an online survey among gymnasium physics teachers were used to assess the current situation of teaching relativity in these schools. Based on the mentioned research, it was decided to develop learning resources focused on General Relativity. Following an analysis of a selection of relativity textbooks on this topic, a study website for interested students as well as a teaching-learning sequence for teachers to adapt in their own teaching were developed. The development and assessment of these materials are described in detail.

Keywords: relativity secondary school physics education study website

Contents

Introduction

Einstein's Theory of Relativity is commonly counted, alongside, for example, quantum mechanics, to be part of the so-called modern physics. This might seem strange considering that Special Relativity (or SR for short, as we will see written many-many times in this work), the first of Einstein's "relativities", was first published in 1905 and General Relativity (GR) some 10 years later. We (meaning humanity) have known relativity in one form or another over a 100 years and we still call it modern. But we do that with most of the physics of the 20th and 21st centuries. Probably because right around the year 1900 was when physics stopped being complicated and became really complicated. But this complication and divergence from our everyday intuition is what intrigues us about those parts of physics so much. Quantum entanglement, time dilation, black holes, those are concepts that are incredibly complicated and not really understood by most of us, but we know them. They have come to us in most cases not from the pages of physics textbooks but science fiction novels, films and TV-shows. Because even though such concepts are (as far as we know) grounded in the workings of the universe, there is a shroud of mystery about them that fascinates us. You would be hard-pressed to find an interesting science fiction novel about the physics of a pulley (although if you do, please don't keep it to yourself).

On a personal note, I find modern physics fascinating, and most of all relativity. It is the reason I started studying physics and even now, as a science educator and, dare I say it, researcher, I keep this fascination with me at all times. I am thankful that I could turn this personal interest into a professional one, and in doing so perhaps help spread that interest around. As we shall see in this work, the argument for the inclusion of modern physics in our schools is getting louder. Just because something interesting is also complicated and technical doesn't mean that we can't find a way to explain it at least in basic terms to others. After all, the ability to explain something is one of the highest forms of understanding.

As the name of this work suggests, our goal is to identify and develop ways to increase the understanding of relativity of secondary school students. But to do that, we must also map the waters we wish to navigate. Consequently, the thesis consists of two distinct halves. The first two chapters are focused on research. Specifically, in Chapter 1 we will present an overview of the existing research regarding teaching of relativity to secondary (and even primary, in some cases) students and show that the educational research on this topic is quite contemporary, especially when it comes to teaching GR. Just the amount of studies available now compared to 2014, when (oh, so long ago) the work on this thesis began, speaks for itself. Another important aspect is what sources of information on relativity are available to young students. Are they abundant and of good quality or non-existent? The second part of Chapter 1 presents what sources of information on relativity suitable for secondary students we found.

Chapter 2 is devoted to the situation of teaching relativity in Czech secondary schools. This is a surprisingly complex issue, not just because different types of schools exist, but also because a purposeful flexibility of the Czech school system. The official curricular documents, in which there is very little relativity, as we will see, state what must be taught but schools can add content according to their own judgment. We therefore conducted a large questionnaire survey among physics teachers to find out if relativity is or is not actually found in our schools, the results of which are described in the second part of the chapter.

The second half, Chapters 3 and 4, deals with the development and testing of materials and teaching sequences that are the main outcomes of this work. For reasons described in the first two chapters, a decision was made not to take the approach of adding to the common physics curriculum, which is already quite packed, and we much more often hear discussions about its reduction rather than expansion. Instead, we created materials in the form of a study website for students interested in relativity, to learn on their own. The reason is that we found a large gap between strictly popular sources on relativity and sources of a highly technical university level. Our goal was to try to fill this gap with a material that is engaging for secondary students and yet rests on firm physical principles. Furthermore, it was found that significantly more content is available on SR. This comes as no surprise because of the two theories, GR has always been regarded as the more abstract and mathematically challenging, and rightly so. After all, one can go through the basic ideas of SR armed with little more than the Pythagorean Theorem, but GR is another story. We therefore decided to focus primarily on learning and understanding GR because that is what we felt was most missing. Chapter 3 describes first the analysis of a number of relativity books in order to identify the best possible approach to teaching GR. Then, the creation, content and evaluation of the website is described.

Finally, Chapter 4 describes the course of developing a workshop for students devoted to explaining the basic ideas of GR. Our effort was to make learning GR go beyond the simple exposition of theory, to find ways to make it hands-on and engaging. The workshop serves as a whole teaching and learning sequence that could be adopted by any secondary physics teacher wishing to include this wonderful topic in their classroom.

There are several attachments at the end of the thesis. Attachment A.1 is a list of schools whose curricular documents were analyzed as part of the research regarding the current state of teaching relativity in Czech secondary schools in Chapter 2. Attachment A.2 shows the English version of the questionnaire as well as the structure of its logic jumps used in the online survey among physics teachers described also in Chapter 2. Attachment A.3 contains the full list of free comments on the teaching of relativity submitted by the participants of the online survey.

1. Literature review

In the first section of this chapter, we present an overview of the current state of educational research concerning the teaching of relativity at the secondary level of education (ISCED 2 and 3). We used two online databases, Scopus and Web of Science, to find relevant papers using the key words (*special relativity* OR *general relativity*) AND (*secondary school* OR *secondary education* OR *high school*). After the initial search, papers unrelated to the searched topics were excluded based on their abstracts and all the findings were grouped from both databases for SR and GR separately (accounting for duplicity). Additional sources were also found using further references in the papers.

The second section deals with information sources on relativity that are readily available to Czech secondary students.

1.1 Educational research regarding teaching relativity in secondary schools

From the perspective of educational research, relativity falls under the umbrella of *modern physics*, typically meaning physics of the 20th century and later (as used, for example, in De Ambrosis and Levrini 2010, Dimitriadi and Halkia 2012, Kamphorst et al. 2019 or Balta et al. 2022) and most commonly referring to relativity (both SR and GR) and quantum physics. Modern physics is also used as a direct distinction from *classical physics*. Studies such as (Angell et al. 2004 or Kaur et al. 2020) have shown that inclusion of modern physics topics can increase students' interest and motivation towards physics.

Another commonly found term, used often rather synonymously with modern physics, is Einsteinian physics, coined by the originally Australian *Einstein-First Project* (EFP 2022). This project is now an international collaboration aimed at introducing selected topics of modern physics (such as *curved space*, *warped time*, *photons*, *black holes* and *quantum entanglement*) in primary and lower secondary schools (see Choudhary et al. 2019, Kaur et al. 2017a, Kaur et al. 2017b and Kaur et al. 2017c). Similarly to the modern vs. classical physics distinction, Einsteinian physics is used as a distinction from Newtonian physics (a commonly used synonym with classical physics). Other results of this project will be mentioned later in this section.

In most cases, teaching and learning of SR and GR are treated by researches separately. This is understandable. Even though GR is a generalization of SR and therefore SR is strictly speaking it's subset, the two theories are most often used in different settings and dealing with different phenomena. Most importantly, their mathematical and conceptual requirements differ greatly, with SR being considered the easier of the two (as is evidenced, for example, by attitudes of Czech gymnasium teachers presented in Chapter 2). In this text, we will also deal with the research concerning the two theories separately.

1.1.1 Student difficulties when learning relativity

(Dimitriadi and Halkia 2012) and especially (Alstein et al. 2021) have extensively mapped available literature on secondary student difficulties in learning SR. The term learning difficulty here refers to any obstacle caused by misconception, misunderstanding or lack of understanding of the content of a given topic, not as an unrelated physical or mental disability generally hindering student learning process. (Alstein et al. 2021) have sorted reported learning difficulties from 15 different studies (with both secondary and undergraduate students[1]) into 3 main categories with altogether 8 subcategories:

1. **Frames of reference:**
   - *General*
   - *Inertial and non-inertial*
   - *Events, observes and simultaneity*
   - *Galilean transformation*
2. **Postulates of SR:**
   - *Principle of Special Relativity*
   - *Light postulate*
3. **Relativistic effects:**
   - *Relativity of simultaneity*
   - *Time dilation, length contraction, and relativistic velocity addition*

It seems that the reported difficulties cover most, if not all, of the basic ideas of SR kinematics. Research suggests that students' learning of Special Relativity or even Galilean relativity can be negatively influenced by their previous knowledge of classical physics. For example, (Villani and Pacca 1987) showed students who had gone through the instruction of SR "*considering that the 'true' speed of light can be observed only in the rest frame of the light source.*" - in other words, using Galilean addition of velocities on light propagation in contradiction with the light postulate of SR. On the other hand, insufficient experience with concepts previously used in classical physics, such as *frames of reference* (Panse et al. 1994) or *inertial vs. non-inertial frames* (Ramadas et al. 1996), can also impede further learning.

(Otero et al. 2015) have designed and implemented a didactic sequence on SR for secondary students. They later analyzed student solutions to 8 particular scenarios. Researchers found that students had difficulties in application even of the basic postulates of SR due to prevailing misconceptions; namely, the "motion is absolute" notion proved to be the main obstacle in successfully applying the Principle of Relativity. Furthermore, even though the students seemed to accept the Light Postulate regarding the constancy of light speed in vacuum for

[1] Although we are dealing with relativity for secondary schools, we consider research results involving undergraduate students also informative as their learning difficulties are likely present with secondary students as well.

inertial observers, due to the speed being very large, some students treated light propagation as instantaneous. Consequently, authors also criticize traditional SR textbook for spending too little time with the postulates and their use and quickly moving to the more ”spectacular” parts of the theory.

An important part of the educational research is working with pre-service and in-service secondary school teachers. (Selcuk 2011) has conducted written questionnaires (created based on textbook analysis) as well as interviews with 185 pre-service physics teachers concerning conceptual knowledge of SR. Partial or complete misunderstanding of some of the concepts was found with a majority of respondents.

(Ozcan 2017) have focused on the understanding of three selected concepts: *time dilation*, *length contraction* and *reference frames* of 14 pre-service physics teachers after they passed the SR part of their undergraduate course. The level of students' understanding was determined using semi-structured interviews followed by a video analysis of the interviews. Based on the analysis by two different researches, student answers were evaluated overall to be in one of three categories: *Complete understanding* (6 students), *Incomplete understanding* (3 students) and *Misunderstanding* (5 students). Therefore, similarly to the previously mentioned study, more than half of pre-service teachers showed at best incomplete understanding. A common occurrence (which also appeared in other studies) was the opinion that time dilation and length contraction are not actual phenomena, but arise solely due to imperfection of measurements (one student used the term ”optical illusion” when describing time dilation).

The two mentioned studies underline the importance of working with pre-service teachers to limit the possibility of them carrying on their physical misconception into the teaching practice. Another way of improving the situation is working with in-service teachers. (De Ambrosis and Levrini 2010) have studied the process of teachers moving from a more traditional approach to teaching SR and adopting a proposed novel approach (in the study represented by the textbook by (Taylor and Wheeler 1992)).

To help with probing students' learning, (Aslanides and Savage 2013) have developed a *Relativity concept inventory*; though despite its name it focuses only on special relativity. It contains 24 questions that cover the basic concepts of SR from the two postulates to mass-energy equivalence. Concept inventories are useful tools for assessing students' conceptual understanding of a given topic but developing one is a complex process requiring sufficient prior research. In contrast to educational research concerning SR in secondary schools, research dealing with GR in secondary schools is relatively young. Most of the found papers on GR have been published in the last 16 years, with the largest group published no sooner than 2014. Therefore, there is not yet much research on conceptual understanding concerning GR available. As a result, there is presently (July 2022) no GR concept inventory available.

In terms of existing research on secondary school GR, mostly exploratory and developmental studies can be found. (Baldy 2007) has shown that some basic aspects of GR can be understood by 9th grade students. (Dua et al. 2020) showed, using an identical pretest/posttest, an improvement in understanding of selected

GR concepts in 31 upper secondary school students after a three-week program of activity-based learning. (Kersting 2019) has studied students' understanding of "movement" through spacetime. In the study, students went through prepared activities in pairs or small groups and recorded their conversations. Subsequent analysis showed that "*While generic movement in spacetime did not pose significant challenges to students, the concept of movement along geodesic curves did. [...] Only few groups were able to connect the geometric description of a geodesic curve as the straightest path in a curved space to the physical state of being in free fall or alternatively, to the state of not being affected by external forces.*"

Unusual behavior of time can also cause a problematic conceptual barrier for students - both in SR as *time dilation* (Hughes and Kersting 2021) and in GR as gravitational time dilation (sometimes called *time curvature* or *time warp*). Papers by (Stannard 2018) and (Kersting et al. 2019) present a geometrical visualization to help mitigate such difficulties.

There are also studies that focused their attention on even younger students, introducing them to Einsteinian physics before they encountered classical topics such as Newtonian gravity, according to the philosophy of the already mentioned Einstein-First project. (Pitts et al. 2014) have conducted teaching programmes for Australian year 6 students (10-11 year-olds) and have found using pre/post questionnaires statistically significant improvement in students' knowledge of Einsteinian physics. A follow-up and more detailed study by (Kaur et al. 2020) has achieved similar results and showed a slight improvement in students' attitudes towards physics being an interesting subject. Furthermore, the students retained gained knowledge even after three years after the teaching programme. A similar kind of exploratory study like the one done by (Pitts et al. 2014) was conducted with Italian year 9 students by (Ruggiero et al. 2021) also showing improvement of the students' conceptual knowledge on the topic. All of these results seem to be in accordance with (Walwema et al. 2016) who have shown that the knowledge of classical mechanics is not a necessary prerequisite for learning modern physics. However, we should add that the result was obtained by comparing test answers of a limited sample of students, so further study is necessary.

1.1.2 Recent development of relativity teaching in secondary schools

Several countries have in recent years added topics from modern physics into their secondary school curriculum. In terms of relativity, the Netherlands added SR into their secondary curriculum as a selective topic in 2014 (Kamphorst et al. 2021), which led to the introduction of a new way of teaching SR using the so-called *event diagrams* (Kamphorst 2021). In 2006 Norway included GR (and other topics of modern physics) into their curriculum for upper secondary physics (Kersting et al. 2018). A direct consequence was the creation of the *ReleQuant project*[2] (University of Oslo 2020) aimed at creating teaching materials and se-

[2]As part of the relativity presentation of the project, in Module 3: Curved spacetime, in section named "Dynamic interplay", there is a short animation created by the author of this thesis (`https://www.viten.no/filarkiv/general-relativity/#/id/5a5b662e61f5dd7a0a6ef72b`).

quences to help upper secondary physics teachers with the new topics (Kersting et al. 2018). Scotland added basics of GR into their Advanced Higher Physics course for final year secondary school students (Farmer 2021). Although the Australian curriculum so far does not contain GR, the already mentioned Einstein-First project has been bearing fruits in the recent years in terms of research and development (referenced above) as well as inspiring similar studies around the world (for example the already mentioned studies by (Dua et al. 2020) and (Ruggiero et al. 2021)). Thanks to the ever-growing cooperation between educational researchers, more information about the inclusion of GR in the secondary curriculum of various countries can be found on the website (Teaching Relativity 2020). The current state of teaching relativity in Czech upper secondary schools is discussed in Chapter 2.

The most significant product of this international collaboration is the publication *Teaching Einstenian Physics in Schools: An Essential Guide for Teachers in Training and Practice* (Kersting and Blair 2021), containing contributions from teachers and educational researchers from around the world. The author of this thesis has had the great pleasure of being able to contribute to this book by describing a relativistic workshop for secondary school students in *Chapter 24: Introducing general relativity without special relativity.* A more detailed description of this workshop will be given in Chapter 4 of this thesis.

Due to the very abstract nature of relativity, a great emphasis is put on the use of models and visualizations. A special attention is given to the rubber sheet model, which enables real life demonstrations of some principles related to gravity (Kersting and Steier 2018, Postiglione and De Angelis 2021). Common tools are computer visualizations (Kraus 2007, Ryston 2019b) as well as real hands-on models (Zahn and Kraus 2014 and Zahn and Kraus 2019, also Ryston 2022a and many others). According to (Ainsworth 2006), using multiple representations when learning complex ideas can be beneficial, making all these types of representation equally important.

Lastly, due to recent scientific discoveries, more advanced topics of GR have also been gathering popularity among students. Consequently, science educators and educational researchers use this popularity in introducing secondary students to modern physics. The most notable examples are *gravitational lensing* (Falbo-Kenkel and Lohre 1996, Huwe and Field 2015), *gravitational waves* (Boyle 2019a and Boyle 2019b, Hendry et al. 2014 or Choudhary et al. 2018) and *cosmology* (Lotze 1995).

The conclusion of this section is that educational research into teaching relativity in secondary schools is a relatively young field (with GR-oriented research being younger still) and, judging by the frequency of published studies, very contemporary. More research is available on SR but numbers of studies concerning teaching of GR in secondary schools have been growing in recent years and this trend is likely to continue because there are several ongoing educational research and development projects, often with international collaboration. Furthermore, even though there are studies focusing on exposition of relativistic topics to students in lower secondary schools (mainly the mentioned Einstein-First project), most of the research and development takes place in the upper secondary classes (as we saw with examples from the Netherlands, Norway and Scotland). We have therefore decided also to focus in this work on upper secondary students. Firstly, because they (at least theoretically) possess wider physical knowledge that can be relied upon when discussing relativity, and secondly because there exists already some tradition of teaching relativity in Czech upper secondary education, unlike in lower secondary or primary education (a more detailed discussion will be given in Chapter 2).

1.2 Sources available to Czech secondary students interested in relativity

Let us now look at sources of information about relativity that are available for secondary school students. We will discuss the current state of teaching relativity in Czech upper secondary schools in detail in Chapter 2 but for now we will assume the role of a student interested in relativity, who wants to learn more on their own. There are basically two main groups of sources, the internet and published books.

We also have to make some assumptions about language. We will, of course, primarily focus on Czech sources, as that would likely be the first choice of (especially younger) Czech students; however, English is also a likely option. For the vast majority of students it is their second language, being taught since primary school and quite often already since nursery school. Of course, it really depends on the student's level and self-efficacy in English, which can be due to exposure to English while watching TV, playing computer games or using the internet actually quite high, especially for upper secondary students. We will not take into account sources in any other language, because the fraction of Czech students capable of using them is likely to be marginal.

The following summary is by no means exhaustive, that is, especially in the case of internet sources, practically impossible. Our goal is a reasonable mapping of the existing possibilities.

1.2.1 Book sources

We have searched for books available in Czech using some of the largest bookshop chains in the country (*Luxor*, *Megaknihy*, *Knihy Dobrovský* and *Knihy ABZ.cz*) as well as the *Prague Municipal Library*. We are not going to include university textbooks such as (Horský 1972 or Dvořák 1984) in the current discussion because they rely on higher mathematics and are not a likely source for secondary school students.

Books available in Czech

In terms of special relativity, there exists a brief SR textbook for upper secondary schools (Bartuška 2010) and we will discuss it more in detail in relation to the extent of teaching relativity in upper secondary schools in Chapter 2. The textbook covers only basic topics without much detail and is therefore more suitable (and probably intended) as a supplement and revision for physics lessons. Besides that, all the other found books were popular in nature, such as (Einstein

2000, Cox and Forshaw 2013 or Ferrán 2020), often with a strong biographical component utilizing Einstein's popularity[3] (Abanese and Parisi 2010, Manly and Parisi 2015 or Vaas 2019). Sadly, there are a number of books on SR that are generally out of stock (Bartuška 1991, Einstein 2016, Ženíšek 2015 - the last book is strictly speaking a university level treatment of SR, but could possibly be used, based on the limited excerpts we found, by some of the more advanced upper secondary students). The fact that some of these books were published relatively recently and can't be obtained anymore tells us that even though there is some demand for books about SR, it is often sufficient only to warrant a limited number of copies and no additional printing. Such is probably the fate of all topical scientific literature.

Search for GR oriented books was far less fruitful. (Ferreira 2015) summarizes the hundred-year-old history of GR, whereas both (Begelman and Rees 2013) and (Zee 2019) discuss the phenomenon of gravitation, so significant parts of the books are devoted to GR. Furthermore, a large number of strictly popular physics books on modern physics, the universe as a whole or more specific topics such as black holes, are not focused solely on relativity but some discussion of it is necessarily present. (Greene 2001, Hawking 2015, Tyson 2020) are just a few examples.

By no means do we want to diminish the importance of popular books on physics. They play a great and essential role in provoking the interest in physics in general population, especially in younger students that might even decide to study physics because of them (a prime example of which is the author of this thesis). However, we would argue that the very popular nature of those books that makes them approachable to a general audience is a hindrance in proper understanding of the topic. Authors purposefully omit technical details in order not to discourage most readers, but those technical details, we believe, are a scaffolding on which our understanding of science is built. The following quote is attributed to Stephen Hawking: *"Someone told me that each equation I included in the book would halve the sales."* (Wikiquote.org 2022). This illustrates the reason why popular physics books, with notable exceptions, stay away from mathematics. Understandable, but in our opinion unfortunate. The consequence is that students interested in relativity have to choose basically from two groups of book sources. On one hand, popular physics books that, interesting as they might be, generally lack concrete and technical examples, and on the other hand university level textbooks that are very hard if not unintelligible for a reader without advanced mathematical knowledge. We feel the lack of middle ground (i.e. of sources that use more elementary mathematics, concrete examples and approachable technical details to promote deeper understanding) is evident. As we will see, the situation is similar but not as "dire" as in the case of book sources available in English.

Books available in English

Understandably, there exist far more books on relativity in English. We have already discussed popular books on physics that only partially relate to relativity, so we will omit this very large group from further discussion and include only

[3]We do not include purely biographical works as we are, for the purposes of this research, interested in the physical theory, not the life of its creator.

those specifically dealing with relativity. For our search, we used the website *Amazon.com* and selected books based on their description and reader reviews. The first and largest group of books was bought in 2015 and a few singular books were discovered and added in the following years. The main problem with the selection process was the ambiguity with which the authors use the word relativity. Some use it just in the meaning of special relativity, such as *An Illustrated Guide to Relativity* (Takeuchi 2010), most authors discussing relativity include both SR and GR, but the space devoted to GR varies significantly, from a single chapter to half a book. Descriptions and reviews posted online are not always helpful because they often share in this ambiguity and the specific content of the book is not always presented. Therefore, we ended up with a varied mix of books, both in structure and approach. We have purposely included books with seemingly elementary approach, possibly suitable to secondary students, as well as some well-reviewed university level textbooks. Our goal was not only to find book sources suitable for secondary students but also to map possible approaches of relativity exposition that would guide us in creating study materials described in Chapters 3 and 4.

We split the selected books into two groups according to their mathematical demands on the reader. Books in the first group (presented below with commentary) use at most upper secondary mathematics such as *simple expressions and equations*, *Pythagorean theorem*, *elementary functions*, and so on. The second group (presented at the beginning of Chapter 3) uses advanced mathematics, by which we mean university level mathematics, such as *differential* and *integral calculus* and ”above”. Even though Czech students might encounter basics of calculus in the last year or two of upper secondary studies, this exposition is often limited in scope and usually to a mathematics seminar[4] (if present at all), we therefore don't consider these topics typical parts of upper secondary mathematics. We will point out two notable exceptions in the first group that technically speaking use differential calculus but try to avoid the mathematical complexity by employing conceptual understanding such as ”close-enough points”. This approach will be crucial in the prepared study text described in Chapter 3.

Books without advanced mathematics:

- *Relativity Visualized* (Epstein 1985)
  - Practically no mathematics, heavy emphasis on geometrical ideas and visualization, although quite abstract in places. 8 chapters on SR, 4 chapters on GR.
- *General Relativity from A to B* (Geroch 1981)
  - Very elementary mathematics but quite abstract reasoning using spacetime diagrams. Includes Galilean relativity followed by GR.

[4]The issue of elective seminars is discussed in detail in Chapter 2.

- *It's About Time: Understanding Einstein's Relativity* (Mermin 2005)
  - Elementary mathematics and simple reasoning. Covers mostly SR kinematics, spacetime interval and energy-mass equation. 12 chapters on SR, 1 short chapter on GR.
- *General Relativity Without Calculus* (Natário 2011)
  - Heavy use of equations using elementary functions. Starts with quick SR overview but focuses on GR. Contains Non-Euclidean geometry with a metric introduced as a distance between two "close enough" points (using the symbol Δ for essentially a differential) without further clarification. Discusses differential equations (again using the Δ symbol for differential) and shows results without solving them. Probably for a more advanced upper secondary student.
- *How to Teach Relativity to Your Dog* (Orzel 2012)
  - Very few mathematical formulas, very approachable and yet detailed and technical. Starts with basic ideas about the description of motion and Galilean relativity. 7 chapters on SR, 3 on GR.
- *Gravity from the Ground up* (Schutz 2003)
  - Long and thorough discussion of gravity starting with the Newtonian view, then a quick review of basics of SR and non-Euclidean geometry to be used for the exposition of GR. Offers two levels of difficulty, the main body of text and more complex (especially mathematically) parts in highlighted boxes. Most used equations involve elementary functions. Avoids differential calculus by using the Δ symbol as both finite difference and a differential (again uses the term "nearby" points without much clarification).
- *The Wonderful World of Relativity* (Steane 2011)
  - Purely SR oriented, uses very elementary mathematics (mostly the Pythagorean theorem and a square root).
- *An Illustrated Guide to Relativity* (Takeuchi 2010)
  - Purely SR oriented, the book is split into two main parts. Kinematics with no equations and dynamics with equations. SR kinematics is prefaced by basic ideas about motion, reference frames and Galilean relativity.
- *Spacetime Physics: Introduction to Special Relativity* (Taylor and Wheeler 1992)
  - Mostly devoted to SR with one final brief chapter on GR, a great emphasis in put on conceptual understanding with a limited use of elementary mathematics (mostly in problems at the ends of chapters). Interestingly, the concept of free-falling local inertial frames is introduced already during the exposition of SR.

- *How Einstein Created Relativity out of Physics and Astronomy* (Topper 2013)
    - Strongly biographical in nature, outlines the creation of relativity starting with Galileo, but mostly showing and discussing particular moments and breakthroughs from Einstein's career. Very accessible due to not using complicated mathematics. Focuses more or less equally on basics of SR and GR, followed by a whole part on cosmology and the "quest for finding a unified field theory".
- *Simply Einstein: Relativity Demystified* (Wolfson 2003)
    - Very limited use of simple mathematical formulas. Mostly focuses on SR (12 chapters) with a limited discussion of basics of GR (3 chapters).

1.2.2 Internet sources

The internet is nowadays arguably the most likely immediate source of information, especially for young people. On one hand, students are presented with a plethora of various sources of information, literally sitting in the palm of their hand(-held devices). However, the reliability of these sources varies significantly, ranging from reviewed and curated websites backed by official institutions such as universities, to erroneous or worse, intentionally misleading personal websites. As with all online activity, careful selection of our sources of information is required. We have conducted our search the same way as a student interested in relativity might most probably do it, simply by typing terms *relativity*, *special relativity* and *general relativity* (in both Czech and English) into the *Google* search engine. What we found corresponds very well with the findings of the previous section about book sources. Websites and other internet sources dealing with SR are more commonly found than for GR. We will now present our findings in more detail.

In the case of Czech websites, a significant number of SR related sites are limited to a rudimentary summary of basic SR kinematics and a little of dynamics (usually *relativistic mass, momentum, energy* and their relationships). We suspect this is related to SR still being taught in some upper secondary schools (which is discussed in detail in Chapter 2) and these summaries are basic revisions of this topic to help students prepare for a test or an exam. They rarely contain any in-depth conceptual knowledge, their content is for the most part identical (Fyzika 007 2022 is one example). One website that stands out among these basic revisionary summaries is (Králová 2007) which goes into more detail and includes a basic discussion of some GR topics as well. Even students not familiar with the Czech Physics Olympics (Fyzikální olympiáda in Czech) and their study texts for competitors might encounter their two texts devoted to SR when searching for possible sources (Fyzikální olympiáda 2022). The advantage is that these study texts are specifically meant for upper secondary students (albeit with above-average skills in mathematics and physics). The two texts differ in approach. One is more akin to the existing upper secondary treatment of SR (as in Bartuška 2010) and the other is closer to a university level course.

The best Czech source on SR we could find is technically an online study text *Základy teorie relativity* (*Basics of the Theory of Relativity*, Novotný et al. 2006), including videos and animations[5], and covering not just the mentioned common selection of topics but also more advanced parts of SR such as dynamics, four-vectors, and others. The text includes derivations of formulas (often moved to appendices for a better flow of the main text) and used mathematics ranges from elementary to simple uses of calculus. It could be therefore said that the text is suitable for upper secondary students for the most part or those more mathematically inclined completely. The main goal of the text is the exposition of SR, so GR is really only briefly discussed in relation to SR. Interestingly, spacetime diagrams are, aside from a brief mention, not really part of the text. The only possible disadvantage of the material is its technological solution of including videos and animations. The whole material is put together as a PDF (which on one hand offers the user an easily obtainable offline version) with links to the videos and animations stored as separate files, creating the need to download the videos or view them outside of the material. Moreover, having to download an .exe file for every animation typically raises a red flag in the mind of a cautious internet user (not to mention their anti-virus protection). Of course, this technical solution made sense 15 years ago, when the study text was put together, but today it can be off-putting for users, especially those using mobile devices, that are used to videos and other interactive content being embedded directly into webpages.

As we mentioned at the beginning of this subsection, Czech websites dealing with GR are much rarer. Besides the already mentioned (Králová 2007), we found two mostly conceptual texts (Dvořák 2019 and Kulhánek et al. 2018 - with the latter including but not relying on some university level mathematics) that could be, in our opinion, understandable for a secondary school student, even though they seem to be primarily meant for undergraduate readers. Students can also encounter basically an online version of the book (Ullmann 1986); however, its scope and used mathematics is most likely not suitable for secondary students.

English sources again offer a wider range of options, but the general theme stays the same. More secondary level sources on SR were found than for GR. For example, basics of SR kinematics are part of *Khan Academy*, a well known website hosting educational videos for home-study, but no GR can be found there. Similarly, website *World Science U* that offers for free video courses on many scientific topics led by renown scientists, offers two courses (one conceptual, the other mathematically oriented) suitable for upper secondary students on SR, but not GR. Both websites can be easily found by their name. These are just two typical examples of how SR is favored among available sources of information, most likely due to GR being perceived as more difficult. As a notable exception, a website run by the (Max Planck Institute for Gravitational Physics 2005), *Einstein Online*, offers an elementary treatment of both SR and GR, which is quite approachable even for younger students (provided they read English) with simple and clear illustrations and animations. However, both sections are quite

[5] In our opinion, moving and interactive elements such as videos, animations and applets are truly the biggest advantage of most digital sources over classical paper or static electronic documents. We will discuss this issue further in Chapter 3.

brief, lacking even simple mathematics and technical details, reminding our arguments about popular scientific books. Furthermore, we have already mentioned the Norwegian *ReleQuant Project* which resulted, among other things, in the creation of a website with learning modules dedicated to GR (Viten.no 2019). The contents of the website are structured according to the requirements of the Norwegian curriculum on GR and the treatment is purely conceptual. Even though this approach is most likely more suitable for general audience, we express doubt whether it cultivates a thorough understanding of the topic.

A big plus of the digital environment are visualizations, which have a special place in relativity as it often predicts very unusual results even about the very appearance of objects. A brief summary of websites with interactive elements visualizing some phenomena predicted by SR can be found in (Soukup 2015). Some simple visualization of GR related phenomena, such as falling to a black hole or the formation of an Einstein ring can be found at (Kraus and Zahn 2022). A more advanced animation of the fall into a black hole based on real calculations can be also found, for example, at (Roussel 2022).

In our summary of internet sources, we should not omit Wikipedia (both Czech and English variant), which is a free community-driven repository of knowledge. Being encyclopedic in nature, it is not meant as a study material for promoting conceptual understanding; however, it is a good first source of supplementary factual information, for example about both historical and contemporary experimental testing of relativity.

Last but not least, educational videos on platforms such as *Youtube.com*[6] are a very popular source of information. There are a number of channels with millions of subscribers that specialize in popularization of science or more specifically (and relevant to our topic) physics. Examples include *Veritasium*, *Minute Physics*, *Physics Girl*, *Steve Mould* and many more. Their videos, watched by literally millions of people all over the world, cover wide variety of topics, relativity included. Nevertheless, Youtube videos have the same issues that we identified with popular science books. Their goal is to attract the widest audience possible; therefore, the videos are relatively short and rarely include technical details. Furthermore, the factual correctness of the videos is not straightforwardly guaranteed by any institution or publishing process and there are seemingly scientific videos online whose claims are simply false. As with any other internet sources, the viewers need to be cautious themselves and think critically about the content that is presented. Overall, popular videos cannot substitute formal education but on the other hand they can generate interest in a given topic. We found two channels that specialize in relativity, both SR and GR, as well as other topics from modern physics, and include more technical and concrete details than other popular videos. These channels are called *PBS Spacetime* (Spacetime 2022) and *ScienceClic English* (EN 2022). Moreover, thanks to popularizing efforts of various universities, one can find full videos of some university lectures on Youtube. For example, anyone, secondary school students included, can watch SR and GR lectures from the Stanford University by Professor Susskind (Stanford 2022), a famous theoretical physicist. These lectures are, of course, not meant primarily

[6]There are other similar websites for hosting videos; however, Youtube is still the largest and most popular one.

for secondary students, but the very fact that they are freely available online offers not just interested students but also members of general public a great opportunity for learning relativity.

In conclusion, we have seen on a number of examples repeated throughout various types of sources that treatment of SR is more commonly found in both Czech and English. Also, in both languages we found SR sources that can be considered both suitable for (at the very least upper) secondary students and of good educational quality. On the other hand, the offer of educationally sound, detailed and reasonably technical sources on GR is very limited, especially in Czech. Students have to mostly pick between purely popular or university level sources.

2. Current state of teaching relativity at upper secondary schools in the Czech Republic

In order to find ways to improve Czech students' understanding of the theory of relativity, it is first necessary to know if and to what extent relativity is taught at schools. In the case of the Czech Republic, we need to first look at the *Framework Education Programmes* (further referred to as FEPs), which are the national curricular documents issued by the Czech Ministry of Education, Youth and Sports (further as MEYS) containing all the mandatory content and expected outcomes of teaching and learning. However, these outcomes are described quite broadly to provide schools with a degree of freedom when creating their own more detailed curriculum called *School Education Programme* (SEP) based on and adding to the mandatory content of FEPs.

As mentioned before, the decision was made to focus throughout this entire work on upper secondary schools (ISCED 3). For the purpose of this research, we can split all Czech upper secondary schools into two groups: specialized and general schools. Specialized schools, as the name suggests, offer their students specific specialization in a given field to help them prepare for a job in that field. On the other hand, general secondary schools, in the Czech Republic and other Central European or German speaking countries called *gymnasium*[1], provide general education and students are typically expected to continue into tertiary education. We will discuss these two groups separately as their respective curricular documents differ greatly when it comes to physics.

Later in this chapter, we present both the creation process and the findings of an online survey used to determine the current state of teaching relativity at gymnasiums.

2.1 Analysis of curricular documents: Specialized upper secondary schools

All the curricular documents can be found online (MEYS 2020b). There exists a large range of study programs students can choose from. However, physics can be specifically found only in those programs that end with the *matura* leaving exam (referred to as *maturita* in Czech) (MEYS 2020a); therefore, we will now focus only on this group. In it there are 108 study programs covering a wide variety of lines of work such as metallurgy, woodcarving and many more. Because of such a high degree of variability between all the programs along with their very different requirements, all FEPs in this group (with one exception of the Medical Lyceum program whose physics curriculum is specifically tied to applications in medicine) contain physics content and expected outcomes in three variants denoted A,B and C.

[1]Some gymnasiums offer also lower secondary education; however, in this work we use the word gymnasium to refer only to the upper secondary branch.

The breadth and detail of these variants is largest for variant A and gradually decrease. To quote directly from the cited FEPs:

"Variant A is meant for programs with high, variant B for intermediate and variant C for lower demands on physics education." "School chooses a variant of physical and chemical education at minimum of the level stated [in this document] (a higher level can therefore be selected)." [translated from the Czech original]

In terms of relativity, only variant A contains the following brief mention of Special Relativity (translated from the original):

6 Special theory of relativity	
- principles of special theory of relativity	- [student] understands the consequences following from the special theory of relativity for understanding space and time;
- basics of relativistic dynamics	- [student] knows the connection between the energy and mass of objects moving with great speed;

Table 2.1: Excerpt mentioning SR content in the FEPs for specialized upper secondary school. The right column contains desired outcomes of teaching.

General relativity is not mentioned. As stated above, the minimal variant is set individually for every program. Out of the 107 programs that have these three variants defined, both A and B are set 29 times, C is set 47 times and for two programs the variant is not specified. That means that in only less than a third of these programs schools are obligated to introduce students to basics of SR.

Another important parameter we should take into consideration is the time allocated to teaching physics. In the FEPs, physics, chemistry and biology are grouped together under the umbrella of *Science Education* and then a minimum number of weekly lessons throughout the whole (usually four-year) study is set for every program individually. As an illustration, let us consider as an example that this given minimal number of lessons is equal to 6 for a particular program. Assuming an even spread of time among the three subjects (which might not be the case), we get two weekly lessons of physics during the whole study. That means for example that students might have one physics lesson per week in the first two years or perhaps two lessons per week in just one year. It is necessary to add that schools are also given 26 more weekly lessons to be allocated freely to various subjects according to their particular needs (that is why all the lesson amounts are specified as minimums); however, it is not likely that a majority of schools would significantly bolster the numbers of physics lessons as physics is often not directly related to that particular field of study (with the exception of for example a Technical Lyceum).

Table 2.2 summarizes the numbers of programs with a given set minimal overall number of weekly physics lessons. Programs are shown in three groups according to their set minimal variant of physics content and outcomes. The two outliers with 20 lessons are Technical Lyceum (variant A) and Science Lyceum (variant B). The only program with more weekly lessons is the already mentioned Medical Lyceum whose FEP doesn't follow the A-C variant system.

# of lessons	**Variant**		
	A	**B**	**C**
4	0	**8**	**43**
5	3	0	0
6	**23**	7	3
7	1	3	0
8	0	6	0
9	0	1	0
10	1	1	0
11	0	1	0
12	0	1	0
13	0	0	1
20	1	1	0
average # of lessons	6.6	7.1	4.4

Table 2.2: Numbers of specialized upper secondary school study programs with a given minimal amount of weekly lessons for Science Education throughout the whole program split into three groups according to the set minimal variant of physics content A,B and C. The most common amounts of lessons for each category are in bold.

We can see from the table that the minimal amount of allocated weekly lessons for Science Education is already quite low and physics constitutes only a fraction of those numbers. Programs with variant A average at 6.6 weekly lessons (median 6). For variant B we get surprisingly a slightly larger average of 7.1 lessons (with median still at 6). Understandably, lowest average of 4.3 belongs to programs with minimal variant C (median 4) most likely because this group consists of programs deemed the least related to physics.

2.2 Analysis of curricular documents: Gymnasiums

Unfortunately, previous findings cannot be straightforwardly compared to those for gymnasiums. The Framework Education Programme for gymnasiums (Balada 2007) defines so-called *Educational Areas* that group certain subjects together. Physics, together with chemistry, biology, geography and geology, is part of the *Man and Nature* area. Another area named *Man and Society* contains history and basics of civics and social sciences. The FEP specifies the minimal time allotment for these two areas together in all four years of upper secondary study to be 36 weekly lessons. Moreover, physics (as well as all the other mentioned subjects) must be included in the first two years of upper secondary education. Their inclusion in the other two years is left optional. As before with the specialized schools, gymnasiums are given 26 extra weekly lessons to be allotted according to their needs. All this gives schools a certain flexibility and autonomy; however, a direct comparison of time spent teaching physics between gymnasiums and specialized schools is practically impossible.

Nevertheless, assuming an even distribution of lessons among the subjects of the two mentioned Educational Areas for gymnasiums and in the Natural Science Education group for specialized upper secondary schools, the minimal number of physics lessons in the former easily surpasses those of the latter. This conclusion is in accordance with the authors personal experience of teaching physics at gymnasiums. To further support this claim, 40 gymnasiums were randomly[2] selected and their SEPs were searched for the actual number of physics lessons in the four years of upper secondary education. The names of these gymnasiums and the respective amounts of physics lessons can be found in Attachment A.1. Figure 2.1 offers a summary of the data:

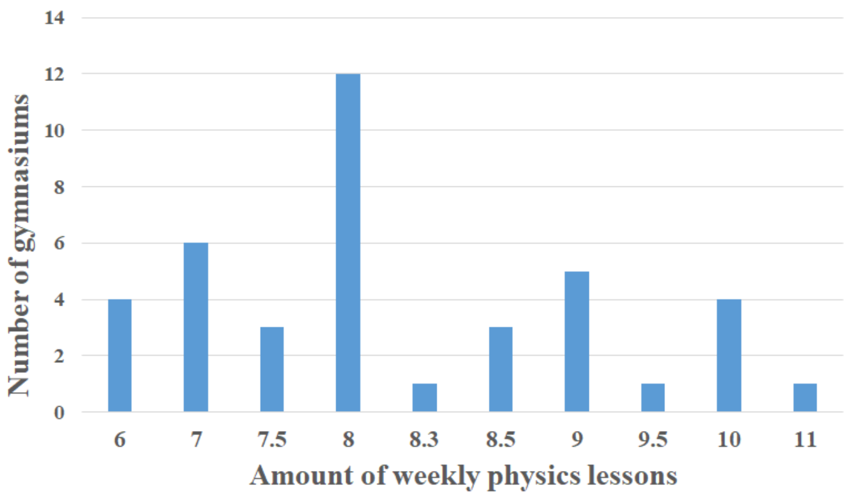


Figure 2.1: Frequencies of amounts of weekly physics lessons throughout the four years of upper secondary stated in the SEPs of 40 randomly selected gymnasiums.

With the average amount of 8.1 physics lessons (median 8), we can conclude that gymnasium students most likely do spend more time learning physics than in most specialized upper secondary schools. Gymnasiums are also generally considered more academically oriented than specialized schools, and thus we expect at least some gymnasium students more likely to be willing to engage in more advanced topics of physics such as GR. Most importantly, it is a common practice at gymnasiums that students in the last two years of upper secondary study choose a few specialized seminars according to their interests and future education plans. The seminars have several purposes: to deepen and broaden

[2]The selection was made using the Google search engine. After searching the term "School Education Programme" (in Czech "Školní vzdělávací program"), the first 40 hits were used. The selection is therefore not actually random due to the inner workings of the search engine; however, it is random from the point of view of the researcher.

the topics covered in normal lessons, to cover new topics that are not covered in normal lessons due to a lack of time or relative difficulty, and finally to help students revise for their maturita exam in that particular subject. The dichotomy of gymnasium physics education because of seminars can be again illustrated by the author's personal teaching experience at a Prague gymnasium, where, for example, SR was taught exclusively at the physics seminar.

Depending on the particular gymnasium, an elective physics seminar can be a supplement to normal physics lessons, or it can be the only physics subject available to students in the last year or two. As we will see later in this chapter in the results of the survey among gymnasium physics teachers, including more advanced topics in the seminar (i.e. with only those students who are interested in physics) is quite common. A similar situation can be found in some other countries. In the previous chapter, we have mentioned that Scottish upper secondary students can elect the *Higher Physics* and *Advanced Higher Physics* courses which include SR and GR respectively (Farmer 2021 or Kersting and Blair 2021, chapter 22). Another already mentioned example is Norway where physics is an elective subject in the last two years of a *General Studies* upper secondary program (Vibli.no 2022 and Henriksen et al. 2014). In both of these cases, the subject of physics (including topics from modern physics) is part of generally oriented study programs (as opposed to vocational or other specialized upper secondary programs) aimed at preparing students for universities, which is comparable in purpose to Czech gymnasiums. On the other hand, the subject of physics itself is not part of a common core, it is elective, therefore resembling gymnasium seminars. For all the above mentioned reasons as well as the inspiration from abroad, we decided to focus our efforts of finding ways to increase the understanding of relativity in gymnasiums rather than in all upper secondary schools. This applies specifically to the survey described in the next section. All the developed material and activities mentioned in the further chapters are fully open to any students interested in relativity regardless of whether they attend a gymnasium or not.

Looking again at the FEP for gymnasiums (Balada 2007), we find there is no mention of relativity of any kind (be it special or general). However, the system of Framework Education Programmes and School Education Programmes as described above started to be used in Czech education relatively recently, in 2007. Prior to that, a more uniform and rigid system of school curriculum was used, where the mandatory content of teaching was specified in detail. In particular, we can find SR in curricular documents of four-year gymnasiums (i.e. the upper secondary gymnasium programs) from the year 1991 (MEYS 1991). This curriculum is split into three parts: *humanities*, *science* and generally oriented gymnasiums. In all three branches, SR is included in the fourth year for a suggested number of 6 lessons. Suggested subtopics are (translated from the Czech original): *Space and time in classical mechanics. Creation of special theory of relativity. Relativity of simultaneity, time dilation, length contraction. Relativistic mass, relationship between mass and energy of a body.* We will see later in the survey section how this selection of subtopics corresponds to which parts of SR are actually taught today. There is no direct mention of GR in the old curriculum. We can find the term *black hole* mentioned together with *final phases of lives of stars* in the *Astro-*

physics topic; however, the curriculum does not specify whether any connection to GR should be made. Therefore, we cannot confidently count this mention as an occurrence of GR, as the topic could be discussed purely from an astronomical (i.e. observational) perspective.

To summarize, even though Special Relativity is no longer part of a mandatory curriculum for gymnasiums, there is an established tradition of teaching the basics of SR in gymnasiums. There also exists an SR physics textbook for gymnasiums (Bartuška 2010). Consequently, SR can still be found at least in some gymnasiums. This is illustrated, for example, by the author's personal experience with teaching physics at a gymnasium in Prague, where SR is still part of the School Education Programme, so it is not only taught but also one of the topics for the final maturita exam in physics.

2.3 Questionnaire survey

To find out to what extent the tradition of teaching SR is still followed in Czech gymnasiums, a questionnaire for physics teachers was prepared. An online form was chosen for simplicity of distribution among teachers as well as data processing. Moreover, simply filling out an online questionnaire was assumed to be easier and less time consuming for teachers, thus increasing the chance of their response. Because most of the teachers were contacted using their school email address (as we will discuss below), we were certain that they all have access to a computer with internet connection at least at their school, so none of the addressees were limited in their ability to fill out the questionnaire.

2.3.1 Design principles

The way we form questions and structure them, whether they are easy to understand or prone to being misunderstood can have a great impact on the accuracy of answers (Stone 2003 or Couper et al. 2001) as well as the return rate. When designing the questionnaire, we tried to adhere to the following principles:

- When possible, we used closed questions as open questions are more likely to be left unanswered (Couper et al. 2001). They may also lead to redundant or irrelevant information as well as take up more of the respondent's time (Cohen et al. 2007, page 322).

- The most important questions were positioned in the early and middle parts of the questionnaire. According to (Galesic and Bosnjak 2009), later items have a higher possibility of being skipped and also produce a lower quality of data as less time is usually spent on them. (Stone 2003) emphasizes a descending order of question difficulty; however, we consider all the questions of our survey to be quite straightforward. This belief is supported by the fact that among the responses, no question stood up as noticeably less answered or with significant signs of confusion on the part of the respondents.

- Questionnaire length has been found to negatively affect response rates of web surveys (Galesic and Bosnjak 2009). Our questionnaire was therefore designed to take no longer than 10 minutes and the expected length was stated at the beginning of the questionnaire (as suggested by Crawford et al. 2001) as well as in the introductory email (Marcus et al. 2007).

- Clear structure and appealing visual design also play an important role (Couper et al. 2001). To achieve this, we have chosen a commercial online survey system *Typeform.com*. The second reason was that the website allows the use of logic jumps in their questionnaires. Using this feature, the respondent is shown only those questions relevant to them (for example, if they answered that they do not teach Special Relativity, the system would skip asking them which topics of SR they teach). This significantly reduces the individual time necessary for completion as well as saves the respondent the frustration of seeing irrelevant questions (and thus increasing the probability of them finishing the survey).

2.3.2 Questionnaire content

All the questionnaire items (translated into English from the Czech original) as well as used logic jumps can be found in Attachment A.2. Questions are sorted into 6 groups:

- **Group 1** consists of only one question regarding the school where the teacher works. This information was used purely for asserting the geographical distribution of the responses (discussed below). Respondents were assured of total anonymity with respect to them and their school because we wished to avoid them being vary of revealing any potentially sensitive information about their particular school.

- **Group 2** is concerned with teaching Special Relativity in regular physics lessons, the extent of covered subtopics and the teacher's personal opinion whether SR should be taught as part of the gymnasium physics curriculum.

- **Group 3** is similar to Group 2 but concerns General Relativity. Even though teaching of GR is less likely than SR, we did not want to assume as much. However, the wording of this group of questions is different and speaks, for example, about "mentioning GR in connection with another topic" as opposed to Group 2 that discusses "teaching SR".

- **Group 4** concerns physics seminars. For the purposes of the survey, we inquired only about physics seminars that are not purely revision-oriented. Questions in this group encompassed teaching of both SR and GR in seminars including the teacher's opinion whether relativity should be part of physics seminars or not.

- **Group 5** deals with the frequency with which teachers refer students to various external sources of information such as books, videos, websites, events, etc.

- **Group 6** is the final group and contains miscellaneous questions. The two last groups of questions are meant to probe whether there is a tendency by students to inquire about topics outside of the curriculum (such as GR) and what percentage of students might be interested in additional sources of information. In the final question, we inquire about the length of the teachers' practice. We hypothesized that possibly the use of internet sources might be affected by the teacher's age; however, (Cohen et al. 2007) warn against using a question that might irritate the respondent. We have therefore chosen to ask for the length of teaching practice as it provides similar information as the age (with the minor exception of teachers who started to teach later in life) but also allows us to differentiate answer's of experienced and novice teachers.

2.3.3 Piloting

The questionnaire design was piloted in two stages. First, it was fulfilled by and then discussed during a seminar with the supervisor and other doctoral students at the Department of Physics Education who all also teach physics in lower and upper secondary schools. This provided valuable insight both from research and teacher perspective and let to the improvement of the questionnaire especially in terms of clarity of the questions.

The main piloting was done with physics teachers from the so-called *faculty schools*. These are primary and secondary schools that have an agreement of cooperation with a specific university or faculty. In case of the Faculty of Mathematics and Physics, faculty schools commonly accept in-training physics teachers for a teaching practice, take part in research among students, etc. They are also typically schools that have "good results" in physics teaching (above-average numbers of students graduating in physics, student successes in physics competitions and so on). Faculty schools were selected for the piloting phase because their teachers were expected to be more willing to participate in the survey and therefore more likely to forgive possible imperfections of the design.

130 physics teachers from 21 gymnasiums (10 of which are in Prague) were asked via email to participate in the survey in January 2017. Their email addresses were found on school websites (it is common for schools to display online which subjects are taught by which teachers). In case a school didn't specify physics teachers, an email was sent to the deputy headmaster inquiring about the names of physics teachers. We chose to write to a deputy headmaster instead of the headmaster, because the latter are typically busier with email communication outside the school; therefore, it was considered more likely to get an answer from a deputy. Knowing the names of physics teachers was sufficient because practically all teacher email addresses follow the *name@school-domain.cz* template (this information is also quite commonly stated on the websites).

Of the 130 addressees, 58 have finished the questionnaire, which corresponds to a response rate of approximately 45 % (comparing that to the return rate of the main phase of 24 %, our hypothesis regarding higher willingness of the faculty school teachers to participate seems to be valid). The analysis of the pilot responses did not reveal any problems with the design. The only addition was question 3b "How much time do you devote to GR?" that appeared to those who selected that they "...talk about GR as a standalone topic." Interestingly, this question stood out during the piloting because a surprising number of faculty teachers responded that they talk about GR as a standalone topic (the phrase "talk about" was used on purpose because "teach" might evoke a deeper level of analysis than it is typically possible in upper secondary education and thus dissuade teachers from selecting this answer). 62 % of faculty school physics teachers answered so, compared to 21 % of respondents in the main phase.

Even though we are dealing with faculty physics teachers, whose teaching might be at least theoretically presumed to be above average due to an expected above average emphasis placed on physics at their school, such a high portion of teachers including relativity in their lessons raises suspicions of a sampling bias. It seems reasonable that teachers who "have more to say about teaching relativity" are more likely to fill out a questionnaire about this topic. And vice versa, teachers who don't teach relativity at all might be more reluctant to admit so and therefore choose not to take part in the survey. We anticipated this possibility and emphasized in the email invitation for the survey (both for piloting and main phase) that we value the responses from all physics teachers regardless of whether they teach relativity or not. Furthermore, we tried to form the questions in a neutral way in order to avoid making teachers who don't teach relativity feel ashamed or irritated (coming back to the suggestion of Cohen et al. 2007). Nevertheless, we have no way of making sure that this actually helped lower the sampling bias.

Because the piloting and final versions of the questionnaire are almost entirely similar, the results of the piloting phase were added to the final results as they constitute a substantial fraction of the overall answers as well as represent some of the most achieved gymnasiums in terms of physics education in the country.

2.3.4 Representability of the responses

After the piloting phase, the main batch of emails was sent in July 2017. The email addresses were found the same way as before. Including the faculty schools in the piloting phase, over 1106 gymnasium physics teachers from 316 gymnasiums were asked to participate in the survey. The exact number of teachers is not known because in a few cases of schools that neither publicly displayed the teacher emails nor the names of physics teachers, and an email was sent to a deputy headmaster, instead of the names of teachers we received a reply that the survey invitation had been passed on to the physics teachers. The schools were found using a commercial website (SeznamŠkol.eu 2022) because no better list of school sorted by their type and region was found. It was later discovered that (Department of Informatics and Statistics MEYS 2022) states the number of gymnasiums for the school year of 2016/2017 that offer the upper secondary program as 358, so we see that we were able to find and contact only approximately 89 % of gymnasiums. The main complication is that some schools are both a specialized upper secondary school and a gymnasium (i.e. they offer both of these programs). Even though some such combined schools were part of the survey (in which case we tried to contact only those teachers who teach at the gymnasium part - as stated on the school website), we were obviously not able to find all the gymnasiums. However, we would argue that the schools we didn't find are more likely to be smaller ones and therefore the missing 11 % of gymnasiums does not correspond to 11 % (or more) of gymnasium teachers.

We were unable to verify this by comparing the number of addressed teachers to the total number of gymnasium teachers in Czechia, as this information was not gathered by the MEYS at the time. In the beginning of 2019, however, the ministry organized a large survey to find out (among other things) the numbers of teachers in all forms of education by directly contacting the school headmasters (Maršíková and Jelen 2019). According to the ministry, 99.9 % of schools took part in the survey. Unfortunately, the survey does not distinguish between types of upper secondary schools, it only states the overall number of upper secondary physics teachers in that year to be 2467. Another information that (Department of Informatics and Statistics MEYS 2022) provides is the total number of upper secondary schools that end with maturita - 1093 for school year 2016/2017. We are considering only the schools with maturita here because, as we have mentioned above, those are the only upper secondary schools with physics education. We see that gymnasiums constitute roughly 33 % of these upper secondary schools, yet the gymnasium physics teachers we addressed represent 45 % of the amount of all upper secondary physics teachers that was found two years later. However, a direct comparison of these percentages has very little meaning. We have mentioned that gymnasiums on average have more physics lessons than specialized upper secondary schools, therefore they need more physics teachers per school. Consequently, we have come to the conclusion that even though we most likely contacted a large majority of gymnasium physics teachers, we have no way of asserting exactly what percentage of gymnasium physics teachers it actually was.

We have received 296 replies to the survey (including the piloting phase), making the overall return rate approximately 27 %. The data can be found as an electronic attachment to this work. Given that less than a third of addressed teachers took part in the survey, our main concern was how well the data represents a wider teacher population. We have already discussed the possible sampling bias. Another issue might be with the spread of responses over the country. Our aim was to try to map the teaching of relativity in gymnasiums in the whole country. We could hardly claim to have done so if the majority of the responses came from a single region (Prague, for example). To analyze the spread of answers, we decided to group them according to region. Czech Republic is divided into 14 regions (in Czech *kraje*) with a certain level of self-governing power. The names of the regions can be found in Table 2.3 (they are named usually after the largest city in the region, for example *Plzeň*, or according to geographical location, e.g. *South Bohemian*).

Unfortunately, we were unable to compare the distribution of answers to the actual numbers of gymnasium physics teachers in each region because, as we already said, such data wasn't yet collected by the MEYS at the time (and the 2019 ministry survey, though it does show regional distribution of physics teachers for upper secondary schools, cannot be used because it doesn't distinguish between types of schools). However, we can use the regional distribution of gymnasium students for the school year 2016/2017, because that data is available (Department of Informatics and Statistics MEYS 2022), and, under the assumption that student and teacher numbers correlate, compare it to the distribution of survey answers. We used the Pearson's χ^2 test (Lehman and Romano 2005) to test our *null hypothesis*, that the observed distribution of teacher responses (denoted n_i, where $i = 1, 2, \ldots, 14$ are the 14 degrees of freedom of our distribution corresponding to 14 regions) does not differ from the distribution of students. We also chose the *significance level* α to be 0.05, which corresponds to the probability of 5%. We first calculated the theoretical (or expected) frequencies of responses under the assumption of the null hypothesis (see Table 2.3).

Regions	**student numbers**		**teacher responses**	
	absolute	p_i (relative)	n_i (actual)	$n{\cdot}p_i$ (expected)
Prague	24331	18.84 %	57	56
Central Bohemian	12484	9.66 %	23	29
South Bohemian	7778	6.02 %	22	18
Plzeň	6168	4.77 %	14	14
Karlovy Vary	3300	2.55 %	5	8
Ústí nad Labem	8150	6.31 %	14	19
Liberec	3984	3.08 %	14	9
Hradec Králové	6683	5.17 %	14	15
Pardubice	5944	4.60 %	18	14
Vysočina	6191	4.79 %	8	14
South Moravian	15460	11.97 %	38	35
Olomouc	8117	6.28 %	19	19
Moravian-Silesian	13395	10.37 %	37	31
Zlín	7192	5.57 %	13	16
sum	**129177**		$n = 296$	

Table 2.3: Distribution of students between 14 regions in the school year 2016/2017, the actual amounts of teacher responses based on the school region and the expected amounts under the null hypothesis that there is no difference between the distributions of students and responses.

If p_i are the relative frequencies of the student distribution and n is the total amount of responses, the expected response frequencies can be calculated simply as $n{\cdot}p_i$. We then calculate the χ^2 statistic according to (Lehman and Romano 2005) as

$$\chi^2 = \sum_{i=1}^{14} \frac{(n_i - n{\cdot}p_i)^2}{n{\cdot}p_i} \doteq 13.189. \tag{2.1}$$

We used the *CHIINV* function in Microsoft Excel to calculate the *critical value* of the χ^2 distribution for the chosen significance level of 0.05 and $k - 1$ degrees of freedom (in our case, 13) to be approximately 22.362. Because the result of equation 2.1 is lower than the found critical value, we conclude that the null hypothesis cannot be rejected. Although this is not a direct prove of statistical similarity, it gives us a certain level of confidence, that the distribution of responses does not single out any particular region or regions. Therefore, even though our sample represents only about a quarter of addressed gymnasium physics teachers, we consider this quarter to be reasonably spread-out across the country, increasing our confidence in its representability of the overall gymnasium physics teacher population.

2.3.5 Survey results

We will now present the results of the survey, starting with questions regarding teaching of SR in both normal physics lessons and seminars. Figure 2.2 shows the main result regarding teaching SR during lessons. 113 teachers responded that they don't teach SR in regular lessons. However, 56 of them used to teach it and 19 plan to teach in the future. Furthermore, as we will see later, some teachers don't teach SR in lessons, but do so in seminars.

Figure 2.3 shows reasons why teachers don't teach SR in lessons. This was obviously asked only of those 113 teachers who picked one of the negative answers to the previous question. Whenever a question was relevant only for part of the respondents and so not all of them saw it due to logic jumps, we display it by stating the relevant sample size. In this case, $N = 49$ for teachers who don't teach SR in lessons (including those who plan to start teaching SR) and $N = 62$ for those who used to teach it (including 6 who used to teach it and plan to do so again in the future). Two teachers didn't fill out this answer. As we can see in the graph, the prevalent answer for not teaching SR in lessons is a lack of time, especially for those not teaching it anymore. One respondent answered that they prefer spending time with another topic, namely quantum mechanics. Overall 16 teachers chose to include their own answer (”SR is taught in seminars” 8x, ”have been teaching only briefly and hadn't got to teach SR” 4x, ”not enough time or not in our SEP” 2x, ”teaches only a few lessons per week or no physics at all” 2x).

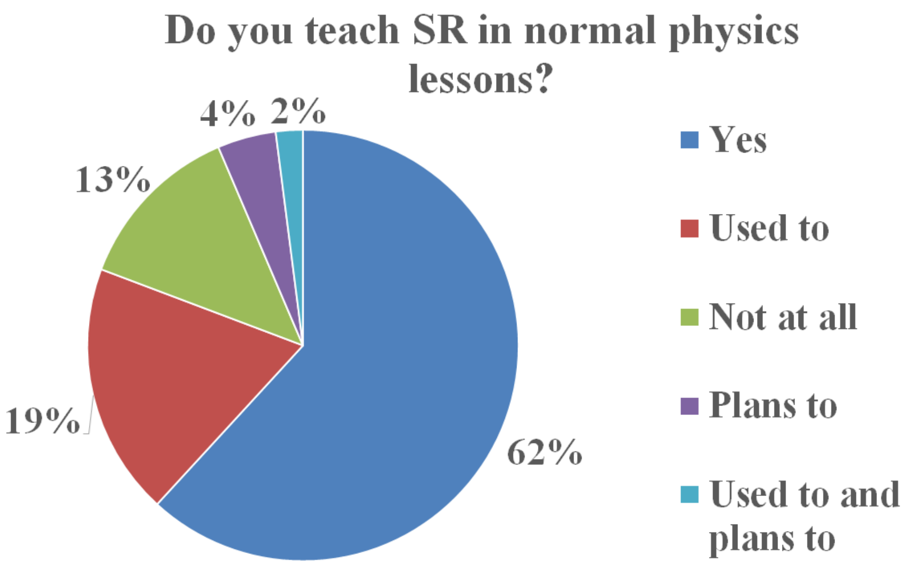


Figure 2.2: Relative numbers of responses to the question whether the respondents teach SR in regular lessons. Respondents were given the option to answer negatively in a more specific way: ”No, but I used to teach SR before.” (labeled as **Used to**), ”No, but I plan to include SR in my teaching.” (**Plans to**), fully negative ”No, SR is not part of my teaching, neither do I plan to include it.” (**Not at all**), or ”Used to teach it and plan to in the future.” (**Used to and plans to**).

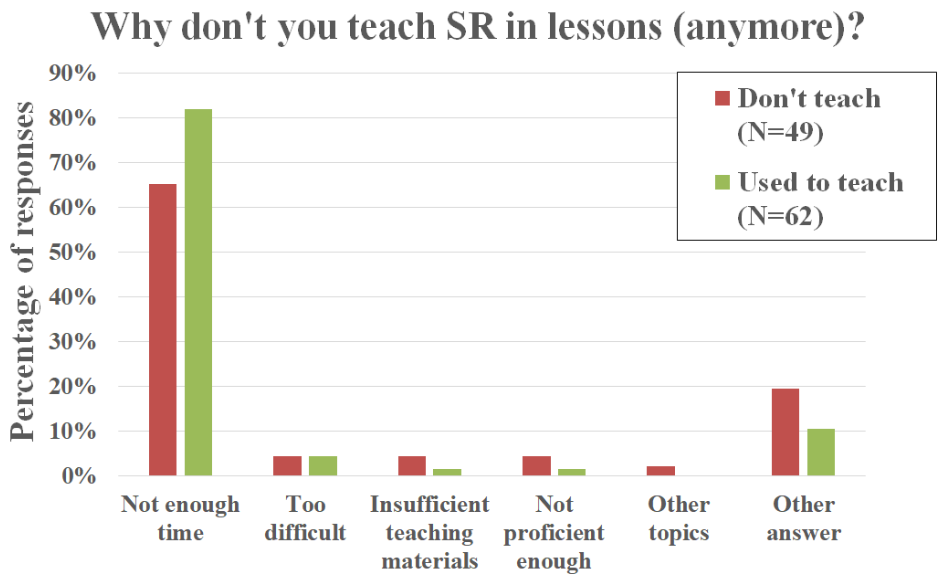


Figure 2.3: Reasons why teachers either don't teach SR in lessons or don't teach it anymore (**Used to**). More than one answer could have been selected and the percentages were calculated from the total sum of answers.

As we can see in Figure 2.4, about 60 % of respondents (177 people) either teach a seminar now or have done so in the past. We purposely included the "used to teach a seminar" option because for example in some schools physics teachers take turns teaching the seminar or just simply some scheduling conflicts may result in a change of the seminar teacher between school years. In other words, we were interested whether someone teaches a seminar long term, not necessarily that particular school year. That is why both of these groups were asked further questions about SR and GR in their seminar and we consider the answers of those not presently teaching the seminar equally valid. Large majority of these respondents do teach SR in their seminars (Figure 2.5).

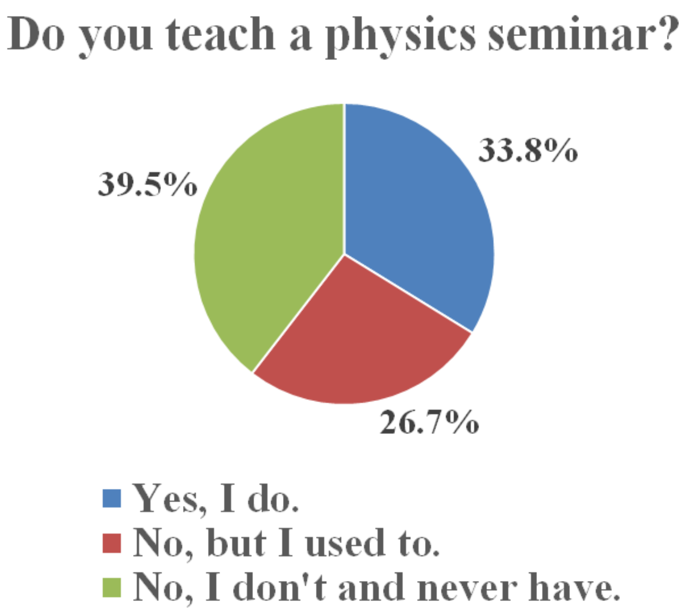


Figure 2.4: Question concerning teaching physics seminars that are not purely focused on revision.

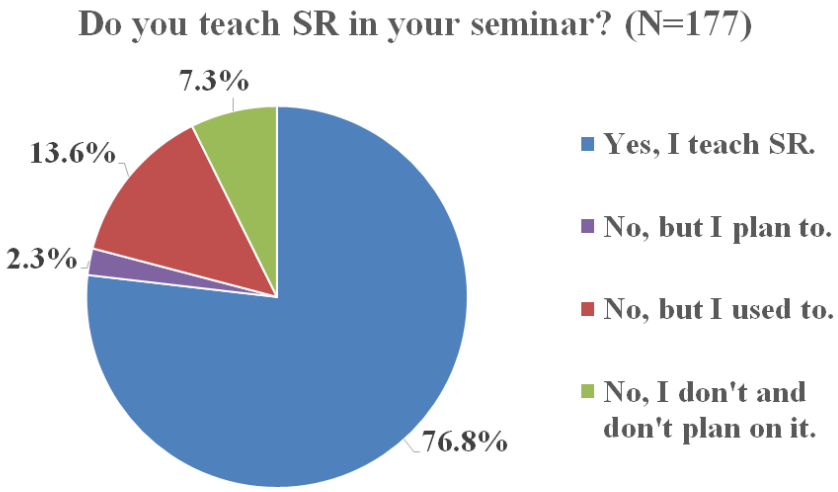


Figure 2.5: Majority of teachers who lead a seminar (177 respondents) include SR there.

We have also combined the answers regarding teaching of SR in lessons and seminars (Figure 2.6) to have a better understanding of the overlaps of different groups in Graphs 2.2 and 2.5. As it turns out, only about a fifth of the respondents don't teach SR in any shape or form because 17 % of respondents (50 teachers) teach SR in seminars and not in regular lessons.

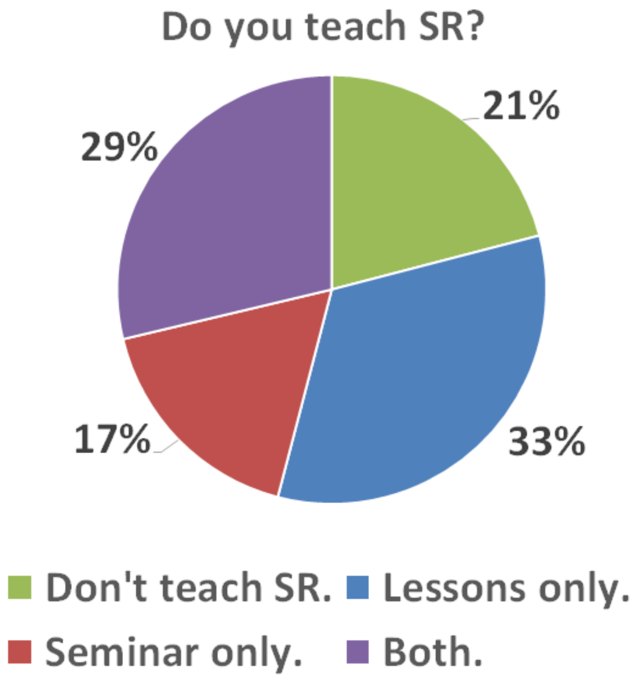


Figure 2.6: Summary of responses concerning teaching SR in both lessons and seminars.

Next, we look at which subtopics of SR are taught. First, Figure 2.7 gives a general summary of the percentage of teachers that include a given subtopic. In addition, we were also interested in the extent and combinations of taught subtopics. Figure 2.8 highlights the most common combinations separately for the three relevant categories from Figure 2.6 - regular lessons only, seminars only and both. The percentages shown at each subtopic combination are relative to the number of respondents in that particular group (98 for purely lessons etc.). The main differences in those combinations are clearly visible, especially regarding the mentioning of the Michelson-Morley experiment. However, all the combinations show a common "core" of subtopics - *time dilation, length contraction* and *relativistic mass/momentum*. We could then conclude that these 3 subtopics are collectively considered essential or the bare minimum of SR by the respondents. This could be caused by the fact that these are the subtopics covered in the mentioned SR textbook for gymnasiums (Bartuška 2010). The book also contains the description of the Michelson-Morley experiment, which is traditionally used as a historical introduction to SR; however, looking at Figure 2.7 we see that only about half of the teachers include it in their teaching.

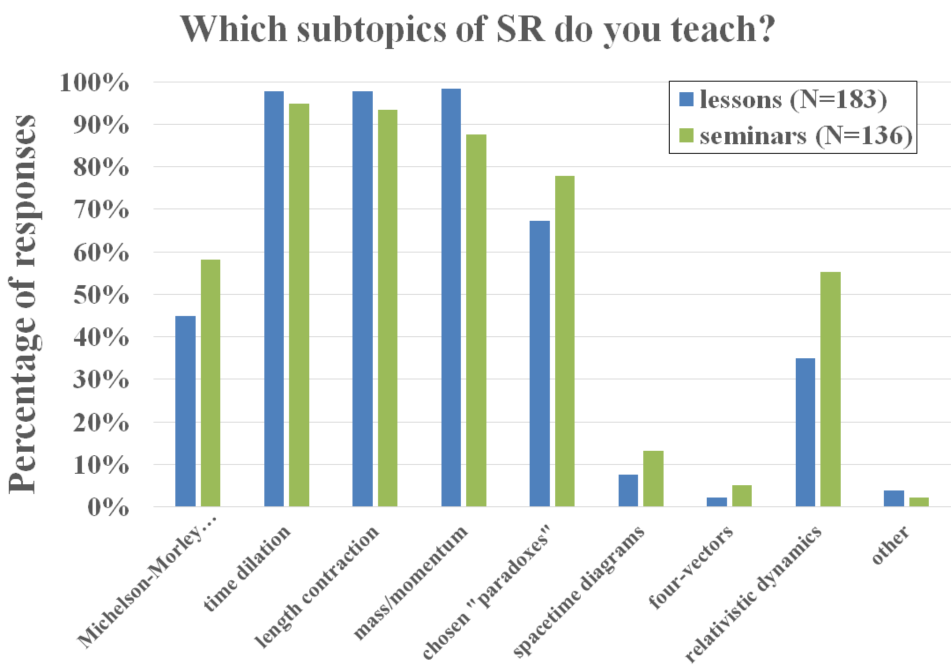


Figure 2.7: Subtopics of SR that are taught separately for lessons and seminars.

Topics	Most common combinations of topics											
	lessons (98)						seminar (51)			both (85)		
Michelson-Morley experiment	×	✓	×	✓	×	✓	✓	×	✓		×	×
time dilation	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
length contraction	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
relativistic mass/momentum	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
chosen "paradoxes"	✓	✓	✓	✓	×	×	✓	✓	×	✓	✓	✓
spacetime diagrams	×	×	×	×	×	×	×	×	×	×	×	×
four-vectors	×	×	×	×	×	×	×	×	×	×	×	×
relativistic dynamics	×	×	✓	✓	×	×	✓	×	✓	×	×	✓
relative frequency	21.4%	15.3%	11.2%	11.2%	10.2%	10.2%	41.2%	23.5%	13.7%	11.8%	9.4%	7.1%

Figure 2.8: The most common combinations of SR subtopics that are taught. The shown percentages are relative to the number of responses for the particular group (lessons, seminars, both). The split cell in the third column from the right signifies that the particular subtopic is taught in regular lessons but not during seminars.

Another interesting note is that *spacetime diagrams* and the idea of *four-vectors* are quite rarely used. They do not appear in the most common combinations of subtopics (Figure 2.8) and are mentioned by only a small minority of teachers (Figure 2.7). This might also be at least partially caused by the absence of these topics in the mentioned textbook (Bartuška 2010).

10 respondents added another subtopic not mentioned in the list, such as: *mass-energy relationship*, *relativity of simultaneity*, *Lorentz transformation*, *connection between SR and global navigation* and *addition of velocities*.

The most common answer to why teachers don't include SR in their seminars is that it is already covered in regular lessons, as can be seen in Figure 2.9. Second most common answer was that either the teacher or the students prefer to spend time with a different topic of physics. These answers suggest a certain level of teacher autonomy regarding the seminar content that might even translate into the students being given a choice in the discussed topics. We further inquired about these topics and got a variety of answers: *biophysics, individual solving of practical problems, mechanics, hydrodynamics, particle physics* or *according to the students' interests*. As their own answer, some teachers wrote ”not enough time”, ”seminar not opened this year”, ”basics of SR in regular lessons”, ”we don't have a physics seminar”, ”the seminar is mostly revision or preparation for university entrance exams”. We can deduce from the individual answers that a few teachers misunderstood or misread the question. For instance, the option ”SR is taught in normal lessons” was already available. However, this is a case of only a small number of responses, so the validity of answers to this particular question is overall not jeopardized.

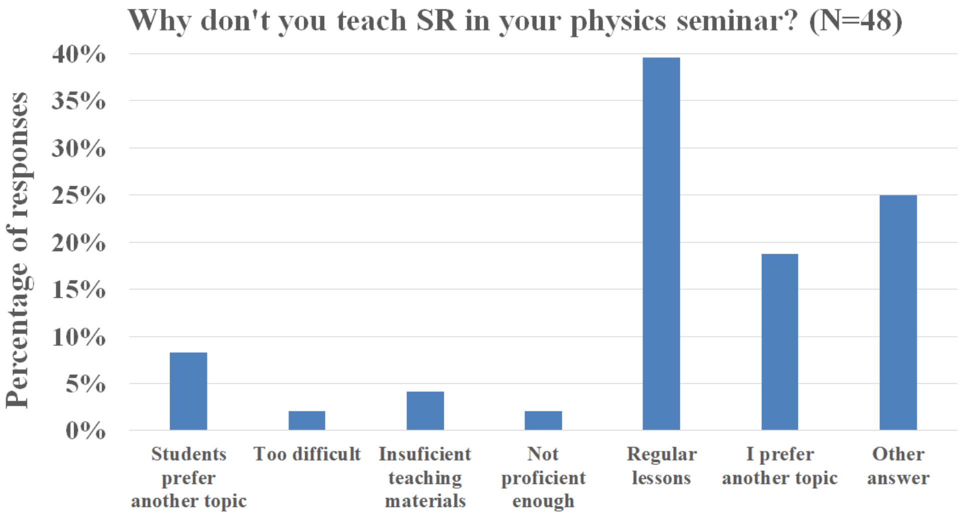


Figure 2.9: Reasons why teachers don't include SR in their seminar. Respondents could select more than one answer, so the shown numbers add up to more than the original amount of teachers that do not teach SR in seminars.

We were also interested in the teachers' opinion regarding whether SR belongs to upper secondary school. Figure 2.10 shows the percentages of answers separately for regular lessons and seminars. As possible answers we used a text version of the four-point Likert scale (Likert 1932) specifically because it lacks the middle (neutral) options and so the respondent was encouraged to ”choose a side”. In case of the seminars, we added one extra option ”normal lessons only” and it was selected by almost 20 % of the respondents. We can see that only about 10 % think that SR should not be part of regular physics lessons and even fewer respondents (around 2 % which constitutes 5 people) say the same thing about seminars. The vast majority of teachers that filled out our survey therefore think that SR indeed belongs to gymnasium physics.

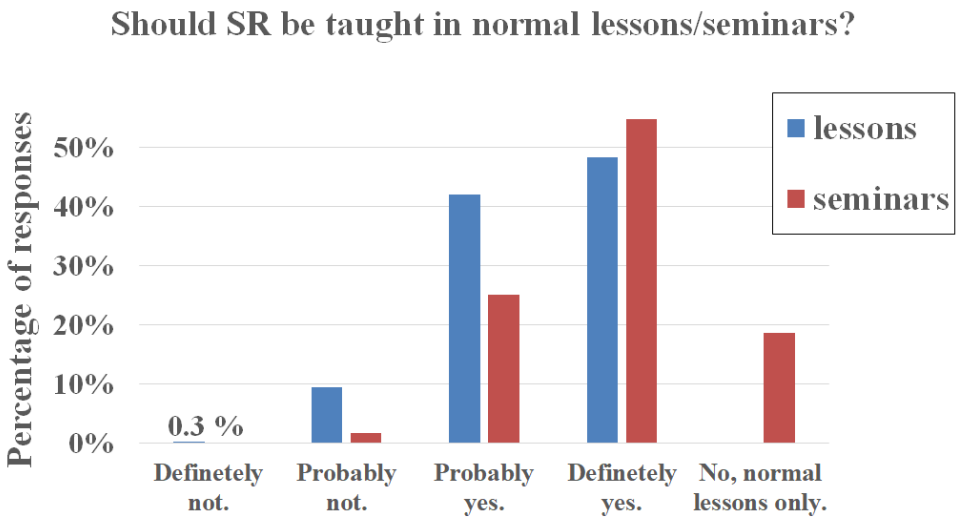


Figure 2.10: Teachers' opinions whether SR should be taught at upper secondary either in regular lessons or seminars.

Let us proceed to the GR related questions. As mentioned before, although the nature of questions was similar to the previous case of SR, we formulated some of the questions differently to better fit the unique situation of GR not being a typical part of physics curriculum. Nevertheless, due to the mentioned similarity, we will refrain from detailed description of the questions and refer reader to the figure captions.

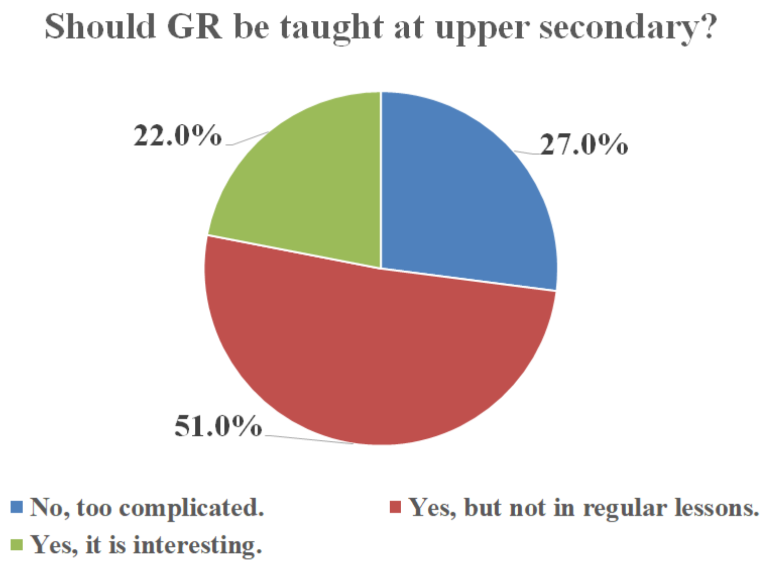


Figure 2.11: Just like with SR, we asked teachers about their opinion whether GR should be taught as part of gymnasium physics.

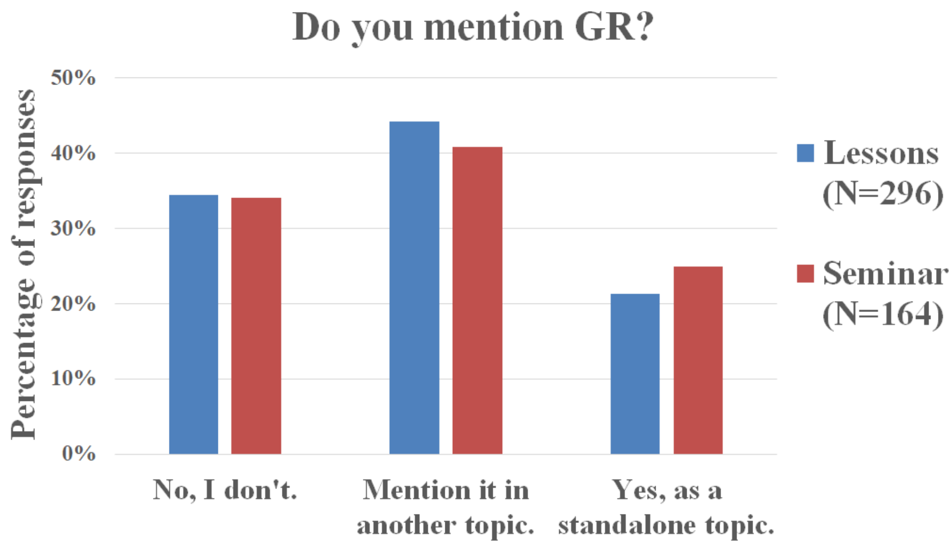


Figure 2.12: Main question regarding ”mentioning” (as opposed to teaching) GR in lessons and seminars. For seminars, the overall number of respondents is lower because it applies only to those who teach a seminar.

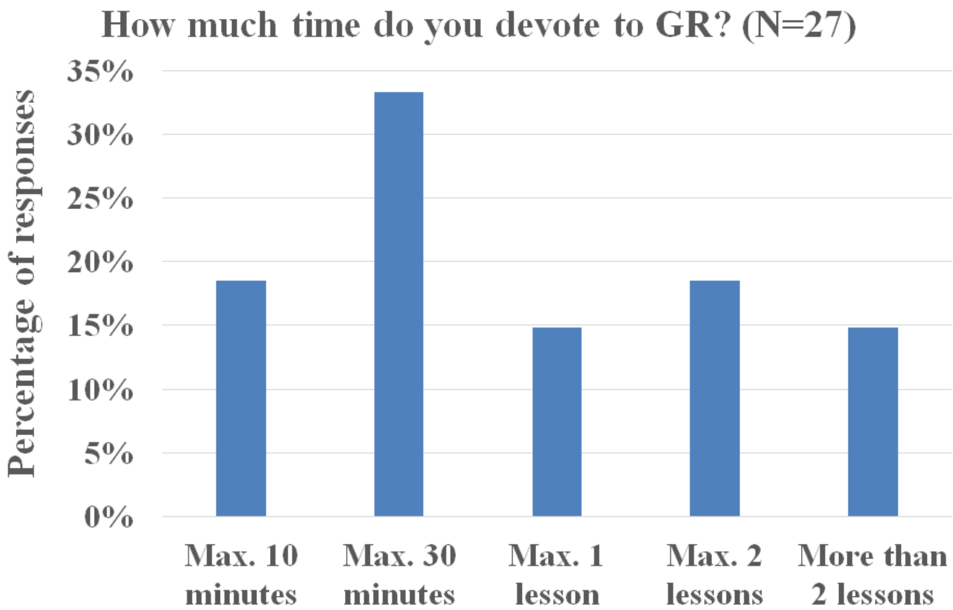


Figure 2.13: Maximum time devoted to GR in lessons for those teachers who treat it as a standalone topic (third column in the previous question).

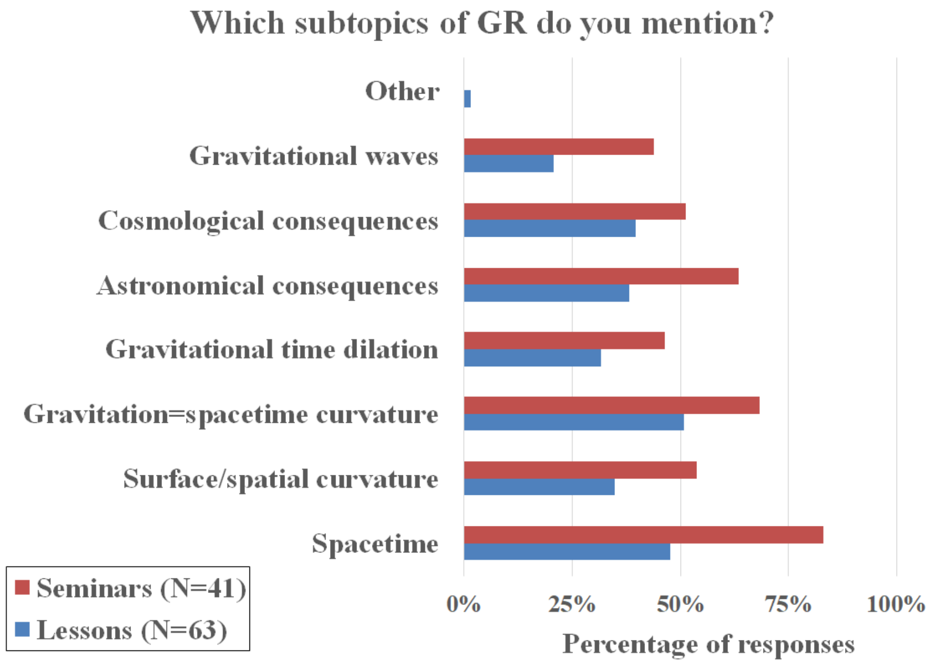


Figure 2.14: GR subtopics mentioned by teachers who treat it as a standalone topic for lessons and seminars. No significant similarities in subtopics selection were found, so the data is visualized only this way. The subtopic mentioned as "Other" was the *equivalence principle*.

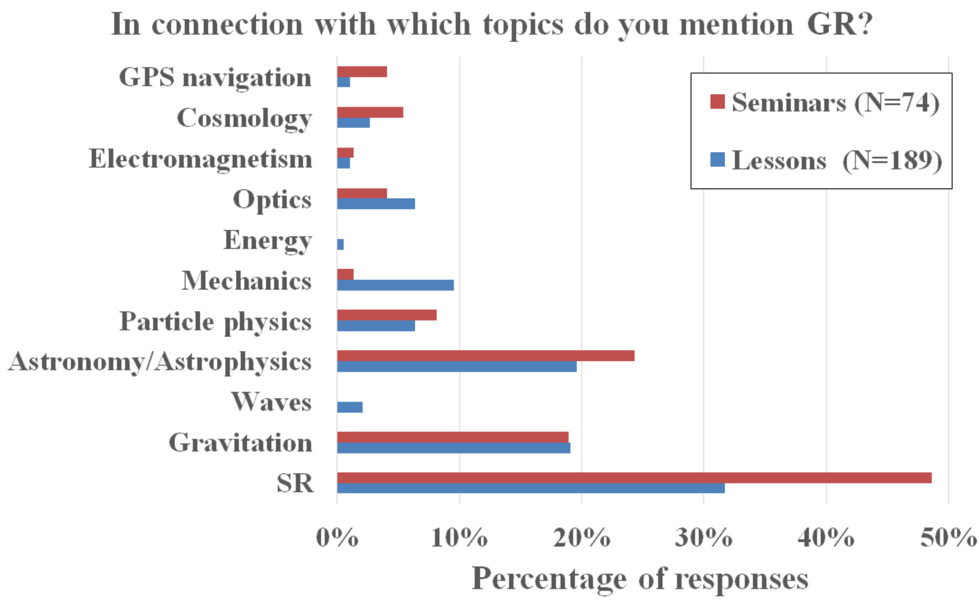


Figure 2.15: Topics in connection with which GR is mentioned (as opposed to being a standalone topic). Respondents could have entered multiple answers (and often did); therefore, the percentage is calculated relative to the number of answers, not respondents.

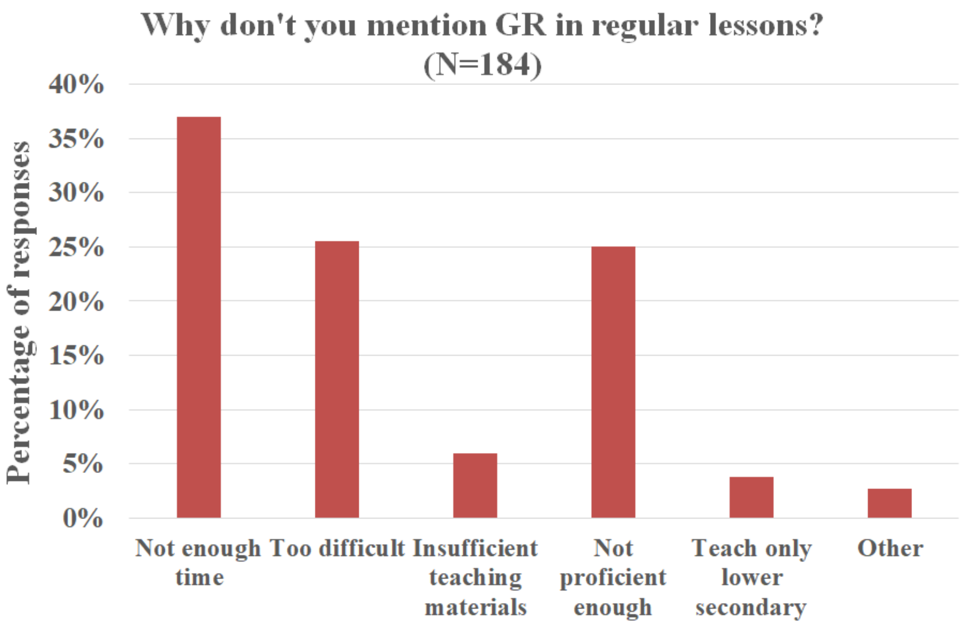


Figure 2.16: Reasons why teachers don't mention GR in lessons plotted separately from seminars because the given options were slightly different. First of all, the option "I teach only lower secondary physics" is not relevant for seminars because they are scheduled typically for the last two years of upper secondary gymnasium. Secondly, as mentioned before, due to the broadening nature of seminars it is much more likely there for the students to have some choice in the particular topics that are treated. Therefore, the option of "Students prefer a different topic." was omitted in case of regular lessons.

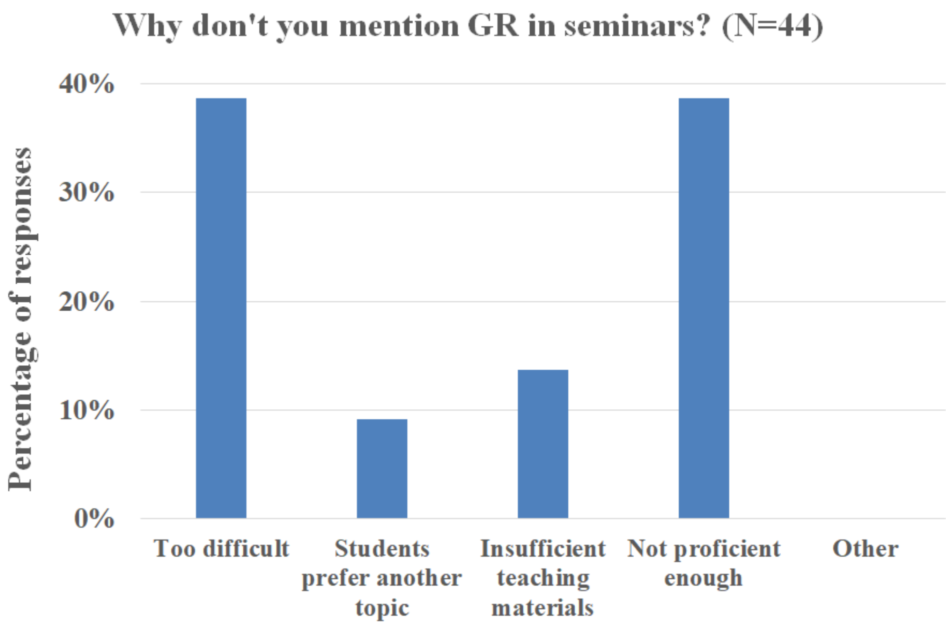


Figure 2.17: Reasons why teachers don't mention GR in seminars.

Lastly, we will present the answers to the group of miscellaneous questions at the end of the questionnaire. In the question regarding referring students to extracurricular sources of information (Figure 2.18), respondents could add their own option. The most notable examples are: *competitions such as Astronomical Olympics or Physics Olympics, excursions, lending physics books to students, inviting guest lecturers to school* or *awarding bonus points for attending physics related events.*

Most questions in this group inquire about frequency of occurrence. For this purpose, we have chosen a four-point scale *Never, Rarely, Sometimes, Often/Fairly regularly.* When the scale is first used in question group 5, these points are described in more detail with *Rarely* as "at most a few times per school year", *Sometimes* as "not more than once a month" and *Often/Fairly regularly* as "at least a few times a month".

We have not found any significant correlation between the length of a teacher's practice and other answers; therefore, we are not showing the data on the practice length, as it appears not to be relevant to the rest of the discussion.

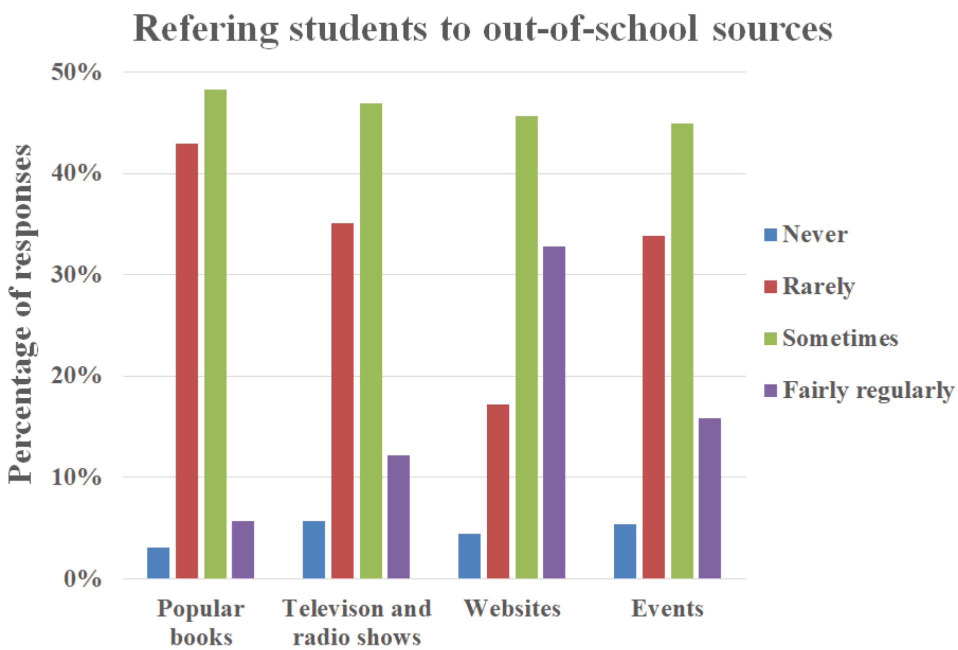


Figure 2.18: How often teachers refer students to various interesting sources of information about physics outside of school.

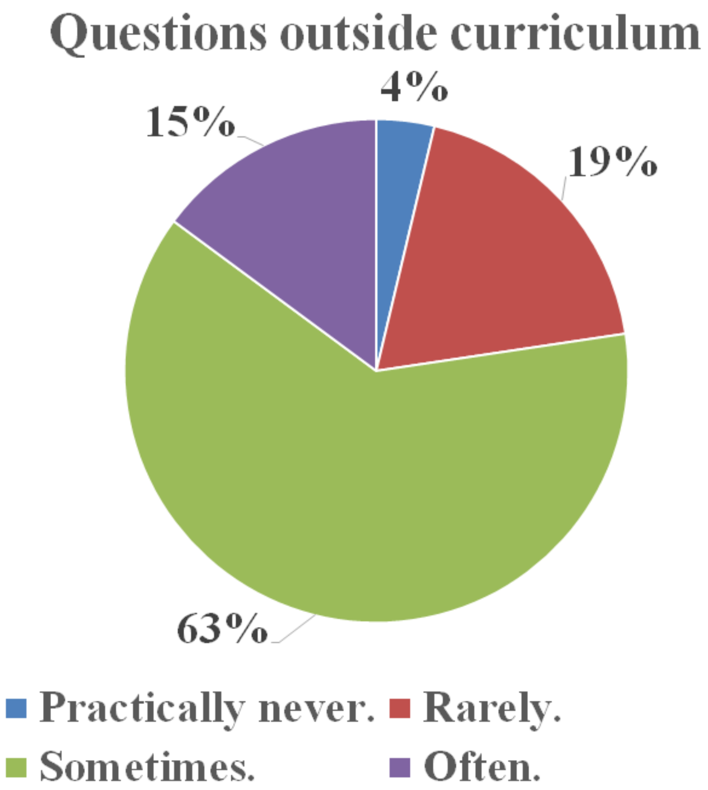


Figure 2.19: How often teachers receive questions from students concerning physics topics outside the curriculum.

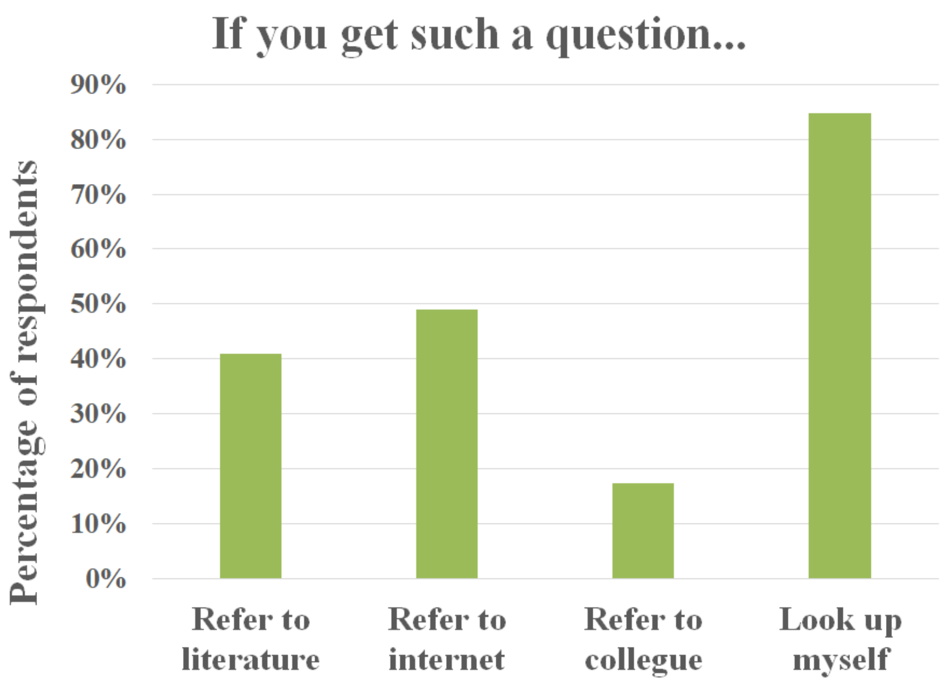


Figure 2.20: How teachers react to receiving a physics related question to which they don't know the answer. More than one option could have been selected.

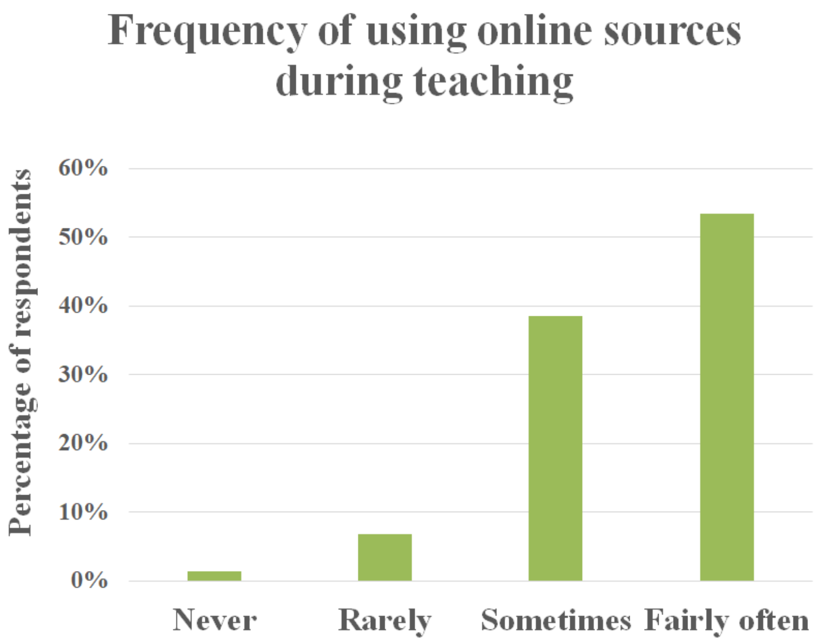


Figure 2.21: Self-reported frequency of using online resources by teachers during their teaching.

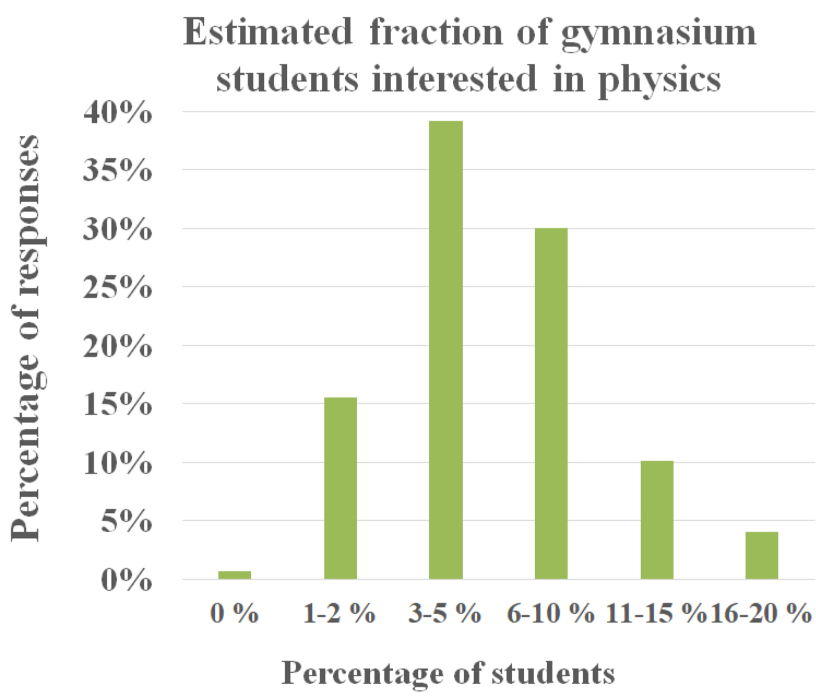


Figure 2.22: Percentages of students interested in physics enough to engage in extracurricular sources of information, as estimated by teachers.

Lastly, we gave respondents the opportunity to add anything they wished to say in connection with teaching relativity. About 25 % of the respondents chose to do so. We present a selection of their answers translated into English (the full list of these answers can be found in Attachment A.3):

- *Usually it is not possible to discuss everything in depth, because SR is included as the last topic of teaching in a very packed year. It is discussed in depth in the physics seminar.*
- *Personally, I would prefer to reduce some chapters so that there is more time left for SR (GR), but at the same time I am often not able to skip the optional chapters of some areas preceding physics. Therefore, we most often encounter the issues of relativity through students' questions.*
- *Due to the number of lessons devoted to physics in compulsory education and also the interest in physics, I consider the teaching of SR to be unnecessary. Seminars also offer more useful topics for further study at universities. I personally teach in a seminar the use of derivatives and integrals in physics.*
- *I don't know how it is in other high schools, but we all teach relativity. Most students are more interested in this topic and enjoy it more than previous "classic" topics. They also come to a chapter for the first time where it is clear that they will only look at the edge and that the real depth of the problem is much greater.*
- *In my judgment, the material does not belong to a general gymnasium at all. It confuses students who have difficulty with high school physics. Students gain the feeling that physics is not only difficult but even absurd.*
- *This topic is interesting, unfortunately it is taught in the fourth year of upper gymnasium, when students have their heads full of maturita and it is very difficult for teachers to excite them for physics in this period.*
- *Not enough time.*
- *I teach SR during labs.*
- *I did not teach SR in a regular class for the first time this year when one physics lesson was removed from the schedule. Colleagues haven't taught SR in a long time. In the next classes, I plan to include SR again and sacrifice something else (probably Electrostatics). I consider the introduction to SR and quantum physics to be essential, because of the difference from the world that the students know from their experience.*
- *The topic is quite interesting for students, but unfortunately due to lack of time considered marginal, so it is not possible to show students the application of SR, for example, in terms of astronomy, astrophysics, quantum physics, and particle physics.*
- *It depends on the "space-time" that the teacher has available to teach SR. The students should leave high school with at least the following three pieces of information on this topic: that things are "a little different" than in everyday life, when they are "different", and why they are "different". They can find out how things are at any time and study it later, it is essential to understand the causes and accept the "otherness".*

- *It would be nice to have some high school ideas for GR reasonably written - and, for example, reasonably linked to astronomy.*
- *I haven't taught SR yet, but I hope to one day. It is also a challenging topic for me, so I will have to study it diligently.*
- *I think the topic is interesting, everyone should at least have an idea about it. Unfortunately, I don't understand enough on my own to be able to answer all the questions.*

2.3.6 Survey conclusions

Based on the presented survey answers, we can see that SR is still being taught at a significant fraction of gymnasiums (Figure 2.6). GR is at least mentioned by approximately two thirds of the respondents, but it is mostly mentioned briefly or in connection with some other topic (Figures 2.12 and 2.15). The majority of teachers agree that SR should be part of gymnasium physics and even part of regular lessons. Figure 2.10 shows that the preference for inclusion of SR in the seminar is understandably stronger but not significantly so. Moreover, about one fifth of the respondents prefer SR to be part of regular lessons only. Again, a majority of respondents think that GR should be part of gymnasium physics, but the prevailing opinion is that it belongs to the seminar only.

As the main hindrance in teaching relativity in general, teachers mention the lack of time the most. A reduction in the number of physics lessons, especially in the fourth year of upper secondary, has been repeatedly mentioned by teachers in their open statements at the end of the survey. Teachers also feel less proficient in GR than in SR (compare Figures 2.3, 2.9, 2.16 and 2.17).

According to the teacher's answers, working with internet sources and referring students to outside sources is quite common. There is also, according to teachers, a small but non-zero percentage of students that are interested in physics enough to seek information about it outside of school in their own time. Universally present lack of time in physics education makes introducing new content difficult and teachers also feel significantly less proficient to teach GR than SR.

As seen in Chapter 1, it is easier for students to find quality extracurricular sources of information regarding SR compared to GR. Most of the existing sources on GR are either purely popular in nature or too complicated for most upper secondary students (university textbooks).

All these findings led us to the decision to

- create an online study text focusing mainly on the introduction of basics of GR to upper secondary students as extracurricular learning materials for interested students to try to fill the gap between existing literature (Chapter 3).
- create a suitable standalone teaching-learning sequence to present basic ideas of GR that teachers can include in their teaching if they choose to (Chapter 4).

3. Study website for students interested in General Relativity

The conclusions of the previous chapters led us to the decision to create a website dedicated to increasing the understanding of GR of interested students. As our starting point, we took a short study text that was created in a previous work (Ryston 2014) based on a limited literature search. We first conducted a more thorough literature analysis in order to identify common ideas, themes and chains of thought in generally well-reviewed textbooks (as described below). Based on our findings, the original text was mostly rewritten, expanded, turned into a website and supplied with interactive elements. The content of the website as well as the review process are described in the second section of this chapter.

3.1 Analysis of literature

As it was mentioned in Chapter 1, we selected a number of relativity textbooks for analysis in order to identify common approaches to the exposition of GR. Group 1 of the books, those that don't use higher mathematics such as differential calculus and are therefore suitable for upper secondary students, was already presented in Section 1.2. Group 2 are books intended for undergraduates or specifically textbooks written for undergraduate courses. The selection process was the same as with the first group, mostly based on positive reader reviews on *Amazon.com*. A few of the undergraduate titles were recommended by a colleague or were found in internet discussions about relativity textbooks. Two of the books are in Czech (Dvořák 1984 and Kulhánek 2020), the rest are in English. Figure 3.1 shows a summary of the analysis. Because our main goal was the exposition of GR, we did not include two books from Group 1 (Takeuchi 2010 and Steane 2011) that contain purely SR.

(Hartle 2006) formulated two distinct approaches to the exposition of GR for undergraduate GR courses. *Math-first* is a deductive approach where: (1) necessary mathematical tools are first developed, (2) a physical problem is formulated in general, (3) a solution for a particular situation is found and (4) applied to make predictions to be compared with experiment. In case of undergraduate GR courses, this is typically and best illustrated by first developing the necessary mathematical description of curvature, then forming Einstein field equations, finding the Schwarzschild solution and then applying it in concrete physical situations such as orbiting of a planet or light deflection. (Hartle 2006) argues that having to develop the whole necessary mathematical apparatus to derive general equations first can discourage mathematically less inclined students from the physics to come and thus create a sort of a barrier in studying relativity. He proposes a *physics-first* approach of (1) stating the simplest physically important solution (in case of GR the simplest physically relevant spacetime, i.e. the Schwarzschild solution), (2) deriving and (3) applying predictions from it, and finally (4) motivating the Einstein equations and solving them to show where the originally presented spacetime came from. Even though these two approaches apply originally to undergraduate GR courses, they can be generalized to other physics education. We see the physics-first approach very commonly in secondary school, where we don't deal with general problems but rather with special cases. It makes sense to start the initial discussion of electric field, for example, with the field of a point charge, not Maxwell equations.

Another clear example of the two approaches from the topic of relativity is in SR. We can often see first the derivation of the Lorentz transformation from the basic postulates of the theory and then deriving predictions of physical phenomena (such as time dilation) from the transformation (a math-first approach). On the other hand, it is possible to focus on physics first using for example results from real-world experiments or thought experiments, come to time dilation and other predictions of SR from the first postulates and only then derive the Lorentz transform to confirm the "previous" conclusions. Of course, these two approaches form two opposite ends of a spectrum, it might be possible to combine them, use math-first in some chapters and physics-first in others. The second column of Figure 3.1 shows that we tried to identify the prevailing approach in the analyzed books. It comes as no surprise that all of the books in Group 1 have been identified to primarily use the physics-first approach (denoted simply as P), as they emphasize the relevant physics, and mathematics is used (if it is used at all) to illustrate given points. On the other hand, most of the books in Group 2 follow the more traditional path of an undergraduate GR course and strongly lean on the side of the math-first approach. The two notable exceptions are (Taylor and Wheeler 2000), which is commonly praised for being very approachable and yet detailed and technical, and (Hartle 2003), which exhibits both approaches in different parts of the book.

We focused on the book contents concerning GR exposition in three main groups. *Topics prior to GR*, *main GR topics* and *applications of GR*. Almost all books, even those solely focused on GR included prior topics to prepare their ground for GR chapters. Of course, some of the books, as described in Section 1.2 focused both on SR and GR more or less equally or rather were more focused on SR with only a few chapters on GR. Still, in those cases the GR chapters were directly linked to previous discussions; therefore, we can think of them, from the perspective of GR, to be preparatory chapters. The three most common preparatory topics for GR we found were *classical or Galilean relativity*, *special relativity* and *non-Euclidean geometry*. The first topic is a common starting point for discussions about SR and its inclusion is usually meant to "set the stage", to make sure that the reader is caught up to the classical worldview with which the discussed view of modern relativity is later confronted. It also serves to let readers know that relativity did not start with Einstein and that it's a much older concept. To add a personal observation, when the author of this text was teaching, as mentioned before, SR at a gymnasium, students were always surprised by that because Galilean relativity is not commonly taught in Czech upper secondary schools. As we can see in Figure 3.1, classical relativity was not included only in some of the advanced and more mathematical books of Group 2. In case of (Taylor and Wheeler 2000), which is essentially a continuation of (Taylor and Wheeler 1992), it could be argued that classical relativity is part of the first book as an introduction to SR, and therefore it is not present in the second one, where

SR is briefly revised in the first chapter for the purposes of the GR discussion.

The inclusion of the other two topics, SR and non-Euclidean geometry is understandable because at least some of their parts are necessary for the discussion of GR. A notable point is the placement of the chapter on non-Euclidean geometry. There are two common options. It either precedes GR chapters completely in a math-first fashion or it forms a geometric "intermezzo" after some initial discussion of GR, typically involving the equivalence principle, resembling the physics-first approach. The only two books that did not include a chapter specifically devoted to non-Euclidean geometry were (Orzel 2012), which was the most popularly written book of the whole selection with a brief GR section (still discussing concepts like spacetime curvature but without spending time with the concept of curvature alone) and (Topper 2013) which is, as it is described in Section 1.2, a strongly biographical book focusing on Einstein's revolutionary breakthroughs with added physics context.

Moving on to the main topics of GR, we identified three most common themes. *The equivalence principle* with the consequential introduction of local inertial frames, *Einstein equations* embodying the key principle that mass(-energy) is the source of spacetime curvature and *Schwarzschild spacetime* as the simplest relevant solution of Einstein equations that serves to illustrate how gravitational phenomena can be derived from a known metric of spacetime. The order of these topics depends on the particular approach used in the book. One of the key reasons for including the equivalence principle is a motivation for the iconic geometrical approach of GR. Interestingly, the four books that do not include the equivalence principle all present the geometrical nature of GR as given without any reasoning on why it is the case.

As mentioned, a typical (and indeed archetypal) example of the math-first approach is first motivating Einstein equations and then deriving the Schwarzschild solution, while in the physics-first approach we first state the Schwarzschild solution without derivation, use it to illustrate "new" physics (a common occurrence is the discussion of the so-called *classical tests of GR*) and only then (but not necessarily) do we solve Einstein equations to obtain the already known solution. The Schwarzschild spacetime, even thought the simplest non-trivial solution of the Einstein equations, can be used to quantitatively demonstrate a significant number of gravitational phenomena, for example *gravitational time dilation*, *bending of light rays in a gravitational field*, *planet orbits*, *pericenter shift* or *black holes*, depending on the level of mathematics that we can involve. Figure 3.1 shows that only two of the Group 1 books (Natário 2011 and Schutz 2003) specifically introduce the Schwarzschild spacetime, most likely because in order to do that, readers need to have at least some understanding of the mathematical description of curved spacetime. Therefore, it is practically not possible to introduce it with purely secondary mathematics. In Section 1.2 we have mentioned that these two books of Group 1 that do include the Schwarzschild solution use expressions from differential calculus and differential geometry (such as metrics) without actually proper mathematical treatment of these topics by introducing concepts such as "distance of two close points" instead of a differential. This approach will also be key in our study materials described later.

			Topics prior to GR			Main GR topics			Application of GR		
	Book	M/P first	Galilean/Classical relativity	Special relativity	Non-Euclidean geometry	Equivalence principle	Schwarzschild spacetime	Einstein equations	Black holes	Cosmology	Gravitational waves
Group 1	(Epstein 1985)	P	✓	✓	✓	×	×	×	✓	×	×
	(Geroch 1981)	P	✓	✓	✓	×	×	✓	✓	×	×
	(Natário2011)	P	✓	✓	✓	✓	✓	✓	✓	✓	×
	(Orzel 2012)	P	✓	✓	×	✓	×	×	✓	✓	×
	(Schutz 2003)	P	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(Taylor, Wheeler 1992)	P	✓	✓	✓	✓	×	✓	✓	×	×
	(Topper 2013)	P	✓	✓	×	✓	×	×	×	✓	×
	(Wolfson 2003)	P	✓	✓	✓	✓	×	×	✓	✓	×
Group 2	(Cheng 2010)	M	✓	✓	✓	✓	✓	✓	✓	✓	×
	(Cheng 2015)	M	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(Dvořák 1984)	M	×	✓	✓	✓	✓	✓	✓	✓	✓
	(Geroch 2013)	M	×	✓	✓	×	✓	✓	✓	✓	×
	(Grøn, Næss 2011)	M	✓	✓	✓	✓	✓	✓	✓	✓	×
	(Hartle 2007)	M/P	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(Kulhánek 2020)	M	×	✓	✓	✓	✓	✓	✓	✓	✓
	(Lieber 2008)	M	×	✓	✓	✓	✓	✓	×	×	×
	(Steane 2012)	M	✓	✓	✓	✓	✓	✓	✓	×	×
	(Taylor, Wheeler 2000)	P	×	✓	✓	×	✓	✓	✓	✓	×
	(Walecka 2007)	M	×	✓	✓	✓	✓	✓	✓	✓	✓

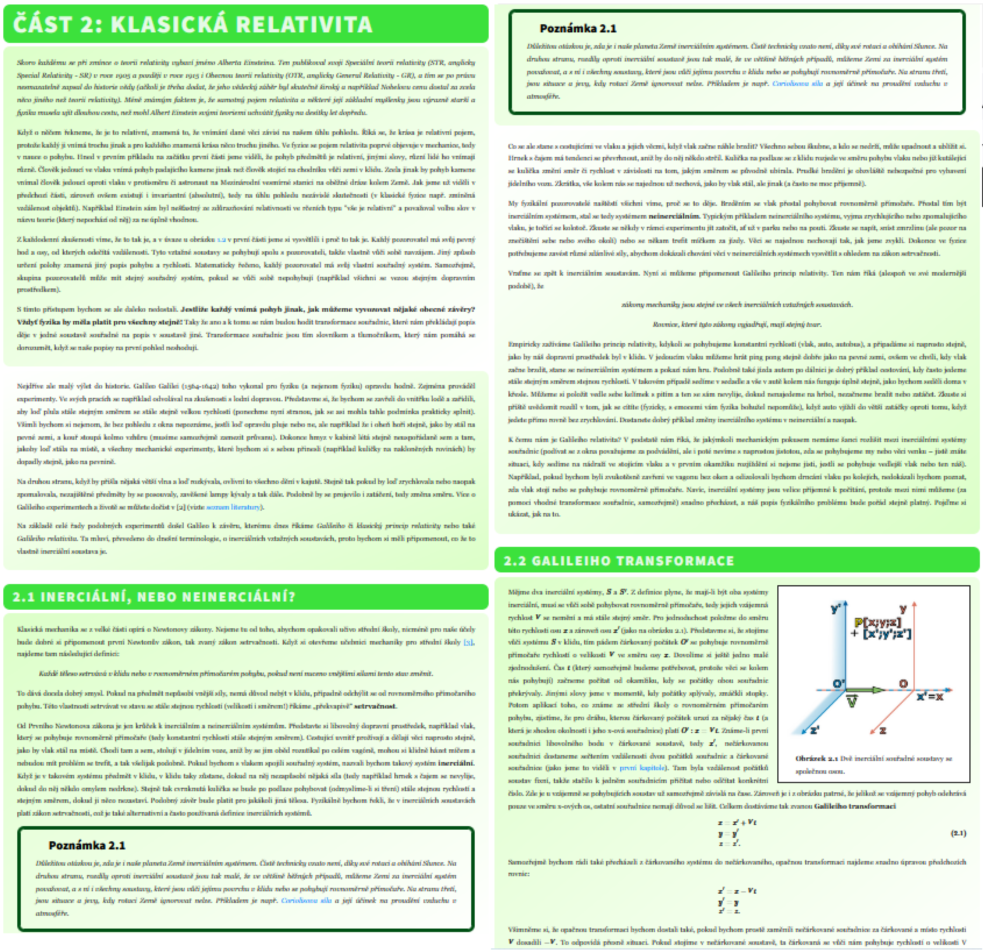
Figure 3.1: Summary of the GR books content analysis. The selected books were divided into two groups according to used mathematics, their overall approach was judged to be either math-first (M) or physics-first (P - see above for the description of these two approaches). Book topics were then split into three groups related to the exposition of GR: introductory topics prior to GR, main GR topics and subsequent chapters on application of GR.

Finally, all the books except (Lieber 2008) included chapters on further astrophysical applications of GR beyond our solar system, most notably *black holes*, *cosmology* and *gravitational waves*. The detail and depth of these chapters varied significantly based on the level of mathematical tools used in a given book. Inclusion of at least some of these chapters is quite common due to their fascinating nature and famous contemporary scientific experiments such as the successful measurement of gravitational waves or the direct observation of black holes.

3.2 Study website

In this section, we present the created study website, its structure and elements that were chosen as a result of the previously described book analysis as well as its review process. The website (Ryston 2022b) is primarily intended for Czech upper secondary students, so it is presently in Czech, but we plan to create also an English version in the future to make it accessible to a wider readership. It can also be downloaded as an offline PDF version, which is identical to the content on the website apart from interactive elements. Instead of applets or videos, there are links to those elements, and instead of animations, still figures are used. When converted to a PDF form, the text currently consists of 182 pages, contains 71 originally made drawings and 6 custom made applets (that will be described more in detail below).

The only education-oriented study on the development of web learning resources we found was (Hadjerrouit 2010), although it does not focus on text-heavy materials, which our website necessarily is. (Hadjerrouit 2010) emphasizes interactivity, the use of multimedia and differentiation of content as the main factors that influence a successful use of web-based study materials and we will address all these elements of our website below; however, the text-heavy nature of our website required further consideration. We took inspiration from websites with popular science articles, such as *sciencenews.org* or *phys.org*. A common way to keep the reader interested in reading (or rather not discourage them) is using short paragraphs to keep the illusion of flow, giving the reader frequent breaks. This is important especially when using mobile devices which have narrower screens and compact the text sideways, making it look even longer. However, this is certainly easier for a news article, but in case of physics study text it might not always be the best choice. One way we tried to balance this issue is that besides chapters and sections, the text is visually divided into smaller portions, usually encompassing one particular idea within the chapter, using frames with diagonally oriented linear colour gradients that are meant to guide the reader gradually from one frame to another (see Figure 3.2). Our aim is to break the long progression of text that can be tedious for the reader especially on a screen, but still emphasize the connection between all the portions, similarly to individual pages in a book.


ČÁST 2: KLASICKÁ RELATIVITA

Poznámka 2.1

2.1 INERCIÁLNÍ, NEBO NEINERCIÁLNÍ?

Poznámka 2.1

2.2 GALILEIHO TRANSFORMACE

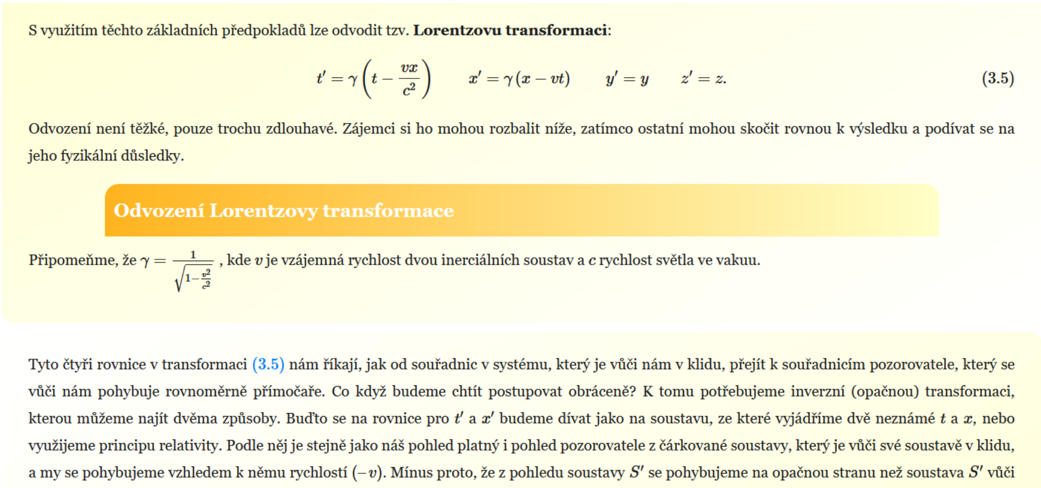
Figure 3.2: A sideways comparison of two adjacent "screens" (with the overlap of the paragraph denoted "Poznámka 2.1" meaning "Sidenote 2.1" framed using a thick dark-green border) showing the text divided not just using numbered sections but also coloured frames, each containing just a few paragraphs related to a single idea.

The text itself is made up of 4 main parts: *The Basics*, *Classical Relativity*, *Special Relativity* and *General Relativity*, with the last part being by far the largest. More specifically, not counting appendices, the four parts have in their PDF forms 13, 10, 30 and 101 pages. The parts are distinguished using a number and different colour schemes. Each part is then divided into chapters, numbered for example 2.1 or 4.3 and so on, and chapters can be divided still into unnumbered sections separated by their headings. Although the main aim of the text is a discussion about GR, just like most authors of the books analyzed in the previous section, we thought it necessary to include the three preceding chapters to make sure every reader was up to speed on all the concepts and topics on the knowledge of which the GR chapter builds and relies. Concepts like *frames of reference* or *coordinate transformations*, not to mention the whole SR, are not part of the FEP for gymnasiums (Balada 2007) and therefore we cannot rely on students being even familiar with them.

Consequently, the requirements on prior knowledge in physics as well as mathematics are quite small. Physics-wise, the reader could benefit from having already gone through basic mechanics such as *linear motion with constant speed or* *constant acceleration, free-fall, Newton's Laws of Motion, etc.* because it is used throughout the text for illustrations and simple examples. It is not, however, the knowledge of definitions and equations in mechanics that is required. Those are all given in the text. It is the familiarity with basic mechanics that might be helpful and contribute to an easier reading. No other physics knowledge is required nor assumed. We therefore consider the text to be suitable for any student who went through upper secondary mechanics (typically the first year of upper secondary gymnasium).

In terms of mathematics, we have already mentioned our intent of creating a study material that is approachable to upper secondary students, yet more technical with concrete formulas than popular physics books. Therefore, equations are used often to quantitatively illustrate the discussed theory. The text expects the reader to have basic mathematical knowledge and skills such as the *Pythagorean Theorem*, *simple equation manipulation and enumeration*, *trigonometric functions*, *powers*, *square roots* and the *scientific notation for numbers.* All the other mathematics is gradually built up in the text. Examples of that include *coordinates, coordinate transformations* and most notably *basics of differential geometry.* The text contains many derivations of formulas or general proofs; however, these tend to be more mathematically involved and we did not want to impair the readability of the text, so the website version contains roll-outs that reveal for example the given derivation if the reader chooses to see it. An example is shown in Figure 3.3. This creates basically two layers of reading difficulty from a mathematical perspective, the main body of the text for the less (but still somewhat) mathematically inclined and the additional mathematical content for the more so. We were inspired by a similar use of roll-outs in the *Collection of Solved Problems in Physics* (Department of Physics Education 2016). We think using these roll-outs makes their content more immediately accessible rather than putting it in an appendix somewhere else (although this solution had to be used for the offline version).

The first three main parts each end with a recapitulation of the main points of the part and some sample exercises for the reader to try. The fourth and largest part has recapitulation and exercises after every chapter. Furthermore, each of the first three parts is located on its own webpage. The fourth part is split over several pages to reduce the loading time, which otherwise would be due to the number of used illustrations, applets and equations quite significant.



S využitím těchto základních předpokladů lze odvodit tzv. **Lorentzovu transformaci**:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad x' = \gamma(x - vt) \qquad y' = y \qquad z' = z. \tag{3.5}$$

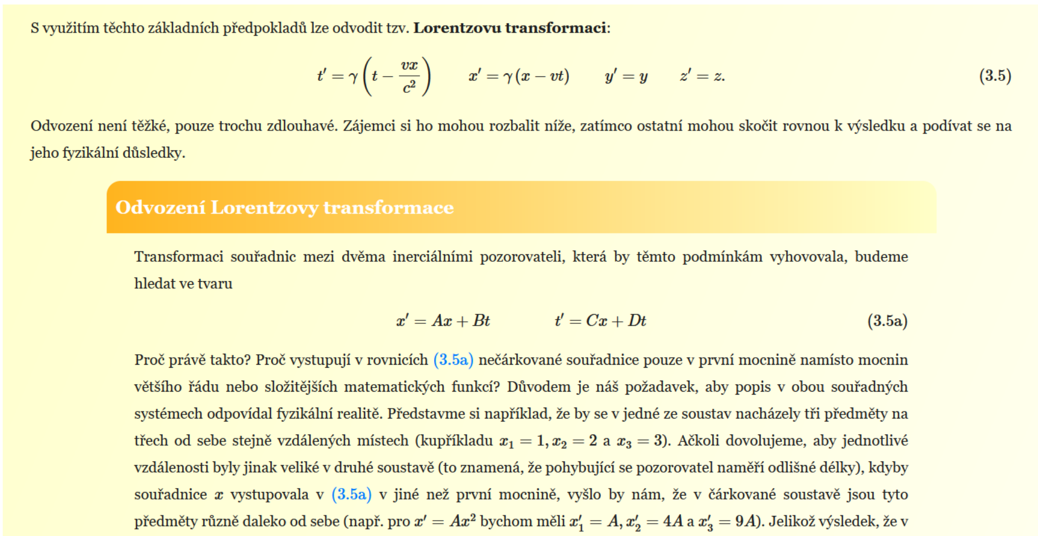
Odvození není těžké, pouze trochu zdlouhavé. Zájemci si ho mohou rozbalit níže, zatímco ostatní mohou skočit rovnou k výsledku a podívat se na jeho fyzikální důsledky.

Odvození Lorentzovy transformace

Připomeňme, že $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, kde v je vzájemná rychlost dvou inerciálních soustav a c rychlost světla ve vakuu.

Tyto čtyři rovnice v transformaci (3.5) nám říkají, jak od souřadnic v systému, který je vůči nám v klidu, přejít k souřadnicím pozorovatele, který se vůči nám pohybuje rovnoměrně přímočaře. Co když budeme chtít postupovat obráceně? K tomu potřebujeme inverzní (opačnou) transformaci, kterou můžeme najít dvěma způsoby. Buďto se na rovnice pro t' a x' budeme dívat jako na soustavu, ze které vyjádříme dvě neznámé t a x, nebo využijeme principu relativity. Podle něj je stejně jako náš pohled platný i pohled pozorovatele z čárkované soustavy, který je vůči své soustavě v klidu, a my se pohybujeme vzhledem k němu rychlostí $(-v)$. Mínus proto, že z pohledu soustavy S' se pohybujeme na opačnou stranu než soustava S' vůči

(a)



S využitím těchto základních předpokladů lze odvodit tzv. **Lorentzovu transformaci**:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad x' = \gamma(x - vt) \qquad y' = y \qquad z' = z. \tag{3.5}$$

Odvození není těžké, pouze trochu zdlouhavé. Zájemci si ho mohou rozbalit níže, zatímco ostatní mohou skočit rovnou k výsledku a podívat se na jeho fyzikální důsledky.

Odvození Lorentzovy transformace

Transformaci souřadnic mezi dvěma inerciálními pozorovateli, která by těmto podmínkám vyhovovala, budeme hledat ve tvaru

$$x' = Ax + Bt \qquad t' = Cx + Dt \tag{3.5a}$$

Proč právě takto? Proč vystupují v rovnicích (3.5a) nečárkované souřadnice pouze v první mocnině namísto mocnin většího řádu nebo složitějších matematických funkcí? Důvodem je náš požadavek, aby popis v obou souřadných systémech odpovídal fyzikální realitě. Představme si například, že by se v jedné ze soustav nacházely tři předměty na třech od sebe stejně vzdálených místech (kupříkladu $x_1 = 1, x_2 = 2$ a $x_3 = 3$). Ačkoli dovolujeme, aby jednotlivé vzdálenosti byly jinak veliké v druhé soustavě (to znamená, že pohybující se pozorovatel naměří odlišné délky), kdyby souřadnice x vystupovala v (3.5a) v jiné než první mocnině, vyšlo by nám, že v čárkované soustavě jsou tyto předměty různě daleko od sebe (např. pro $x' = Ax^2$ bychom měli $x'_1 = A, x'_2 = 4A$ a $x'_3 = 9A$). Jelikož výsledek, že v

(b)

Figure 3.3: Example of a roll-out for the derivation of the Lorentz transformation in the SR chapter. Figure **(a)** shows the section closed and figure **(b)** its beginning after being clicked on.

3.2.1 Detailed description of the website content

Part One: The Basics

The main purpose of this part is drawing some basic points about the nature of time and distance measurement, the use of reference frames and Cartesian coordinate systems in three dimensions. A very elementary coordinate transformation between two shifted but stationary frames of reference is gradually constructed to help readers get used to such a concept. The setup of two friends surveying a garden in two different ways was taken and slightly adapted from (Taylor and Wheeler 1992). The most important result of this chapter is that even though two points can be labeled with different coordinates, the physical reality of their distance is coordinate-independent or invariant. Also, a Pythagorean Theorem in three dimensions is introduced.

Part Two: Classical Relativity

The second part adds motion to the mix. We derive the Galilean transformation for two inertial frames moving with respect to each other and consequently the classical rule of velocity addition (in one dimension). The notion of invariance of the Newton's Law of Motion and therefore the whole of mechanics under the Galilean transformation is discussed.

Another topic that will be important later are inertial and non-inertial frames of reference and the related notion of fictitious forces. It is worth spending some time with these topics here because, similar to other already mentioned concepts useful for the discussion of GR, it is not at all likely that the students encountered such notions before. The issue is not part of the gymnasium FEP (Balada 2007) and even though it can be still found in the most commonly used gymnasium textbook for mechanics (Svoboda et al. 2020), its inclusion in physics lessons is not guaranteed.

Part Three: Special Relativity

Because our primary goal is the GR chapter, we did not want to spend too much time and effort with SR, only the required minimum necessary for the GR discussion. We have therefore chosen a quite common math-first approach of deriving the Lorentz transformation from the two postulates of SR and then "discovering" the well-known consequences of the transformation: *time dilation*, *length contraction*, *relativistic velocity addition* and so on. Other known SR results such as the *energy-mass equation* were not included to keep the part short; however, to give validity to our theoretical claims, real life experiments verifying the existence of such relativistic phenomena are discussed.

The most important concept of this part is *spacetime*. We show, similarly to the first part, that even though two inertial observers might not agree about the position and time of a given pair of events, they do agree on the spacetime interval between. A connection is drawn between the three-dimensional distance in space, whose invariance under simple coordinate transformations we saw in the first part, and the invariance of a spacetime interval under the Lorentz transformation. The interplay between time and space and the practicality of defining spacetime is emphasized.

A chapter on superluminal motion is included for two reasons. To talk more about causal structure of spacetime, which also comes into play in GR. To include a topic that is perhaps more popular in nature then the previous technical chapters, in order to give the reader a chance to ”rest” before entering the GR discussion. And at the same time to answer a question about the possibility or rather impossibility of actual superluminal motion that arises quite naturally from the SR chapter. Indeed, in our experience, it very often comes from students during teaching SR in gymnasium.

Part Four: General Relativity

With the GR part being the longest, we will describe individual chapters. Chapter 4.1 deals with basic qualitative discussion, starting from the Newton's Law of Gravitation and its incompatibility with SR. Non-inertial frames from Chapter 2 are then supplemented with the notion of fictitious forces. The weak equivalence principle is discussed and used to present the idea of local inertial frames. As we saw in the book analysis, the equivalence principle (or rather equivalence principles, as there are multiple versions) is present in most of the literature. Therefore, rather than invent some novel approach, we chose to follow this quite traditional one but tried to spend sufficient time on all the necessary logical steps and theoretical constructions, because we felt that in case of some of the analyzed books, not enough time was devoted to these crucial ideas. Moving ahead, a parallel is drawn between a gravitational force and fictitious forces due to both being directly proportional to the mass of a body on which they are acting. We finish this chapter with a historical side note, likening our ”current” situation to the Einstein's, who allegedly needed to apply the approach of differential geometry in order to progress with his theory.

Chapter 4.2 deals almost entirely with geometry. Recall that in the book analysis we found two approaches to the introduction of geometry in GR. We adopted the ”geometrical intermezzo” option, because we consider it to be more in line with the physics-first approach. This way, chapter 4.1 provides some initial physical considerations as well as a little motivation as to why we are now spending time talking about geometry. This chapter is basically divided into three thematic parts. The first part deals with the idea of curvature and non-Euclidean geometry qualitatively. We will not go into detail now because the content of this part is almost identical to the workshop on non-Euclidean geometry described in detail in Chapter 4. Suffice it to say, the goal of this part is to make reader aware of non-Euclidean geometry using curved surfaces and show that the Euclidean geometry we are taught at schools is a special case and many of its claims, such as that the sum of internal angles of a triangle is equal to 180°, do not hold true for example on a sphere. A sphere is a main example used in this chapter because students are quite familiar with the shape; however, other shapes, such as a hyperbolic paraboloid are used as well. Finally, we arrive at the important concept of a *geodesic* using a practical definition as the straightest possible connection between two points on a curved surface rather than a rigorous mathematical definition.

The geometric exposition is then paused for a moment to draw further connection and parallels between geometry and gravitation. We thought it necessary after a long passage focused on non-Euclidean geometry to remind the reader of our endeavor to come up with a relativistic description of gravity and perhaps rekindle their motivation before the upcoming quantitative part. We show that, at least qualitatively, the behaviour of objects in a gravitational field can be likened to the motion on curved surfaces, sowing the seeds of the geometrical approach to gravity of GR. Moving on to the quantitative part, we revisit surface curvature but start involving mathematics. Our aim here is the concept of a metric and here also lies the greatest challenge. If we want to let the students have a look "under the hood" of GR, to really give them the opportunity to see even a glimpse of the inner workings of the theory, we need to do some calculations. Using the full mathematical apparatus of GR with calculus and differential geometry is, of course, out of the question for upper secondary students. We have therefore implemented a similar approach to (Schutz 2003 and Natário 2011) who bypass the mathematically rigorous use of differentials by introducing the concept of "close points" as a mathematical model where the distance formula between two points on, for example, a sphere can be significantly simplified when we neglect terms of higher order. We use the term "sufficiently close points" to emphasize the limit-like nature of this approach, again using a practical and operational definition rather than mathematical rigor. Two points are sufficiently close to each other, if the error we make by treating the curved distances as straight lines is below the precision of our measurement. We also adopt the differential notation, where instead of, for example, a finite distance Δx we use $\mathrm{d}x$ for sufficiently close points. This way, the used mathematical expressions look the same as in rigorous textbooks. We thus obtain the metric for a sphere and from this point onwards the concept of a metric is at the centre of our attention. We use two-dimensional Cartesian coordinates and polar coordinates to show that two different metrics can describe the same surface (further supporting the notion that coordinates do not correspond to physical reality but merely help us work with it) and we show that non-diagonal (or rather mixed terms, because we do not work with matrix notation) in the metric arise when the coordinates are not orthogonal. All of our claims are supported with mathematical derivations hidden in the roll-outs. Many of the derivations here and further require calculus or other more sophisticated mathematical tools. In such a case, the roll-out always states what mathematics is required to understand it. The reader is also reminded that this extra mathematical content is strictly optional and one should not be discouraged if they do not yet posses the mathematical tools to understand it. On the other hand, readers who happen to already be familiar with for example simple derivatives (which might be the case of some fourth-year upper secondary students, for example) could benefit from this additional mathematical insight. We think this differentiation of content makes the text more flexible for the reader.

Finally, we try to lead the reader through the conceptually difficult but necessary generalization of the discussed description of surface curvature to the curvature of space (three dimensions) and even spacetime (four dimensions). We stress that trying to imagine such things as curved space is extremely difficult and one should not risk discouragement doing it. Rather we let the workhorse of mathematics carry this burden for us. We just simply have to allow our equations to count to more than two (coordinates). We also remind the reader, that in Part 3 we have already encountered the metric of flat spacetime, the spacetime interval.

The final chapter 4.3 is devoted to Schwarzschild spacetime. In adherence to the physics-first approach, the Einstein equations are just very briefly discussed and the Schwarzschild solution is presented without derivation. Furthermore, due to the spherical symmetry of the situation and the consequential planar motion of any body moving solely under the influence of gravity, we focus only on the equatorial plane of the spacetime, reducing the situation from four to three dimensions (one temporal, two spatial) and thus also reducing its abstractness without any loss of information. Such an approach can be found for example in (Taylor and Wheeler 2000). A lot of time and effort is spent on understanding the meaning of used Schwarzschild coordinates (t, r, φ). To visualize and understand more closely the spatial curvature hidden in the metric, we introduce the *embedding diagram* of the Schwarzschild spacetime, which results in the shape called the *Flamm's paraboloid*. Students might be familiar with the shape (or something similar) from many existing depictions related to GR, spacetime curvature, etc. (Ryston 2019a). We consider it important to include it in our text, because students might also come across a demonstration of throwing marbles on a deformed elastic sheet, which is directly related to the problematic of the embedding diagram and yet there are various conceptual problems with the demonstration that the student should be aware of. We use the paraboloid (either as a real 3D-printed surface or as a part of an applet - see below in the next subsection) to demonstrate how curved space curves the trajectories of objects but we also show how the description of spatial curvature is not itself sufficient because it does not, for example, explain simple free fall and cannot produce closed orbits. This line of reasoning is also used in the relativistic workshop described in Chapter 4. To improve our description, we need to add the time component, leading us first to the gravitational time dilation. We use a real life experiment, the Hafele-Keating experiment, to prove the existence of gravitational time dilation and then use our knowledge of the Schwarzschild metric to obtain formulas that allow us to calculate time dilation for a satellite in the Galileo navigational satellite system. The calculation is a direct example of how relativistic corrections need to be used in the global navigation systems for them to be accurate.

The rest of the chapter is focused on astrophysical applications of the Schwarzschild metric. We go through the so-called "classical" experiments of GR - the perihelion shift of Mercury, bending of light (and the consequential phenomenon of gravitational lenses), already mentioned gravitational time dilation (or red shift) and the Shapiro effect. We state the formulas for the perihelion shift and bending of light without derivation from the Schwarzschild metric, because we consider solving differential equations to be above even the advanced level of mathematics we have aimed for. Instead, we use again animations (perihelion shift) and direct numerical solutions (light bending) to support the reader's understanding. We also supplement the theory with historical experiments, such as the famous Eddington experiment of 1919, with plenty of links to further reading for the interested reader, to help ground the theory in reality.

The second to last chapter is devoted to black holes, which is arguably the most mysterious and fascinating result of GR that is commonly known. We saw in the book analysis that almost all the books included a chapter on black holes, and we felt, similarly to the chapter on superluminal speeds, that we should address this issue, especially because it almost immediately arises when an even slightly mathematically trained eye looks at the Schwarzschild metric and the apparent singular behaviour of the r coordinate and also because of the recent well-received photographs of black holes by the *Event Horizon Telescope* project.

The final brief chapter comments on the relationship between the Newtonian picture of gravity and the relativistic one. Also, it is meant to "repair" the reputation of the Newtonian gravity, because we have just basically spend over a hundred pages trying to persuade the reader that the relativistic approach is better (in the sense that it agrees with experiment in more general situations). The reader might then object why do we spend time learning about the classical approach in schools, if we know it to be the inferior one. We therefore felt the need to comment on the issue and stress the practicality of using Newtonian gravity where it suffices.

We could have of course included further chapters - cosmology, gravitational waves, etc. as we have seen them in the analyzed books. However, we decided that for now the text is lengthy enough. In the future, after the publishing of the text and if we see that the text is indeed being used, we will probably add further chapters, not just to the GR part, but also SR or even the previous ones.

3.2.2 Interactive elements

A great advantage of the online version of the text are interactive elements. The use of an internet browser as an environment enables us to use not just illustrations, but also animations or videos, embed external videos (for example from Youtube) and include direct links to other sources of information. Most important are the applets, i.e. interactive animations that enable to set up or change the parameters of the given physical situation or a mathematical demonstration. We have created six applets (described below) to accompany the text using the programming library Visual Python (Ryston 2019b). All the applets are embedded in the structure of the website itself, so no additional software or add-on except the already used browser is necessary. Sample screenshots of the applets (without the interactive elements) are shown in Figure 3.4.

Applets used in order of appearance in the text:

- **Lines of Latitude on a Globe** shows that lines of latitude are not generally (except the equator) geodesics. After the user's click, the program chooses two random points on a globe with the same latitude and connects them using the corresponding segment of the line of latitude as well as the corresponding geodesic (a great circle intersecting both points). The lengths of both segments are calculated to add a concrete numerical result to the visual presentation.

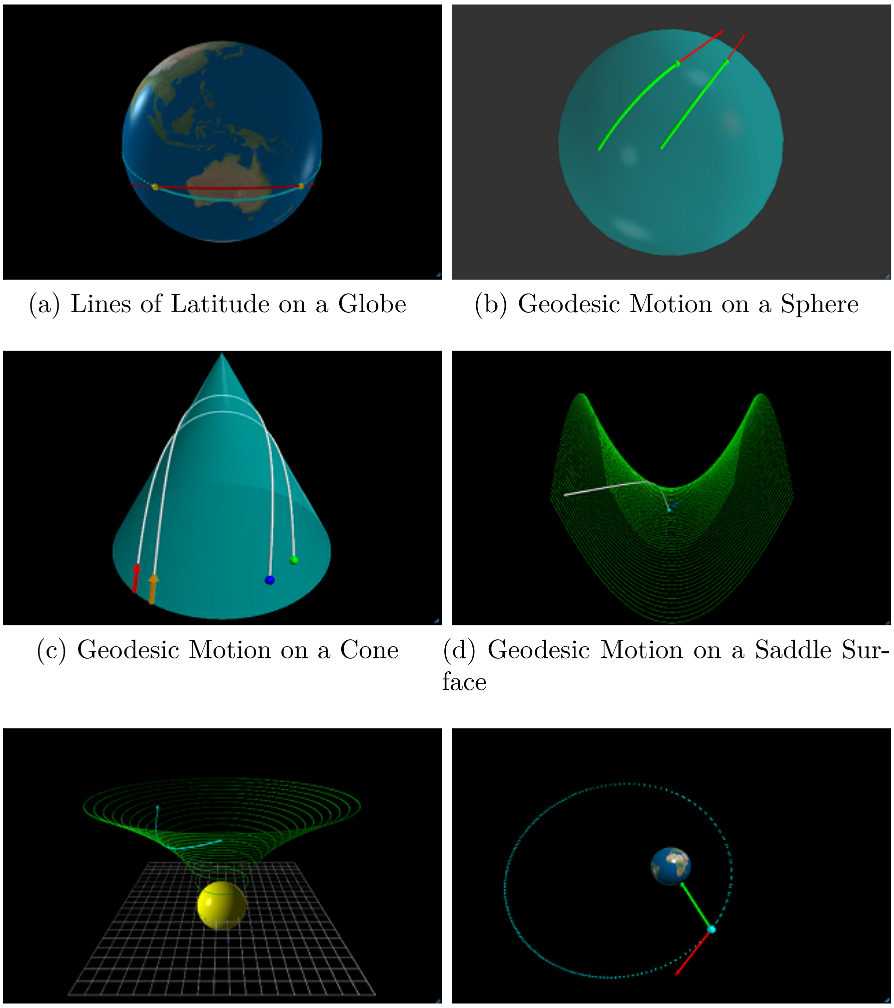
- **Geodesic Motion on a Sphere** contains a few modes of operation showcasing spherical geometry. It allows users to send small individual balls into a geodesic movement on a sphere, as well as a pair of balls on initially approximately parallel trajectories and we see them eventually cross. It also allows the user to select two points on the sphere and then calculates the great circle connecting them (a segment of which is the geodesic between the two points). The final mode does the same thing but with three points, effectively creating a triangle on the sphere, so that we can, for example, observe its internal angles.

- **Geodesic Motion on a Cone** allows to send particles (small balls) on the surface of a cone. The initial position and velocity of the particles can be modified. The applet demonstrates that the curvature of the cone surface is flat everywhere except the singular point at the top. When two particles are sent initially parallel but each on one side from the top point, their trajectories cross. When both are sent on the same side, their trajectories keep the same distance throughout the motion.
- **Geodesic Motion on a Saddle Surface** is similar to the previous applet but the motion takes place on a saddle surface (hyperbolic paraboloid). A particle can be send along the surface visualizing the geodetic movement.
- **Flamm's Paraboloid** lets users send particles on the surface of the Flamm's paraboloid. Parameters such as the initial position and curvature of the paraboloid can be modified. The motion can be observed in 3D but the scene can also be rotated to give the view from above, which enforces the desired perspective of watching a planar motion. A planar projection of the particle and its trajectory can also be switched on to further help this perception.
- **Kepler's Problem** is a simple movement of a particle in a central gravitational field calculated using the classical law of gravitation.

3.2.3 Review of the website

The website content was reviewed and commented on by RNDr. Otakar Svítek, Ph.D. from the Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University as a representative of experts on GR, by Mgr. Martin Malachov as a representative of gymnasium physics teachers, and also by doc. RNDr. Leoš Dvořák, CSc., who is a supervisor of this thesis and an author of one of the books analyzed in section 3.1 (Dvořák 1984). Their comments were most helpful from both technical and educational points and were worked into the text. We thank all three gentlemen for their time and effort. Any possibly prevailing imperfections in the text are to be blamed solely on the author of this work.

Dr. Svítek and Mr. Malachov were also asked to summarize their opinion on the created website. Dr. Svítek wrote: *"The prepared online study tool will provide highly valuable resource to users with high-school level background in mathematics and physics who are looking for introduction into basic general relativistic concepts and fundamental solutions of Einstein equations. The goal is accomplished by first reviewing those parts of classical physics that are later*



(a) Lines of Latitude on a Globe

(b) Geodesic Motion on a Sphere

(c) Geodesic Motion on a Cone

(d) Geodesic Motion on a Saddle Surface

(e) Flamm's Paraboloid

(f) Kepler's Problem

Figure 3.4: Illustrative images from the used applets.

important in building relativistic grasp of the world, like Galilean transformation. Necessarily, this is followed by an extensive exposition of special relativity focusing on effects of Lorentz transformations and, most importantly, on a detailed explanation of the geometry of Minkowski spacetime. The curved geometry is introduced using the surface of a sphere as a simple example, the equivalence principle is explained and an understanding of metric structure is gradually and intuitively built. Finally, Schwarzschild solution of a black hole is analyzed and the main effects of general relativity laid out.

The writing style is very engaging and the text is enriched by plentiful illustrations. Mathematical and physical concepts are carefully motivated and analyzed in detail. Mathematically more involved topics are cleverly hidden for those not wishing to delve into them. The web format allowed the inclusion of numerous external links to resources extending the coverage of a given topic and also makes the work "live" since it provides easy opportunity for future modifications and additions. The webpage is technically very well prepared.

Thus this extensive study tool will surely help any interested reader to better understand one of the most important development of the 20th century physics and might prove extremely useful for high-school teachers."

Mr. Malachov wrote: *"The webpage materials encompassing basic ideas of general relativity form a novel approach thanks to particular didactical transformation of the topic. Materials are multimedial and well-structured so they can be used for motivation and learning a basic overview (high school student, keen amateur) as well as for a deeper understanding of the topic. I highly appreciate that the materials are suitable for the beginner as well as for the university student thanks to the voluntary calculus exercises etc. too. The materials balance very well on the edge between mathematical formalism, physical (experimental) approach and do so in a comprehensible manner without loss of physical/mathematical focus. Some parts of the materials (namely about non-Euclidean geometry) fill the gap between the secondary and tertiary education. This valuable feature serves as a good motivation, propaedeutic and as a support for university students who find standard formal literature too difficult. Another positive aspect of the materials is the language which is very friendly, readable with specific sense of humour. To conclude, the materials are professionally elaborated yet easy to understand. The novel approach is successfully developed. I find the materials very attractive, detailed yet simple and I highly recommend them to all teachers, students and laic fans of general relativity. I would recommend to propagate the materials over schools and consider using various web channels and social media to spread them. Personally, I will be looking forward for next extension of the materials, e.g. with gravitational waves etc."*

We also looked for volunteers among upper secondary students to read through the website. A group of students attending a physics seminar at the Gymnasium Nad Štolou, where the author of this thesis taught physics at the time, showed interest in reading through the website in 2019. They were asked to comment on anything that they find difficult to understand, confusing or if they miss some piece of information in the text, if the illustrations are clear, basically to report any instance of having difficulty with reading the text or using the website. Their comments were mostly minor, they pointed out a few technical issues with the website, picked up on misspellings but overall had very few comments regarding the content of the website. We theorize that when they encountered some minor obstacle in understanding, they looked for the "fault" within themselves and not the text itself, but this theory comes solely from personal observation of students. We do think that any major problem with understanding would be reported by the students if they encountered it. Unfortunately, the COVID-19 pandemic and the consequential lockdown has caused a problem in getting further feedback from the students. Quite understandably, because they had to spend so much time during lockdown in front of their screens and most likely had more work due to online teaching, all of them stopped reading the text and in the essentially two years of school lockdown, they finished school and left for universities, where they were likely very busy with their new studies, so our contact will all of them stopped. Furthermore, we were unable to find volunteers this school year of 2021/2022 even among the visitors of our relativity workshop, which is described in Chapter 4. Consequently, we received no student feedback for chapters 4.2 and 4.3, i.e. the longest and most important ones. Nevertheless, due to the revisions by the two experts and the teacher mentioned above, we feel confident about the content of those chapters. The website contains an email address for the readers to send any suggestion regarding the improvement of the content of the technical solution of the website.

The finished website will be presented at a physics teacher conference at the end of August 2022 to spread the awareness of its existence among physics teachers. We also plan send an information email to the participants of our online survey described in Chapter 2 at the start of the following school year. Online tools such as *Google Analytics* will be used to monitor the amount of visitors to the website, to gauge whether it is actually being used.

4. General Relativity Workshop for Secondary Students

In addition to the study website described in the previous chapter that is meant as an extra-curricular source of information for students interested in GR, we have also prepared a teaching learning sequence, which we call a ”GR workshop”. Teacher answers to our survey in Chapter 2 showed that a non-trivial fraction of teachers consider GR to be an interesting topic that could and should be included in upper secondary education at least as a part of a physics seminar. On the other hand, there are obvious obstacles to this inclusion, as we saw in Figures 2.16 and 2.17. From the standpoint of this thesis, there is not much we can do with the most frequently mentioned lack of time other than reflect it in the design process (see below). Regarding the other obstacles, the ”traditional” viewpoint that GR is too complicated for secondary schools is actually challenged by the very design of this workshop as well as most of the research studies mentioned in Chapter 1. The last two main reasons were doubts whether the teachers’ own knowledge of GR is sufficient enough to teach even the basics of this topic and insufficient teaching materials. The study website could help mitigate these reasons, as it can be used also by teachers (or anyone with basic upper secondary knowledge of mathematics and physics) interested in improving their knowledge of GR and it can also be an inspiration on how to approach teaching the topic. However, it is commonly known (and our own experience with teaching secondary physics confirms this) that teachers are quite busy with their everyday duties, so we can hardly expect most teachers to study GR on their own and at the same time work on how to incorporate it in their teaching. We therefore decided to facilitate the possible inclusion of GR in secondary physics by developing and testing activities and learning sequences that can be readily used in the classroom.

Even before starting the design process itself, we have set a few guiding principles that we wanted the workshop to adhere to:

Depth over breadth: As we mentioned, an obvious conclusion from the teacher survey in Chapter 2 was that one of the largest obstacles to introducing topics that are not part of the FEP is a lack of time. Individual open answers from teachers confirmed that the numbers of physics lessons have been decreasing at schools and teachers have to carefully consider which topics to include and to what extent. Therefore, our proposed learning sequence can hardly be, for example, 10 lessons long. We need to be economic with time and focus only on the most important aspects of GR. After going through the literature search described in Chapter 1, we considered the most distilled down basic message of GR to be that *Gravity is spacetime curvature*. We chose this sentence as the main theme of our workshop and its understanding to be our primary goal.

Hands-on and practical, but guided: GR is by its nature a very abstract and mathematically convoluted theory but just like with the study website, we wanted to find ways to make it engaging and understandable for students. We consider the best way to achieve that is, as in all of physics education, to use practical hands-on activities where possible, to have students, for example, calculate something, generally speaking to let them be part of the process. That is where the word workshop comes from. On the other hand, there is the already mentioned issue of time. We considered handing out worksheets and let students be more in charge of the activities; however, most of what we wished to discuss with students in this workshop, for example non-Euclidean geometry, turned out to be usually completely new to them which significantly increases the time they need to orient themselves in the activities without direct guidance (unlike for example when learning about the physics of motion, where students can lean on some prior experience). As we shall see, the final proposed version of the workshop is a series of guided activities with some lecturing parts in between to keep students apprised on what is going on, why we are doing what we are doing as well as some interesting facts to enliven the workshop.

Flexibility: It follows from the previously said that the situation of every physics teacher regarding teaching of relativity at their school can be different. Some might organize the workshop as a one-time afternoon event (which is a model that worked best for us), some might have the option to spend a few physics lessons on GR following the topic of SR, or even without it just because students expressed interest in GR. We felt the need to reflect this possible intricacy by making the workshop design flexible. Instead of a rigid structure, it is comprised of a series of activities and smaller discussions held together by the overarching scheme. Not all of the proposed activities have to be included (although some are essential and should not be omitted). Most notably, in the largest section on curvature and non-Euclidean geometry, there are purposefully various activities showing essentially the same ideas. If possible, we try to include all of them, because multiple representation of the same situation are beneficial to learning, especially in groups (Ainsworth 2006), but if there are tighter time constrains, some of the activities can be left out or suggested to students to be tried at home. A similar argument can be said for the part about gravitational time dilation, where the discussion about the Hafele-Keating experiment can be cut short if necessary. The following calculation regarding satellite navigation can also be approached in different ways. Either students look for necessary values themselves or we supply them, students can do all of the necessary calculations or just the main one, etc. To give concrete examples, we have done the workshop in a narrowed-down version for other physics teachers in 90 minutes (which admittedly limits especially the hands-on component) and we also once spent 3.5 hours (small breaks included) with it. In our opinion, the ideal time lies somewhere between at 3-4 standard 45-minute lessons time.

No prior knowledge of SR required: We did not originally have this guideline in mind, but it was added soon after the first trial and it became equally important in the workshop design as the previous ones. GR is commonly taught after SR. As we have seen in the book analysis in Chapter 3, practically all the books include at least some basic review of SR that is necessary for the later GR discussion and we have done the same in the previously described study website. After all, the concept of spacetime, that appears in our *Gravity is spacetime curvature* theme, is naturally build-up in SR. However, such a prerequisite severely limits the possible audience of the workshop. As we discussed in Chapter 2, if SR is taught in upper secondary physics, it is usually taught in the last year or in a physics seminar. Moreover, with the recent cutback in the number of regular physics lessons at the gymnasium where the author of this work taught at the time and the consequential moving of some topics from the regular lessons to the physics seminar, we were hard-pressed to find the necessary extra time to implement the workshop. We therefore decided to make the workshop a one-time afternoon event offered for those students interested in GR and in this form the workshop has been implemented ever since (although as we have mentioned, it can be easily reproduced as a short series of traditional lessons). Then the problem of previous SR knowledge arose. We knew we did not want to spend additional time of the workshop on basics of SR. On the other hand, relying on prior knowledge of SR from the audience would mean that the workshop would be suitable only for the fourth-year students, severely limiting the number of possible attendees. We therefore decided to challenge the notion that GR cannot be discussed without prior knowledge of SR and open the seminar to all upper secondary students[1]. As we will see in the description of the workshop structure, we try to motivate the necessity of the notion of spacetime for relativistic description of gravity in a different way. Lastly, we should note that this does not mean SR references cannot be part of the workshop. If the students have gone through basic introduction of SR beforehand, they would surely benefit from tying these two theories together. In our proposed version of the workshop, SR is just not required, and it is up to the teacher to amend the structure according to the particular situation that applies to them and their students.

When developing the workshop, we chose the *design research approach* (Bakker 2018) where an educational design is tested and improved using multiple cycles. The first try took place in June 2016 and since then the seminar has taken place every year (with the exception of the 2019/2020 and 2020/2021 school years due to the Covid-19 lockdown) altogether 6 times at four different gymnasiums, usually with students attending the physics seminar (typically third- and fourth-year students of upper secondary, so between 17 and 19 years old) joined by other volunteers of mixed age. Each group consisted of approximately 15 to 20 participants. A shortened version was also tried altogether 3 times with physics teachers at two different physics teacher conferences. Once with a large group of

[1] Actually, over the years we had also a few lower secondary students in attendance (an 8-year gymnasium covers both lower and upper secondary education). They seemed to do well during the workshop and their feedback, that we gather at the end of the workshop, was mostly positive. Yet, they admitted that some of the discussed concepts, such as gravitational time dilation, was too abstract for them. We can hardly blame them for that, when some of these concepts can be too abstract even for some adults. We still present the workshop as suggested for upper secondary students, but all interested students are welcome. Nevertheless, their attendance and mostly positive reception of the workshop seems to indicate some agreement with the already mentioned studies on the possibility of teaching Einsteinian physics to younger students (Kaur et al. 2020).

about 30 teachers at the ”Elixír do škol” 2019 conference, the other two times with smaller groups of 6-8 participants at the ”Heureka Workshops” 2019 conference. Initially, the design would be improved after every instance based on the overall course of the workshop, participant reactions and most importantly their feedback, that was taken after each workshop. We will discuss the feedback in detail at the end of this chapter because it relates to the content of the workshop. The last two instances, however, were practically identical due to positive feedback from participants, which resulted in our confidence in the current workshop design.

The workshop design has been already published as a series of four Czech articles in a semi-popular *Czechoslovakian Magazine for Physics* (Ryston 2020a, Ryston 2020b, Ryston 2020c and Ryston 2020d) and in a shortened English version as a chapter in an already mentioned international publication *Teaching Einsteinian Physics in Schools: An Essential Guide for Teachers in Training and Practice* (Ryston 2021). It will also be featured as part of the study website described in Chapter 3, so that all the resources are at one place. As we have already mentioned, the website (and consequently the offered workshop design) will be presented at a physics teacher conference in August 2022.

4.1 Workshop design

At the start of each workshop, we present our goal to be understanding the sentence *Gravity is spacetime curvature*. That basically means that we need to understand the last two words, *spacetime* and *curvature*, and also what they mean together. As we mentioned, spacetime is a concept already developed in SR and if students are familiar with it, we can use that familiarity and remind them about some relevant basics of the theory, especially the concept of spacetime (however, that is not the approach that we take here). The basic scheme of the workshop is depicted in Figure 4.1. Even with some prior notion of spacetime, we can hardly imagine such a thing as curved spacetime. We therefore need to start with something simpler. Taking away the temporal dimension doesn't help much, because our brains are not equipped to visualize curved space. Simplifying once more, we arrive at surface curvature. That is something we can actually work with. We then start with the first and largest section of the workshop - non-Euclidean geometry.

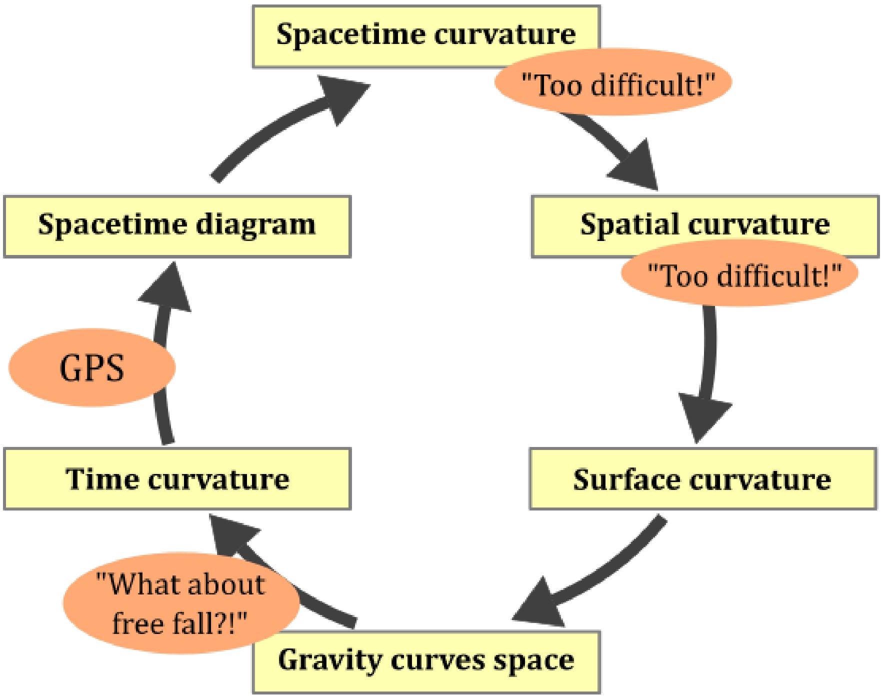


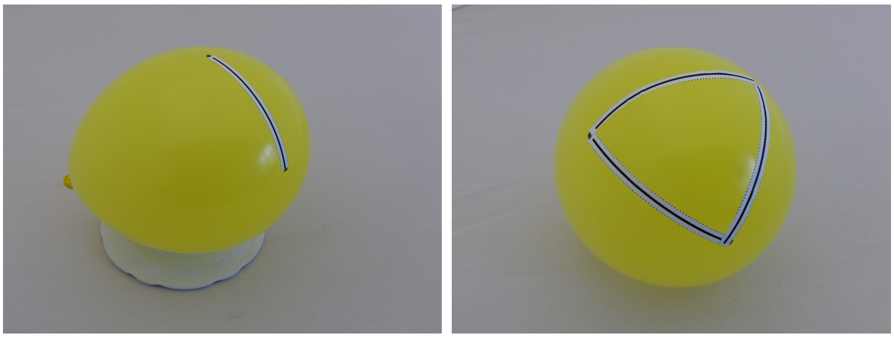
Figure 4.1: Diagram of the overall scheme of the workshop. We start at the top and continue clockwise through less complicated concepts eventually coming full circle back to spacetime curvature.

4.1.1 Non-Euclidean geometry

In this part of the workshop, we use hands-on activities to introduce students to the concepts of *surface curvature* and especially *geodesic*. Because non-Euclidean geometry is almost universally something completely new to the students, we start with something familiar, a few well-known geometrical theorems valid in the Euclidean space (this approach as well as some other parts of the workshop are also used on the study website), namely:

- The shortest connection of two points is a line segment.
- The sum of all internal angles of a triangle is equal to 180°.
- Two parallel lines never meet nor change their respective distance.
- The relationship between the circumference c of a circle and its radius r is $c = 2\pi r$.

We then proceed to test these theorems on a surface of a sphere. Students are given balloons (as a cheap, easily transportable and replaceable alternative to balls or other round objects) and markers. They are then asked to make two points on the balloon and find their shortest connection. To help them, we supply also long thin pieces of paper, which can be used as a guide for drawing the shortest connection. It is easily seen that on a sphere the shortest connection between two points is a part of a circle. Students are then asked to try out the rest of the theorems. They create triangles (connecting three points using the shortest paths) and can easily verify that the sum of internal angles is always larger than



(a) (b)

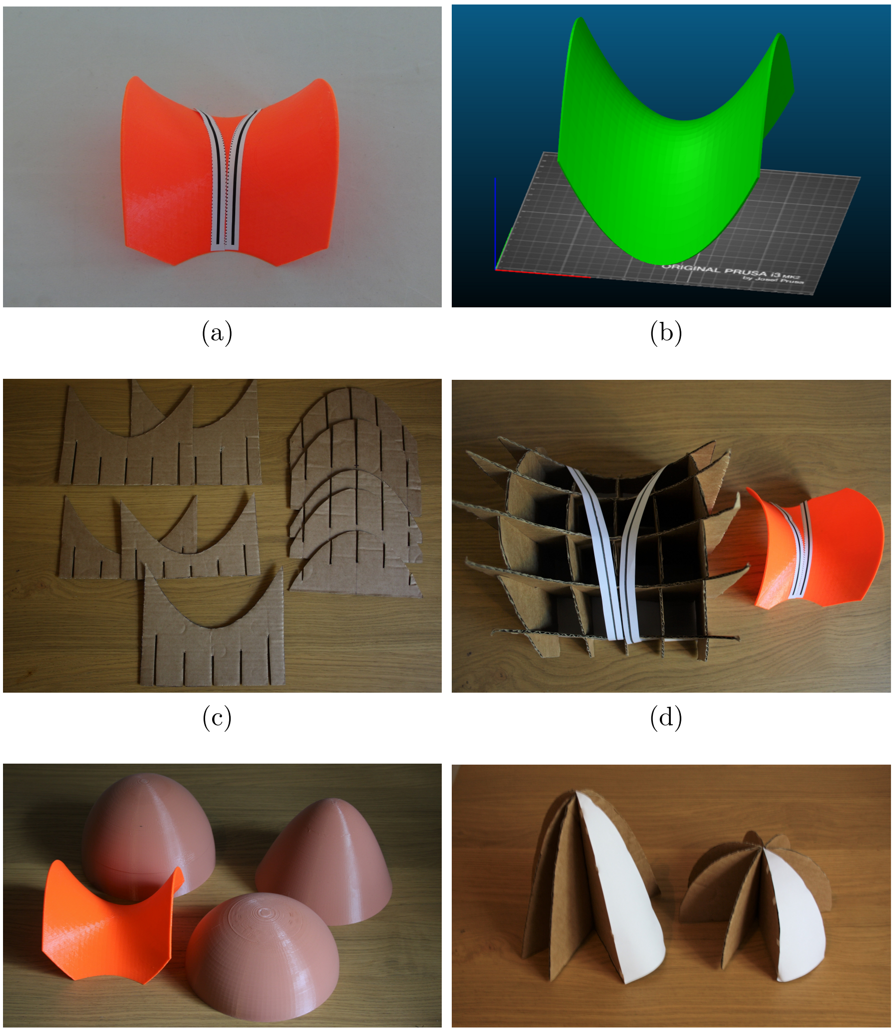
Figure 4.2: Simple activities using spherical geometry with strips of paper representing geodesics.

180°. We make an observation, in both activities we have been using then thin paper strips as a sort of ”next best thing” to a line on a flat surface. The reason is that the paper strips are originally straight but due to their flexibility, they can be laid on a curved surface but they still go as straight as possible, yielding only to the curvature of the surface. They represent a *geodesic*, a generalized idea of a straight line for curved surfaces. In other words, a geodesic is what our trajectory on a curved surface would look like if we went forward without turning right or left (we opt for such a practical definition instead of a technical one). Geodesics play an important role in GR, so we will encounter them a lot. To further improve our understanding of geodesics, we can ask the students if a line of constant latitude on a globe is a geodesic. To help students with this question, we always bring a large inflatable globe. They can readily verify using paper strips that lines of latitude are in fact not geodesics. We can also use the applet introduced in Section 3.2.2 to supplement this practical activity with an alternative representation.

Moving on, we cannot create parallel lines on a sphere but we can choose two curved equivalents - geodesics - that initially start parallel from two different but not too distant points. For example, two meridians on a globe starting on the equator are great examples of two geodesics (verifiable using the paper strips) that go initially in the same direction, for example northward, and keep approaching each other until they cross at the North Pole. The forth theorem gives also a different result to what we are used to. Choosing a centrepoint and a given length, we can draw a circle made of points on a sphere using the strips of paper and directly measure its circumference and radius. We find that $2\pi r$ is actually larger than the measured circumference. Examples of these activities with spherical geometry are shown in Figure 4.2.

All these results suggest that the geometry of a sphere is somehow different from the geometry in a plane that we are used to. For completion, we present another type of surface, a hyperbolic paraboloid also known as a *saddle surface* (Figure 4.3), called for its similarity to a horse saddle. It is very unlikely that a teacher would have access to a real saddle nor that most students have direct experience with it; therefore, we must find another way to create the surface. A very practical option is 3D-printing. If we have the option, we can print multiple versions for the students to work with. Alternatively, a close enough version can be made using cardboard cut-outs (Figure 4.3(c-d)). Templates for the cut-outs as well as all the used 3D-models for printing are available at (Ryston 2022b). With enough saddle surfaces for students to work at least in groups,they can again try the geometrical theorems above. This time it is a bit more complicated, especially creating a circle, but it is possible to verify that all four situations give again different results. The shortest path between two points is not part of a circle but either a parabola or a hyperbola. The sum of internal angles in a triangle is less than 180°. Initially parallel geodesics diverge and the circumference of any circle is larger than 2π times the corresponding radius. In some sense, this geometry is somehow opposite to the spherical one. A sphere is an example of a *positive* curvature or positively curved surface, whereas the saddle shape is a typical example of *negative* curvature or negatively curved surface. The nomenclature comes from differential geometry and is related to the concept of *Gaussian curvature*, but we don't spend time discussing it because it is a complicated mathematical quantity that we do not need to know. We can, however, at least mention that curvature can be calculated for every point of the surface, it can therefore change throughout the surface (it is well-known that a sphere has constant curvature, on the other hand the curvature of the saddle shape is obviously not the same everywhere) and can be a positive or a negative number (with flat surface having zero curvature). If time permits, we can try other surfaces with positive curvature, for example an ellipsoid or a paraboloid - both exhibit the same kind of geometric properties as the sphere. A rotational hyperboloid (most known as the cooling towers seen in nuclear power plants) is another exemplar of a negatively curved surface. All of these can be again 3D-printed or approximated using cardboard cut-outs (Figure 4.3 (e-f)). There are also surfaces that have both types of curvature in different places. Two easiest examples are a torus or a banana (which essentially resembles a distorted section of a torus). Note: Both the geometry of a sphere and of a saddle shape can be further showcased using the corresponding applets from the study website (see Section 3.2.2 of this work).

Before we move on to another activity to help us get more acquainted with positive and negative curvature, we ask the students: "What does it mean when something is curved?" The most common answer is along the lines of "something is curved if it is not flat", which is an understandable viewpoint coming from our everyday life. However, by asking this question, we wish to draw their attention to the difference between *intrinsic curvature* and *extrinsic curvature*. A perfect surface to show the difference is a cylindrical surface (the importance of this distinction when learning about curvature is argued, for example, in Junius 2008). "Is it curved?", we ask the students. Well, from the standpoint of the previous answer - yes, it is not flat; therefore it is curved. However, we can easily show that on this surface, geometry behaves very similarly to a flat plane. Draw a triangle on a flat piece of paper, the sum of its internal angles is guaranteed to be 180°. Now bend the paper to form a cylindrical surface. The paper is not damaged or crumpled in any way, so the internal angles of the triangle surely remain the same and so does their sum. Draw two parallel lines on the flat paper and bend it again. Even though the lines are not straight anymore, they keep their "parallelness", their distance never changes (see example in Figure 4.4). The same goes with the



(a) (b)

(c) (d)

(e) (f)

Figure 4.3: The saddle surface can be 3D-printed **(a-b)** or approximated using cardboard cut-outs **(c-d)**. The same goes for other common geometric shapes **(e-f)**.

circumference of a circle. The cylindrical surface is curved from the outside point of view, we see it curved because we perceive it as embedded in 3D space, i.e. it has extrinsic curvature. However, "inside" the surface, geometry is still the same as on a flat piece of paper. It has zero intrinsic curvature. And for our purposes of exploring basics of GR, we care only about the intrinsic curvature. The reason being that we wish to eventually investigate the curvature of spacetime but we cannot look at it from some hypothetical higher dimension just as we look at the two-dimensional cylindrical surface from a third dimension.

Consequently, we have just stumbled upon a useful practical test to see whether some surface has non-zero intrinsic curvature. If it can be wrapped in a piece of paper without the paper tearing or crumbling, it has zero intrinsic curvature just like the sheet of paper. Students can easily try wrapping a sheet of paper around their balloons, it is guaranteed that they will not succeed. This is also the reason why cartographers have such difficulty accurately representing the surface of our planet on a flat map and have to resort to various kinds of distortions.

Figure 4.4: Two "parallel" curves on a cylindrical surface.

We include another activity to showcase the main difference between positive and negative curvature and how it relates to flat surface. It comes from (Henderson and Taimina 2004). We give students prepared copies of a hexagonal net and ask them to cut it out along the dashed lines (Figure 4.5 (a), to safe time we could have the copies cut beforehand). The template as well as all the other used in this workshop can be found at the workshop section of the study website (Ryston 2022b). In a flat state, six hexagons fit perfectly to form a "flower" (image (b) shows the net of four such flowers interconnected). To introduce positive curvature, we need to remove one of the hexagons from each flower. We could tape or glue two adjacent hexagons over each other, but we added half cuts that allow us to easily slot them in one another. Very quickly, the whole net starts to resemble a sphere, more specifically a football (c). It therefore truly exhibits positive curvature and to make it happen, we had to "remove some material" from a flat surface. We can also try the opposite, adding a seventh hexagon into the flowers. To do that, we have additional triplets of hexagons, the two on the sides having again slots for easier insertion. The result resembles the saddle shape quite nicely (d). The resemblance would be even better if we used smaller

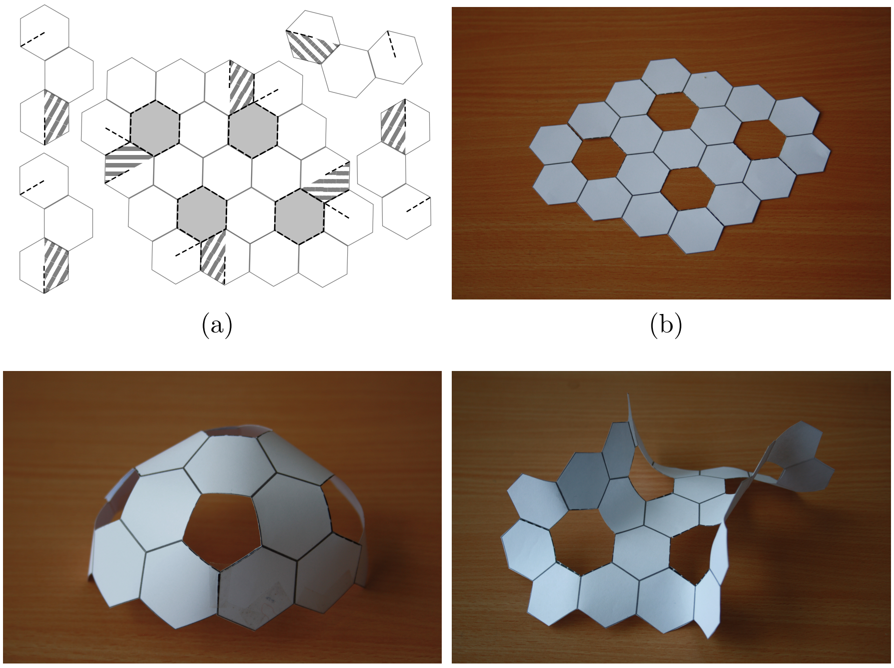

(a) (b) (c) (d)

Figure 4.5: Comparison of positive and negative geometry using a net of hexagons. Image **(a)** shows the full copy that we give students with added guiding striped sections for easier slotting of hexagons. The following photos show the flat **(b)**, positively curved **(c)** and negatively curved **(d)** states.

hexagons.

4.1.2 Gravity as curvature

We move on to the next section of the workshop, the geometrical outlook on gravity. We use a simple activity with a cone found, for example, in (Epstein 1985). We have prepared another copy for students to cut and use, this time of a cone template (Figure 4.6 (a)). We ask the students to draw a straight line across the flat cone section going close to the middle point but not through it. The straight line represents a trajectory of a freely moving body in empty space (represented by the two-dimensional surface) without the presence of a gravitational field. We then imagine a gravitating body in the middle of the flat region (a star for example). According to GR, the space around the gravitating body is curved.[2] Even though GR describes gravity as curvature of spacetime and therefore sole curvature of space is not strictly speaking physically correct, we don't talk about spacetime yet. To help students understand better, we have split the problem of spacetime curvature into two parts, first curvature of space,

[2]Ideally, this idea should not come out of nothing. In the study text we try to motivate it with a discussion following the principle of equivalence. In the workshop, however, we do not talk about the principle of equivalence because that would probably double its length. We therefore have to take this information as purely descriptive. In a sense, we are saying: "That's how it is done in GR, let's understand what it means. Why it is done so is a story for another time."

then "curvature" of time and then finally we combine them. As we are forming the cone, we see the straight trajectory curving around the middle similarly to an object influenced by gravity.

The idea of a curved space is very difficult to imagine for anyone, and it might give students trouble. We can do two things to help. Firstly, it helps not to try to imagine what a curved space looks like. Instead, focus on the differences between flat and curved surfaces we saw earlier and try to extrapolate a similar relationship between a flat Euclidean space and a curved one. Secondly, we know that, for example, the motion of planets around the Sun is planar. Therefore, it is physically relevant to focus indeed on a motion in one plane instead of the whole space. In any case, it is important to stress out that the trajectory we have originally drawn is still in the original plane of motion. The forming of the cone, the drawing of the shape into the third dimension, is purely for the purposes of visualization. Therefore, we should look at the curved trajectory from the top of the cone as seen in Figure 4.6 (b). This very simple visualization is in our experience the most effective in terms of student engagement (judging by their reactions and comments).

We should confess to the students that our previous comments are a bit misleading and perhaps some of the more perceptive of students might pick up on it. How can the cone surface be curved when we can actually lay a sheet of paper directly on it? Or, better yet, when it is made from a sheet of paper where most of the paper is completely intact? The truth is that, indeed, the cone surface has zero curvature with one very important exception, the singular point in the middle. There, the curvature is actually infinite because of the sharp edge. Consequently, if we focused on any region excluding the middle point, geometry is flat just like on a sheet of paper. We can, for example, send two initially parallel trajectories using our makeshift geodesics and if both travel to the same side of the middle, their distance remains the same (Figure 4.6 (c) shows this demonstration using the supplementary applet for better visibility). Only if each trajectory travels on one side, do we get the same behaviour of crossing lines as on a surface with positive curvature. It is the singular point in the middle that causes this illusion of curvature.

Finally, we can relate this activity to the previous section by transforming the cone into a seemingly negatively curved surface. Just like with the hexagons, we "removed material" to create, in this case an illusion of, positive curvature. Therefore, we can also try "adding material" by gluing the second part of the copy to the cone section. By doing so, we are forcing the paper to bulge in a way that reminds us of the saddle shape (Figure 4.6 (d)). The only exception is again the middle point. If time permits, students can verify that the geometry of the surface is still flat unless we include the middle point.

The previous demonstration is interesting, but some students might be disappointed by its simplicity. Another shape that we introduce is the Flamm's paraboloid. At the study website, we show in detail how the shape of the surface is derived to be an embedding diagram for the equatorial plane of Schwarzschild spacetime, but here we do not have the time nor mathematical means to do so. Instead, we introduce it as a surface whose curvature comes directly from relativistic equations and then the following is very similar to the previous ge-

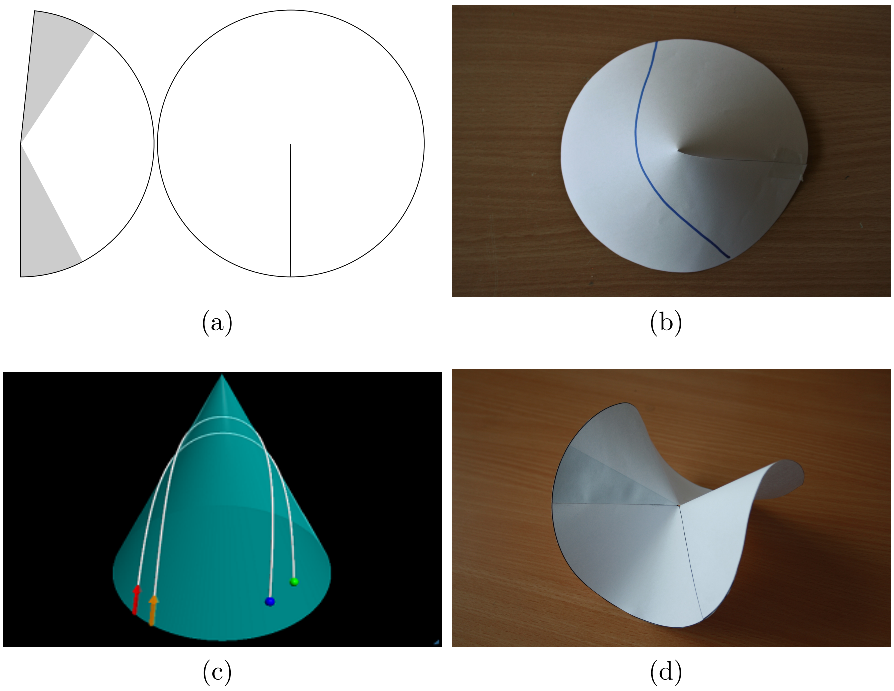


Figure 4.6: Geometrical activities with a cone. Students are given a simple template **(a)** and show that by introducing ”curvature”, the originally straight line is curved **(b)** much like in the presence of gravity. **(c)** shows the supplementary applet with two ”parallel” particle trajectories. We can similarly made something resembling the saddle shape by adding more of the cone surface using the other part of the template **(d)**.

ometrical activities. We again 3D-printed sufficient number of the paraboloids so that students can work with them and use their new knowledge to determine the sign of the Flamm's paraboloid curvature as well as observe how the curvature influences trajectories (Figure 4.7). It is again important to remind that we should look at the curved trajectories from the top because, just like in the cone, the third dimension is added purely for the purposes of visualization. Again, we use the corresponding applet mentioned in Section 3.2.2 to visually supplement this activity.

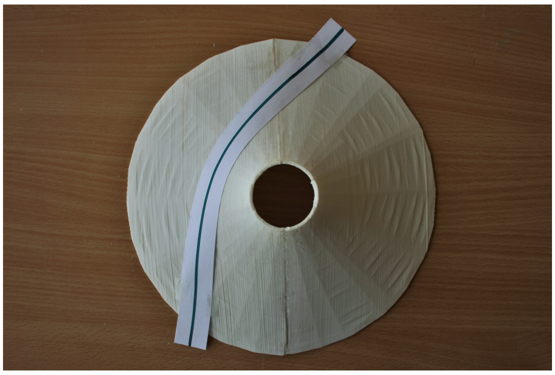


Figure 4.7: A geodesic represented by a thin piece of paper on the surface of a Flamm's paraboloid.

4.1.3 Gravitational time dilation

Now comes an essential moment in our workshop scheme. We have just discussed how the geodesic motion on curved surfaces (and by extension in curved space) resembles the trajectories of bodies in a gravitational field. However, this resemblance can be disrupted using a specific simple example. What if a body is let go from rest? Using the Flamm's paraboloid model, it is like putting a small marble on it. In reality, the marble would of course fall off but that is because it is acted upon by real gravity and that is not something we want here. We are trying to model gravity using the curvature of the surface itself. Neglecting this real gravitational influence, the marble stays where it is let go. There is no reason for it to start moving. This is a problem because our reasoning with spatial curvature cannot explain the simplest motion under gravity - the free fall.

We can present this conundrum to the students or sometimes, they come up with this objection themselves. In either case, the answer to it is that our description of gravity is not complete. Remember that our theme says that gravity is the curvature of spacetime, not just space itself. Before we get to the curvature of spacetime, however, we will talk about its second part, the "curvature" of time. This is quite a perplexing name, though (and we admit we use it specifically to peak the students' interest). How can time be curved? It is therefore better to explain right away. What we mean by that is that in the presence of gravity time does not behave the same way everywhere. This effect is called gravitational time dilation.

Instead of having a lengthy theoretical treatment, we dive right in by presenting the Hafele-Keating experiment and its results, which showed time discrepancy between originally synchronized atomic clocks, where one clock was left at the ground and the other, in fact several clocks, were flown in an airplane around the globe. We will not go into detail here, because the experiment is described in detail on the study website. We will note, however, that this section and the following calculation are the only situations where we cannot hide from the existence of SR. Planes in the experiment fly, that is they move relative to the ground, and so there is not just the gravitational time dilation where time flows more quickly in a weaker gravitational field (i.e. in higher altitude) but also the kinematic time dilation known from SR caused by the relative motion. We freely admit this to students. Either they have already gone through the basis of SR and this comes as no surprise to them or they didn't and we just have to acknowledge the existence of this SR effect. It doesn't really change the course of the workshop and, in our experience, some students stated that after hearing this they are looking forward to learning about SR in their physics seminar.

To make this section more engaging, we propose a calculation regarding this time dilation effect. We briefly discuss the basic principles of satellite navigation systems and how precise time measurement is crucial for their correct operation. That produces a problem because according to the Hafele-Keating experiment, time should flow differently for the satellites in the orbit than it does for us on the surface. We verify this with a calculation. To do so, we need a formula to compare the increments of time for the satellite and for us. For this calculation we chose a satellite from the Galileo system, because it is European and its headquarters are in Prague (we thought it might be an interesting discovery for some students). To simplify the situation, we assume that we are standing on the equator, rotating with Earth with velocity $\vec{v_E}$ and the satellite is orbiting on a circular orbit with velocity $\vec{v_G}$. We then show and explain the following formula that can be derived from the Schwarzschild metric, thought we do not explicitly talk about this metric (or any other, for that matter) during the workshop:

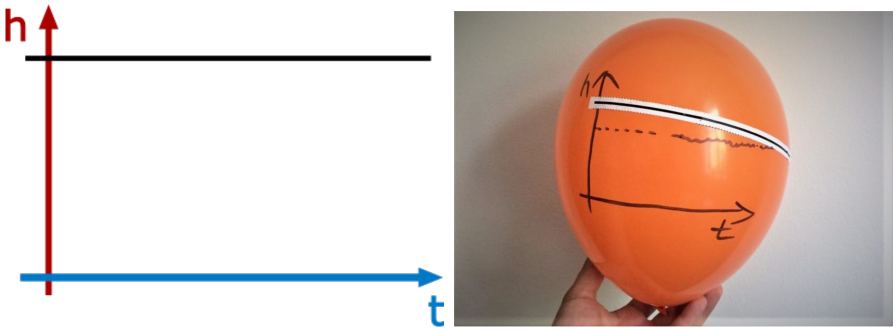
$$\frac{\mathrm{d}\tau_{\mathrm{E}}}{\mathrm{d}\tau_{\mathrm{G}}} = \frac{\sqrt{1 - \frac{r_{\mathrm{S}}}{r_{\mathrm{E}}} - \frac{v_{\mathrm{E}}^2}{c^2}}}{\sqrt{1 - \frac{r_{\mathrm{S}}}{r_{\mathrm{G}}} - \frac{v_{\mathrm{G}}^2}{c^2}}}, \tag{4.1}$$

where $\mathrm{d}\tau_{\mathrm{E}}$ and $\mathrm{d}\tau_{\mathrm{G}}$ are the time increments for us on Earth and the satellite respectively, r_{S} is the Schwarzschild radius (this parameter equal to $2GM/c^2$, where G is the Newton's gravitational constant, M is the mass of the central gravitation body - in this case the Earth - and c is the speed of light, is discussed before this calculation), r_E is the radius of Earth and r_G is the radius of the satellite orbit. The derivation of this formula can be found in (Taylor and Wheeler 2000) or our study website, including a discussion on further approximations made here when we are using a non-rotating spacetime metric for rotating Earth. If it is the case that our listeners are familiar with basics of SR, they might recognize the terms v^2/c^2 in the formula. It in fact nicely visually connects gravitational and kinematic time dilation showing not only that, from the experimental point of view, there is only one time dilation caused by two different phenomena, but also that GR contains SR, it is truly the generalization of its predecessor.

All the parameters in formula 4.1 can be easily found online. In case of the speeds, we can actually go one step further and recall with students the formula for the speed of a circular motion from mechanics, which can be used for both motions to calculate the speed from known periods and radii. Depending on available time, this calculation can be done in many different ways in terms of student participation. Again, the full calculation can be found at the study website. Our final result is, that due to the discrepancy in time measurement on the surface and at the satellite, in one day the error in determining the distance between us and the satellite (which is used to calculate our position) is 12 km. That is why all the navigation systems have to implement relativistic corrections in their calculations and without our understanding of time dilation these systems would not work correctly.

4.1.4 Curvature of spacetime

We can finally combine what we have learned to arrive at our goal. We will take the example of a stone which is located at rest at some height above ground. As a thought experiment, we imagine that the gravity is switched off, so that the stone hangs in the air. We ask the students to draw a graph of the stone's height-time dependance. They will most likely have no problem with this task, as the graph is a simple constant function (Figure 4.8 (a)). We then inform them that they have just drawn a spacetime diagram, which is typically a graph used in relativity with one spatial and one temporal dimension. The diagrams are used to represent spacetime, the unification of time and space, in simplified situations where only one spatial dimension is relevant to make the graph just two-dimensional. As we are interested only in the stone's height, that is our case as well. Usually, the temporal dimension is in these diagrams oriented vertically and in our graph it is, as is normal for a dependence on time, horizontal, but that does not change anything. We introduce the concept of a *worldline*, a trajectory of an object through spacetime, which we have already drawn as the line of the graph. Because we drew the diagram most likely on a piece of paper, it represents a flat spacetime, without curvature. According to the original theme of the workshop, to get gravity we need to curve this spacetime. To do so, we will redraw the diagram on the surface of the balloons that we still have from the beginning (a similar activity is mentioned, for example, in Farmer 2021). The perpendicular axes of the diagram can be created using the thin pieces of paper as guidelines. Finally, we mark the initial point (actually an event) of the stone starting to free-fall. Now comes the main idea. We have already seen a few instances when an imagined free object (i.e. influenced by nothing but gravity) was moving on a curved surface (a 2D situation) or through curved space (a 3D situation) and its trajectory was a geodesic. But we were missing the time component, actually the time dimension. Therefore, to truly get the right influence of gravity on the object, we must go one (this time temporal) dimension even higher and find its geodesic movement through spacetime. We take the thin piece of paper that represents the geodesic and use it to find the stone's trajectory through curved spacetime represented by the balloon surface. What we get is that due to the curvature of the balloon, the worldline of the stone approaches the axis on the diagram that corresponds to the ground (Figure 4.8 (b)). The stone starts moving downwards - it falls. This activity is, of course, just a simplified model; however, it visualizes the main idea of GR. The stone is not pulled by a force, it moves because its worldline is not going purely in the temporal direction. Due to the



(a) (b)

Figure 4.8: Final geometrical activity. Image (a) shows the spacetime diagram representing a stone stationary in a flat spacetime. To visualize the effects of spacetime curvature (gravity), we redraw the diagram on a balloon surface and find the worldline of the stone as a geodesic in that spacetime. As a result, we see that the stone starts falling towards the ground.

curvature of spacetime, it curves also into the spatial direction, thus moving the stone. Gravity is the curvature of spacetime.

That is the final moment of the workshop. As we said at the beginning, our goal was to try to understand a single sentence and we spend quite some time on it, trying our best to get the students involved in the process as much as possible.

4.1.5 Student feedback

As we mentioned at the beginning of this section, after every instance of the workshop, student feedback was gathered using a simple quick written questionnaire. We purposely made it very brief in order not to overload the students who have just gone through an intensive and lengthy cognitive exercise (remember all the installments of the workshop we did were as a single event). The questionnaire consisted of two parts. The first part contained three questions with possible answers as a four-point Likert scale (4 – Definitely not. 3 – Probably not. 2 – Probably yes. 1 – Definitely yes.). These questions served as a quick tool for overall assessment of the seminar. The questions together with gained average answers over all seminars are:

- Do you think that after the seminar you know more about General Relativity than before? (average answer 1.12)
- Do you think that after the seminar you know more about the basic ideas of General Relativity than before? (1.30)
- Would you like General Relativity to be taught at upper secondary school even though there is a lack of time for introducing new topics? (1.63)

We include only the averaged values because we did not observe any significant changes in the averages for separate workshops. We need to bear in mind that these answers came from students who voluntarily attended the workshop;

therefore, they were most likely interested in the subject more than an average student. The design of the workshop was mostly influenced by the second part of the questionnaire with two open questions asking the student to write something they found positive about the workshop (well-made points, interesting activities etc.) as well as something negative (something missing, badly explained parts, boring parts, etc.).

Starting with the positives, students wrote many different aspects of the workshop. Namely *3D-printed models, curvature on the globe, visualizing curvature, making paper models, differences between Euclidean and non-Euclidean geometries, using formulas, applets,* and so on. Basically, almost every aspect of the workshop was found positive by someone. More importantly, different students praised different representations of the same activities (3D-printed surfaces vs. paper models vs. applets), adding validity to the already mentioned claim by (Ainsworth 2006) that using different representations of the same problem is beneficial in learning. In this case, different people respond positively to different representations because of the variety in their learning styles and overall preferences.

Concerning the negative comments, we have already mentioned that the largest change to the design of the workshop came after the first try. Originally, we included references to SR and somewhat relied on the students previously at least hearing something about it. While that might be true in some cases, the most common negative comment was that the particular student did not know anything about SR yet and he/she was lost when we referenced it. This led to the important decision to omit the SR reliance completely (with the little exception during the satellite calculation but we thought that discussing a real, if somewhat simplified, situation would be more engaging to the students then engineering a different scenario where the kinematic time dilation is not present). Besides that, other comments were quite minor and did not lead to any major change in the design. We do not count here singular negative comments that ran counter to the positive ones. For example, we had a negative comment from probably a mathematically oriented student who complained that we spent too much time with models and not enough time actually calculating. This again shows that everyone is different and it is impossible to suit everyone's needs perfectly. Most students appreciate the models and hands-on activities in their responses, so we are confident this workshop could be implemented by other teachers to the benefit of their students and their interest in physics.

Conclusion

We believe the main goals of this work were achieved. Based on research of available sources on relativity suitable for secondary students and curricular documents related to teaching physics in secondary schools as well as a survey among gymnasium physics teachers regarding the current state of teaching relativity at their schools, we identified two possible ways of developing students' understanding of relativity. However, due to a found prevalent lack of sources on GR, we decided to focus primarily on the general theory.

As a result, a study website for students interested in general relativity was created as well as a GR workshop that can be implemented by physics teachers either all at once or as a series of 3-4 lessons. The created materials are grounded in existing literature and informed by the teaching experience of both the author of this work and other more renown science educators. The design of the materials was tested and continually improved over the course of several years. The important step is now spreading the knowledge of their existence among physics teachers. This will be primarily done via physics teacher conferences, but other means will be looked for as well.

The extent of the materials is most likely not final. Both the website and the workshop could be easily enlarged, and there are already plans to include particular topics. In case of the website, we have already mentioned gravitational waves or cosmology (both classic GR chapters related to contemporary scientific findings). Translating the website into English will certainly enable its use worldwide. Our research into the projects aimed at GR education for primary and secondary schools in other countries detailed in Chapter 1 (including the Norwegian ReleQuant project to which we had the pleasure to contribute an animation), show the majority of them focus on simple conceptual understanding without any of the more technical details (which is an approach for which we argue in Chapter 3). We therefore think that translating our website into English could be useful for English-speaking upper secondary students worldwide. And not just students, physics teachers or indeed anyone with a little experience with upper secondary mathematics and physics, who is interested in a more detailed treatment of GR than a typical popular physics book but at the same time reluctant or unable to use university textbooks might benefit from using this website.

However, we would first like to see if the materials will be indeed used by teachers and students, for example, by monitoring user access to the website. Another possible extension of the website would be the inclusion of chapters for undergraduate students, especially in connection with a recently created electable seminar General Relativity for Teachers offered to future physics teachers at the Department of Physics Education and lead by the author of this work.

We have already mentioned a successful project created and maintained by the Department of Physics Education - the Collection of Solved Problems in Physics - which is used worldwide by students and teachers. Currently, there are no solved problems related to relativity, be it SR or GR. Creating a selection of such solved problems and linking them with the prepared study website would surely be an additional benefit to students of relativity.

To conclude the conclusion, this thesis might be at its end but the work on finding better ways to help students learn about relativity never is.

Bibliography

[Abanese and Parisi 2010] ABANESE, L. ; PARISI, A.: *Einstein a teorie relativity.* Praha : CooBoo, 2010 (1st ed.). – ISBN 978-80-00-01874-4

[Ainsworth 2006] AINSWORTH, S.: DeFT: A conceptual framework for learning with multiple representations. Learning and Instruction, 16, 183-198. In: *Learning and Instruction* 16 (2006), 06, P. 183–198

[Alstein et al. 2021] ALSTEIN, Paul ; KRIJTENBURG-LEWERISSA, Kim ; JOOLINGEN, Wouter R. van: Teaching and learning special relativity theory in secondary and lower undergraduate education: A literature review. In: *Phys. Rev. Phys. Educ. Res.* 17 (2021), Jul, P. 023101. – URL `https://link.aps.org/doi/10.1103/PhysRevPhysEducRes.17.023101`

[Angell et al. 2004] ANGELL, C. ; GUTTERSRUD, Ø. ; HENRIKSEN, E. K. ; ISNES, A.: Physics: Frightful, but fun. Pupils' and teachers' views of physics and physics teaching. In: *Science Education* 88 (2004), Nr. 5, P. 683–706

[Aslanides and Savage 2013] ASLANIDES, J. S. ; SAVAGE, C. M.: Relativity concept inventory: Development, analysis, and results. In: *Phys. Rev. ST Phys. Educ. Res.* 9 (2013), May, P. 010118. – URL `https://link.aps.org/doi/10.1103/PhysRevSTPER.9.010118`

[Bakker 2018] BAKKER, A.: *Design Research in Education: A Practical Guide for Early Career Researchers.* London : Routlege, 2018 (1st ed.). – ISBN 9780203701010

[Balada 2007] BALADA, J.: *Framework Education Programme for Secondary General Education: (Grammar Schools): FEP SGE.* Prague : Research Institute of Education in Prague, 2007. – ISBN 978-80-87000-23-6

[Baldy 2007] BALDY, E.: A new educational perspective for teaching gravity. In: *International Journal of Science Education* 29 (2007), Nr. 14, P. 1767 – 1788

[Balta et al. 2022] BALTA, N. ; ERYILMAZ, A. ; OLIVEIRA, A. W.: Increasing the presence of Einsteinian physics in high school: the impact of a professional development program on teacher knowledge and practice. In: *TEACHER DEVELOPMENT* 26 (2022), Nr. 2, P. 166–188. – ISSN 1366-4530

[Bartuška 1991] BARTUŠKA, K.: *Kapitoly ze speciální teorie relativity: Pomocná kniha pro žáky gymnázií.* Praha : SPN, 1991. – ISBN 80-042-2915-8

[Bartuška 2010] BARTUŠKA, K.: *Fyzika pro gymnázia: Speciální teorie relativity.* Prometheus, 2010 (4th ed.). – ISBN 978-80-7196-388-2

[Begelman and Rees 2013] BEGELMAN, M. ; REES, M. J.: *Osudová přitažlivost gravitace: černé díry ve vesmíru.* Praha : Argo, 2013 (Dokořán). – ISBN 978-80-257-0806-4

[Boyle 2019a] BOYLE, J.: Teaching gravitational waves in the lower secondary school. Part I. A teaching module. In: *Physics Education* 54 (2019), Nr. 2

[Boyle 2019b] BOYLE, Julie: Teaching gravitational waves in the lower secondary school. Part II. A model for a STEM enrichment programme. In: *Physics Education* 54 (2019), Nr. 2

[Choudhary et al. 2019] CHOUDHARY, R. ; KRAUS, U. ; KERSTING, M. ; BLAIR, D. ; ZAHN, C. ; ZADNIK, M. ; MEAGHER, R.: Einsteinian Physics in the Classroom: Integrating Physical and Digital Learning Resources in the Context of an International Research Collaboration. In: *The Physics Educator* 1 (2019), Nr. 4

[Choudhary et al. 2018] CHOUDHARY, R. K. ; FOPPOLI, A. ; KAUR, T. ; BLAIR, D. ; ZADNIK, M. ; MEAGHER, R.: Can a short intervention focused on gravitational waves and quantum physics improve students' understanding and attitude? In: *Physics Education* 53 (2018), Nr. 6, P. 065020

[Cohen et al. 2007] COHEN, L. ; MANION, L. ; K., Morrison: *Research Methods in Education*. Abingdon : Routledge, 2007 (6th edition). – ISBN 978-113-8209-886

[Couper et al. 2001] COUPER, M. P. et al.: Web Survey Design and Administration. In: *Public Opinion Quarterly* 65(2) (2001), P. 230–253. – URL `https://doi.org/10.1086/322199`

[Cox and Forshaw 2013] COX, B. ; FORSHAW, J. R.: *Proč platí E=mc²: a proč by nás to mělo zajímat*. Praha : Argo, 2013 (Dokořán). – ISBN 978-80-7363-528-2

[Crawford et al. 2001] CRAWFORD, S. D. et al.: Web Surveys: Perceptions of Burden. In: *Social Science Computer Review* 19(2) (2001), P. 146–162. – URL `https://doi.org/10.1177/089443930101900202`

[De Ambrosis and Levrini 2010] DE AMBROSIS, A. ; LEVRINI, O.: How physics teachers approach innovation: An empirical study for reconstructing the appropriation path in the case of special relativity. In: *PHYSICAL REVIEW SPECIAL TOPICS-PHYSICS EDUCATION RESEARCH* 6 (2010), Nr. 2. – ISSN 1554-9178

[Department of Informatics and Statistics MEYS 2022] DEPARTMENT OF INFORMATICS AND STATISTICS MEYS: *Statistická ročenka školství*. 2022. – URL `https://statis.msmt.cz/rocenka/rocenka.asp`. – Date of access: 2022-06-05

[Department of Physics Education 2016] DEPARTMENT OF PHYSICS EDUCATION: *Collection of Solved Problems in Physics*. 2016. – URL `https://physicstasks.eu/en`. – Date of access: 2022-07-25

[Dimitriadi and Halkia 2012] DIMITRIADI, K. ; HALKIA, K.: Secondary Students' Understanding of Basic Ideas of Special Relativity. In: *International Journal of Science Education* 34 (2012), Nr. 16, P. 2565–2582. – URL `https://doi.org/10.1080/09500693.2012.705048`

[Dua et al. 2020] DUA, Y.S. ; BLAIR, D.G. ; KAUR, T. ; CHOUDHARY, R.K.: Can Einstein's theory of general relativity be taught to Indonesian high school students? In: *Jurnal Pendidikan IPA Indonesia* 9 (2020), Nr. 1, P. 50 – 58

[Dvořák 1984] DVOŘÁK, L.: *Obecná teorie relativity a moderní fyzikální obraz vesmíru*. 1st ed. Praha : Státní pedagogické nakladatelství, 1984

[Dvořák 2019] DVOŘÁK, L.: *Obecná teorie relativity: nástin východisek a jejich důsledků*. 2019. – URL `https://kdf.mff.cuni.cz/vyuka/FOS/FOS_03_OTR_Vychodiska_Dusledky_ver_01.pdf`. – Date of access: 2022-07-18

[EFP 2022] EFP: *THE EINSTEIN-FIRST*. 2022. – URL `https://www.einsteinianphysics.com/`. – Date of access: 2022-07-06

[Einstein 2000] EINSTEIN, A.: *Teorie relativity a jiné eseje*. Praha : Pragma, 2000. – ISBN 80-720-5596-8

[Einstein 2016] EINSTEIN, A.: *Smysl relativity*. Praha : Vyšehrad, 2016. – ISBN 978-80-7429-537-9

[EN 2022] EN, ScienceClic: *ScienceClic EN*. 2022. – URL `https://www.youtube.com/c/ScienceClicEN`. – Date of access: 2022-07-18

[Epstein 1985] EPSTEIN, L.C.: *Relativity Visualized*. Insight Press, 1985. – URL `https://books.google.cz/books?id=lwZBAQAAIAAJ`. – ISBN 9780935218053

[Falbo-Kenkel and Lohre 1996] FALBO-KENKEL, M. ; LOHRE, J.: Simple gravitational lens demonstrations. In: *The Physics Teacher* 34 (1996), Nr. 9, P. 555–557

[Farmer 2021] FARMER, S.: Developing a teacher professional learning workshop on the general theory of relativity. In: *J. Phys.: Conf. Ser.* 1929 (2021), Nr. 012057

[Ferreira 2015] FERREIRA, P. G.: *Nádherná teorie: sto let obecné teorie relativity*. Praha : Vyšehrad, 2015 (Spektrum). – ISBN 978-80-7252-584-3

[Ferrán 2020] FERRÁN, S. Kaid-Salah: *Moje první kniha o teorii relativity*. Praha : Bambook, 2020. – ISBN 978-80-271-1759-8

[Fyzika 007 2022] FYZIKA 007: *Speciální teorie relativity*. 2022. – URL `https://www.fyzika007.cz/speci%C3%A1ln%C3%AD-teorie-relativity`. – Date of access: 2022-07-18

[Fyzikální olympiáda 2022] FYZIKÁLNÍ OLYMPIÁDA: *Přehled studijních textů z Knihovničky Fyzikální olympiády*. 2022. – URL `http://fyzikalniolympiada.cz/studijni-texty`. – Date of access: 2022-07-18

[Galesic and Bosnjak 2009] GALESIC, M. ; BOSNJAK, M.: Effects of Questionnaire Length on Participation and Iindicators of Response Quality in a Web Survey. In: *Public Opinion Quarterly* 73(2) (2009), P. 349–360. – URL `https://doi.org/10.1093/poq/nfp031`

[Geroch 1981] Geroch, R.: *General Relativity from A to B.* Chicago : The University of Chicago Press, 1981 (1st ed.). – ISBN 978-0-226-28864-2

[Greene 2001] Greene, B.: *Elegantní vesmír: superstruny, skryté rozměry a hledání finální teorie.* Praha : Mladá fronta, 2001 (Kolumbus). – ISBN 80-204-0882-7

[Hadjerrouit 2010] Hadjerrouit, S.: A Conceptual Framework for Using and Evaluating Web-Based Learning Resources in School Education. In: *Journal of Information Technology Education: Research* 9 (2010), Nr. 1, P. 53–79. – URL `https://www.learntechlib.org/p/111356`. – ISSN 1539-3585

[Hartle 2003] Hartle, J. B.: *Gravity: An Introduction to Einstein's General Relativity.* Addison Wesley, 2003 (1st ed.). – ISBN 0-8053-8662-9

[Hartle 2006] Hartle, J. B.: General relativity in the undergraduate physics curriculum. In: *American Journal of Physics* 74 (2006), Nr. 1, P. 14–21

[Hawking 2015] Hawking, S.: *Stručná historie času: od velkého třesku k černým dírám.* Praha : Argo, 2015 (3rd ed.). – ISBN 978-80-257-1527-7

[Henderson and Taimina 2004] Henderson, D. W. ; Taimina, D.: *Experiencing Geometry.* 3. Upper Saddle River, NJ : Pearson, 2004

[Hendry et al. 2014] Hendry, M. ; Bradaschia, C. ; Audley, H. ; Barke, S. ; Blair, D.G. ; Christensen, N. ; Danzmann, K. ; Freise, A. ; Gerberding, O. ; Knispel, B. ; Lieser, M. ; Mandel, I. ; Moore, T. ; Stuver, A. ; Whiting, B.: Education and public outreach on gravitational-wave astronomy. In: *General Relativity and Gravitation* 46 (2014), Nr. 8, P. 1 – 11

[Henriksen et al. 2014] Henriksen, E. K. et al.: Relativity, quantum physics and philosophy in the upper secondary curriculum: challenges, opportunities and proposed approaches. In: *Phys. Educ.* 49 (2014), Nr. 678

[Horský 1972] Horský, J.: *Speciální teorie relativity.* 1st ed. Praha : Státní pedagogické nakladatelství, 1972

[Hughes and Kersting 2021] Hughes, T. ; Kersting, M.: The invisibility of time dilation. In: *Physics Education* 56 (2021), 01, P. 025011

[Huwe and Field 2015] Huwe, P. ; Field, S.: Modern Gravitational Lens Cosmology for Introductory Physics and Astronomy Students. In: *The Physics Teacher* 53 (2015), Nr. 5, P. 266–270

[Junius 2008] Junius, P.: A case example of insect gymnastics: how is non-Euclidean geometry learned? In: *International Journal of Mathematical Education in Science and Technology* 39 (2008), Nr. 8, P. 987–1002

[Kamphorst 2021] Kamphorst, F.: *Introducing Special Relativity in Secondary Education*, Utrecht University, Dissertation, 2021

[Kamphorst et al. 2019] Kamphorst, F. ; Vollebregt, M. J. ; Savelsbergh, E. R. ; Joolingen, W. R. van: Students' preinstructional reasoning with the speed of light in relativistic situations. In: *PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH* 15 (2019), Nr. 2. – ISSN 2469-9896

[Kamphorst et al. 2021] Kamphorst, F. ; Vollebregt, M. J. ; Savelsbergh, E. R. ; van Joolingen, W.: An Educational Reconstruction of Special Relativity Theory for Secondary Education. In: *Science & Education* (2021)

[Kaur et al. 2017a] Kaur, T. et al.: Teaching Einsteinian physics at schools: part 1, models and analogies for relativity. In: *Phys. Educ.* 52 (2017), Nr. 065012

[Kaur et al. 2017b] Kaur, T. et al.: Teaching Einsteinian physics at schools: part 2, models and analogies for quantum physics. In: *Phys. Educ.* 52 (2017), Nr. 065013

[Kaur et al. 2017c] Kaur, T. et al.: Teaching Einsteinian physics at schools: part 3, review of research outcomes. In: *Phys. Educ.* 52 (2017), Nr. 065014

[Kaur et al. 2020] Kaur, T. ; Blair, D. ; Stannard, W. et al.: Determining the Intelligibility of Einsteinian Concepts with Middle School Students. In: *Res Sci Educ* 50 (2020), P. 2505–2532

[Kersting 2019] Kersting, M.: Navigating four dimensions – upper secondary students' understanding of movement in spacetime. In: *Journal of Physics: Conference Series* 1287 (2019), 08, P. 012007

[Kersting and Blair 2021] Kersting, M. ; Blair, D.: *Teaching Einsteinian Physics in Schools: An Essential Guide for Teachers in Training and Practice*. Routlege, 08 2021. – ISBN 9781003161721

[Kersting et al. 2018] Kersting, M. ; Henriksen, E. ; Bøe, M. ; Angell, C.: General relativity in upper secondary school: Design and evaluation of an online learning environment using the model of educational reconstruction. In: *Physical Review Physics Education Research* 14 (2018), 05

[Kersting and Steier 2018] Kersting, M. ; Steier, R.: Understanding Curved Spacetime: The Role of the Rubber Sheet Analogy in Learning General Relativity. In: *SCIENCE & EDUCATION* 27 (2018), Nr. 7-8, P. 593–623. – ISSN 0926-7220

[Kersting et al. 2019] Kersting, M. ; Toellner, R. ; Blair, D. ; Burman, R.: Gravity and warped time—clarifying conceptual confusions in general relativity. In: *Physics Education* 55 (2019), Nr. 1, P. 015023

[Kraus 2007] Kraus, U.: First-person visualizations of the special and general theory of relativity. In: *European Journal of Physics* 29 (2007), Nr. 1, P. 1–13

[Kraus and Zahn 2022] Kraus, U. ; Zahn, C.: *Space Time Travel*. 2022. – URL `https://www.viten.no/filarkiv/general-relativity/#/`. – Date of access: 2022-07-18

[Králová 2007] Králová, M.: *Teorie relativity.* 2007. – URL http://edu.techmania.cz/cs/encyklopedie/fyzika/teorie-relativity. – Date of access: 2022-07-18

[Kulhánek 2020] Kulhánek, P.: *Vybrané kapitoly z teoretické fyziky.* Praha : AGA, 2020. – ISBN 978-80-906638-3-1

[Kulhánek et al. 2018] Kulhánek, P. ; Havlíček, I. ; Rozehnal, J. ; Břeň, D.: *Gravitace.* 2018. – URL https://www.aldebaran.cz/astrofyzika/gravitace/. – Date of access: 2022-07-18

[Lehman and Romano 2005] Lehman, E. L. ; Romano, J. P.: *Testing Statistical Hypotheses.* New York : Springer, 2005 (3rd edition). – ISBN 0-387-98864-5

[Lieber 2008] Lieber, L. R.: *The Einstein Theory of Relativity: A Trip to the Fourth Dimension.* Philadelphia : Paul Dry Books, 2008. – ISBN 978-1-58988-044-3

[Likert 1932] Likert, R.: A technique for the measurement of attitudes. In: *Archives of Psychology* 140 (1932)

[Lotze 1995] Lotze, K.-H.: *Special and General Relativity and Cosmology for Teachers and High-School Students.* P. 335–353. In: Bernardini, C. (Hrsg.) ; Tarsitani, C. (Hrsg.) ; Vicentini, M. (Hrsg.): *Thinking Physics for Teaching.* Boston : Springer US, 1995. – ISBN 978-1-4615-1921-8

[Manly and Parisi 2015] Manly, S. ; Parisi, A.: *Teorie relativity a kvantová fyzika pro začátečníky.* Praha : Práh, 2015 (1st ed.). – ISBN 978-80-7252-584-3

[Marcus et al. 2007] Marcus, B. et al.: Compensating for Low Topic Interest and Long Surveys A Field Experiment on Nonresponse in Web Surveys. In: *Social Science Computer Review* 25(3) (2007), P. 372–383

[Maršíková and Jelen 2019] Maršíková, M. ; Jelen, V.: *Hlavní výstupy z Mimořádného šetření ke stavu zajištění výuky učiteli v MŠ, ZŠ, SŠ a VOŠ.* 2019. – URL https://www.msmt.cz/file/50371_1_1/download/. – Date of access: 2022-06-22

[Max Planck Institute for Gravitational Physics 2005] Max Planck Institute for Gravitational Physics: *Einstein Online.* 2005. – URL https://www.einstein-online.info/en/. – Date of access: 2022-07-18

[Mermin 2005] Mermin, D. N.: *It's About Time: Understanding Einstein's Relativity.* New Jersey : Princeton University Press, 2005. – ISBN 978-0-691-14127-5

[MEYS 1991] MEYS: *Učební osnovy čtyřletého gymnázia: Fyzika (Four-Year Gymnasium Curriculum: Physics).* Prague : Fortuna, 1991

[MEYS 2020a] MEYS: *Obory L0 a M.* 2020. – URL https://www.edu.cz/rvp-ramcove-vzdelavaci-programy/ramcove-vzdelavaci-programy-stredniho-odborneho-vzdelavani-rvp-sov/obory-l-a-m/. – Date of access: 2022-05-27

[MEYS 2020b] MEYS: *Rámcové vzdělávací programy středního odborného vzdělávání (RVP SOV).* 2020. – URL https://www.edu.cz/rvp-ramcove-vzdelavaci-programy/ramcove-vzdelavaci-programy-stredniho-odborneho-vzdelavani-rvp-sov/. – Date of access: 2022-05-27

[Natário 2011] Natário, J.: *General Relativity Without Calculus.* Springer, 2011 (1st ed.). – ISBN 978-3-642-21451-6

[Novotný et al. 2006] Novotný, J. ; Jurmanová, J. ; Geršl, J. ; Svobodová, M.: *Základy teorie relativity.* 2006. – URL https://is.muni.cz/elportal/?id=703391. – Date of access: 2022-07-18

[Orzel 2012] Orzel, C.: *How to teach relativity to your dog.* New York : Basic Books, 2012. – ISBN 978-0-465-02331-8

[Otero et al. 2015] Otero, M. R. ; Arlego, M. ; Prodanoff, F.: Teaching the basic concepts of the Special Relativity in the secondary school in the framework of the Theory of Conceptual Fields of Vergnaud. In: *NUOVO CIMENTO C-COLLOQUIA AND COMMUNICATIONS IN PHYSICS* 38 (2015), Nr. 3. – ISSN 2037-4909

[Ozcan 2017] Ozcan, O.: Examining the Students' Understanding Level Towards the Concepts of Special Theory of Relativity. In: *Problems of Education in the 21st Century* 75 (2017), Nr. 3, P. 263–269. – ISSN 1822-7864

[Panse et al. 1994] Panse, S. ; Ramadas, J. ; Kumar, A.: Alternative conceptions in Galilean relativity: frames of reference. In: *International Journal of Science Education* 16 (1994), Nr. 1, P. 63–82. – URL https://doi.org/10.1080/0950069940160105

[Pitts et al. 2014] Pitts, M. ; Venville, G. ; Blair, D. ; Zadnik, M.: An Exploratory Study to Investigate the Impact of an Enrichment Program on Aspects of Einsteinian Physics on Year 6 Students. In: *Research in Science Education* 44 (2014), 06

[Postiglione and De Angelis 2021] Postiglione, A. ; De Angelis, I.: Experience gravity in the classroom using the rubber sheet: an educational proposal from the collaboration between university and school. In: *Physics Education* 56 (2021), Nr. 2, P. 025019

[Ramadas et al. 1996] Ramadas, J. ; Barve, S. ; Kumar, A.: Alternative conceptions in Galilean relativity: inertial and non-inertial observers. In: *International Journal of Science Education* 18 (1996), Nr. 5, P. 615–629. – URL https://doi.org/10.1080/0950069960180509

[Roussel 2022] Roussel, A.: *Falling into a realistic Black Hole (VR 360°).* 2022. – URL https://youtu.be/17tEg_uTF_A. – Date of access: 2022-07-18

[Ruggiero et al. 2021] Ruggiero, M. L. ; Mattiello, S. ; Leone, M.: Physics for the masses: teaching Einsteinian gravity in primary school. In: *Physics Education* 56 (2021), Nr. 6, P. 065011

[Ryston 2014] RYSTON, M.: *Možnosti elementárního výkladu obecné teorie relativity*, Charles University, Faculty of Mathematics and Physics, Master's Thesis, 2014

[Ryston 2019a] RYSTON, M.: Embedding Diagrams - a Hands-on Activity for Understanding Spatial Curvature. In: *Journal of physics. Conference series* 1287 (2019), Nr. 1. – ISSN 1742-6588

[Ryston 2019b] RYSTON, M.: Interactive animations as a tool in teaching general relativity to upper secondary school students. In: *Journal of Physics: Conference Series* 1286 (2019)

[Ryston 2020a] RYSTON, M.: Obecná teorie relativity středoškolsky – 1. část: Neeukleidovská geometrie. In: *Československý časopis pro fyziku* 70 (2020), Nr. 2, P. 135

[Ryston 2020b] RYSTON, M.: Obecná teorie relativity středoškolsky – 2. část: Geometrický pohled na gravitaci. In: *Československý časopis pro fyziku* 70 (2020), Nr. 3, P. 198

[Ryston 2020c] RYSTON, M.: Obecná teorie relativity středoškolsky – 3. část: Dilatace času a GPS. In: *Československý časopis pro fyziku* 70 (2020), Nr. 4, P. 254

[Ryston 2020d] RYSTON, M.: Obecná teorie relativity středoškolsky – 4. část: Gravitace jako zakřivení prostoročasu. In: *Československý časopis pro fyziku* 70 (2020), Nr. 5, P. 332

[Ryston 2021] RYSTON, M.: *Introducing General Relativity Without Special Relativity*. Chap. 24, P. 361–370. In: KERSTING, M. (Hrsg.) ; BLAIR, D. (Hrsg.): *Teaching Einsteinian Physics in Schools: An Essential Guide for Teachers in Training and Practice*, Routlege, 2021. – ISBN 9781003161721

[Ryston 2022a] RYSTON, M.: Homemade Geometrical Models - Teaching General Relativity at the Secondary School Level with Activities. In: *AIP Conference Proceedings* 2458 (2022)

[Ryston 2022b] RYSTON, M.: *Relativita: Proč a jak?* 2022. – URL `https://kdf.mff.cuni.cz/relativita-stredoskolsky`. – Date of access: 2022-07-25

[Schutz 2003] SCHUTZ, B.: *Gravity from the Ground up*. Cambridge University Press, 2003 (1st ed.). – ISBN 978-0-521-45506-0

[Selcuk 2011] SELCUK, G. S.: Addressing pre-service teachers' understandings and difficulties with some core concepts in the special theory of relativity. In: *EUROPEAN JOURNAL OF PHYSICS* 32 (2011), Nr. 1, P. 1–13. – ISSN 0143-0807

[SeznamŠkol.eu 2022] SEZNAMŠKOL.EU: *Seznam škol v ČR*. 2022. – URL `http://www.seznamskol.eu/`. – Date of access: 2022-06-18

[Soukup 2015] Soukup, M.: *Možnosti prezentace základů speciální teorie relativity*, Charles University, Faculty of Mathematics and Physics, Master's Thesis, 2015

[Spacetime 2022] Spacetime, PBS: *PBS Spacetime*. 2022. – URL https://www.youtube.com/c/pbsspacetime. – Date of access: 2022-07-18

[Stanford 2022] Stanford: *Leonard Susskind - All Stanford physics lectures in order*. 2022. – URL https://youtube.com/playlist?list=PL6i60qoDQhQGaGbbg-4aSwXJvxOqO6o5e. – Date of access: 2022-07-18

[Stannard 2018] Stannard, W.: A new model of special relativity and the relationship between the time warps of general and special relativity. In: *Physics Education* 53 (2018), 05, P. 035013

[Steane 2011] Steane, A. M.: *The Wonderful World of Relativity: a Precise Guide for the General Reader*. New York : Oxford University Press, 2011. – ISBN 978-0-19-969461-7

[Stone 2003] Stone, M. H.: Substantive Scale Construction. In: *Journal of applied measurment* 4(3) (2003), P. 282–297

[Svoboda et al. 2020] Svoboda, E. ; Bednařík, M. ; Široká, M.: *Fyzika pro gymnázia: Mechanika*. Praha : Prometheus, 2020 (6. vydání). – ISBN 978-80-7196-482-7

[Takeuchi 2010] Takeuchi, T.: *An Illustrated Guide to Relativity*. New York : Cambridge University Press, 2010. – ISBN 978-0-521-14100-0

[Taylor and Wheeler 1992] Taylor, E. F. ; Wheeler, J. A.: *Spacetime Physics: Introduction to Special Relativity*. New York : Freeman and Company, 1992 (2nd ed.). – ISBN 978-0716723271

[Taylor and Wheeler 2000] Taylor, E. F. ; Wheeler, J. A.: *Exploring Black Holes: Introduction to General Relativity*. San Francisco : Addison-Wesley, 2000. – ISBN 0-201-38423-X

[Teaching Relativity 2020] Teaching Relativity: *On the Status Quo of General Relativity in School Curricula*. 2020. – URL https://teaching-relativity.org/GRCurriculum. – Date of access: 2022-07-07

[Topper 2013] Topper, D. R.: *How Einstein Created Relativity out of Physics and Astronomy*. Springer, 2013. – ISBN 978-1-4614-4781

[Tyson 2020] Tyson, N. d.: *Astrofyzika pro lidi ve spěchu*. Jota, 2020. – ISBN 978-80-7565-645-2

[Ullmann 1986] Ullmann, V.: *Gravitace, černé díry a fyzika prostoročasu*. Československá astronomická společnost, 1986. – URL https://astronuklfyzika.cz/GravitCerneDiry.htm

[University of Oslo 2020] University of Oslo: *The ReleQuant Project.* 2020. – URL `https://www.mn.uio.no/fysikk/english/research/projects/relequant/index.html`. – Date of access: 2022-07-07

[Vaas 2019] Vaas, R.: *Prostě Einstein!: geniální myšlenky vtipně a jasně.* Praha : Grada, 2019 (1st ed.). – ISBN 978-80-271-2061-1

[Vibli.no 2022] Vibli.no: *Natural Science and Mathematics.* 2022. – URL `https://www.vilbli.no/en/en/no/natural-science-and-mathematics/program/v.st/v.strea3----/p1`. – Date of access: 2022-06-30

[Villani and Pacca 1987] Villani, A. ; Pacca, J. L. A.: Students' spontaneous ideas about the speed of light. In: *International Journal of Science Education* 9 (1987), Nr. 1, P. 55–66. – URL `https://doi.org/10.1080/0950069870090107`

[Viten.no 2019] Viten.no: *General Relativity.* 2019. – URL `https://www.viten.no/filarkiv/general-relativity/#/`. – Date of access: 2022-07-18

[Walwema et al. 2016] Walwema, G. B. et al.: Is classical mechanics a prerequisite for learning physics of the 20th century? In: *Phys. Educ.* 51 (2016), Nr. 065022

[Wikiquote.org 2022] Wikiquote.org: *Stephen Hawking Quotes.* 2022. – URL `https://en.wikiquote.org/wiki/Stephen_Hawking#A_Brief_History_of_Time_(1988)`. – Date of access: 2022-07-15

[Wolfson 2003] Wolfson, R.: *Simply Einstein: Relativity Demystified.* New York : W. W. Norton & Company, 2003. – ISBN 978-0-393-32507-2

[Zahn and Kraus 2014] Zahn, C. ; Kraus, U.: Sector models - A toolkit for teaching general relativity: I. Curved spaces and spacetimes. In: *European Journal of Physics* 35 (2014), Nr. 5

[Zahn and Kraus 2019] Zahn, C. ; Kraus, U.: Sector models - A toolkit for teaching general relativity: II. Geodesics. In: *European Journal of Physics* 40 (2019), Nr. 1

[Zee 2019] Zee, A.: *O gravitaci: krátké seznámení se závažným oborem.* Praha : Argo, 2019 (Dokořán). – ISBN 978-80-257-2862-8

[Ženíšek 2015] Ženíšek, A.: *Relativita do kapsy.* Brno : Masarykova univerzita, 2015. – ISBN 978-80-210-8043-0

List of Figures

List of Tables

List of Abbreviations

FEP - Framework Education Programme

GR - General Relativity

MEYS - Ministry of Education, Youth and Sports

SEP - School Education Programme

SR - Special Relativity

List of Publications

(Ryston 2014) Ryston, M.: *Možnosti elementárního výkladu obecné teorie relativity*, Charles University, Faculty of Mathematics and Physics, Master's Thesis, 2014

(Ryston 2019a) Ryston, M.: Embedding Diagrams - a Hands-on Activity for Understanding Spatial Curvature. In: *Journal of physics. Conference series* 1287 (2019), Nr. 1, S. 12008–. – ISSN 1742-6588

(Ryston 2019b) Ryston, M.: Interactive animations as a tool in teaching general relativity to upper secondary school students. In: *Journal of Physics: Conference Series* 1286 (2019)

(Ryston 2021) Ryston, M.: *Introducing General Relativity Without Special Relativity*. Kap. 24, S. 361–370. In: Kersting, M. (Hrsg.) ; Blair, D. (Hrsg.): *Teaching Einsteinian Physics in Schools: An Essential Guide for Teachers in Training and Practice*, Routlege, 2021. – ISBN 9781003161721

(Ryston 2022a) Ryston, M.: Homemade Geometrical Models - Teaching General Relativity at the Secondary School Level with Activities. In: *AIP Conference Proceedings* 2458 (2022)

A. Attachments

A.1 Number of Weekly Physics Lessons of 40 Randomly Selected Gymnasiums

Below is the list of randomly selected gymnasiums and the amounts of weekly physics lessons throughout the upper secondary study program according to their School Education Programme. The selection of schools is described in more detail in subsection 2.2.

Gymnasium	**Lessons**	**Gymnasium**	**Lessons**
Botičská, Praha	8	Jiráskovo, Náchod	8
Česká Třebová	8	Tanvald	10
Slovanské, Olomouc	10	Frýdlant	9
Opatov, Praha	8	Šumperk	9
Jeseník	7	Mariánské Lázně	7.5
Písek	7	B. Hrabala, Nymburk	6
Frenštát pod Radhoštěm	10	Prachatice	7.5
Hladnov	8	Děčín	8
Vysoké Mýto	7	Rokycany	8.5
Voděradská, Praha	6	J. Barranda, Beroun	8
Omská, Praha	7	Česká, České Budějovice	8.5
Postupická, Praha	8	Jihlava	7.5
Olomouc - Hejčín	11	Stříbro	8.5
Arcibiskupské, Praha	6	Doctrina, Liberec	8
Křenová, Brno	8	F. Palackého, Val. Meziříčí	9
Polička	8	Jiřího Ortena, Kutná hora	6
Mikulášské, Plzeň	10	Masarykovo, Plzeň	9
Karlínské, Praha	7	Uničov	8
Svitavy	9	J. Vrchlického, Klatovy	9.5
Kodaňská, Praha	7	Ústí nad Orlicí	8.3

A.2 Questionnaire for Physics Teachers

Logic jumps were used in the questionnaire to assure that only questions relevant to a particular respondent based on their previous answers were displayed. Following figures demonstrate these logic jumps (images were generated directly by the questionnaire-providing website Typeform.com). The displayed questions and response options have been shortened for an easier visualization. The full list of questions with all possible answers in case of closed questions follows after the images.

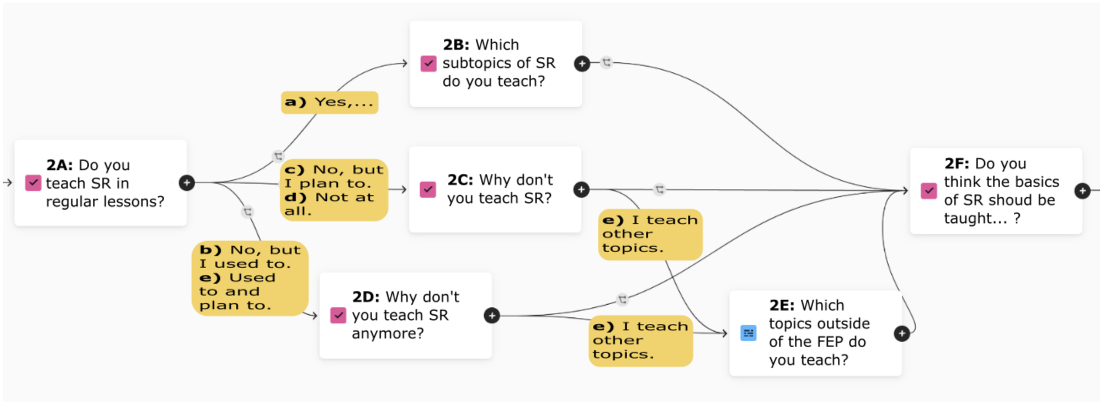


Figure A.1: Logic jumps for group 2 of the survey questions.

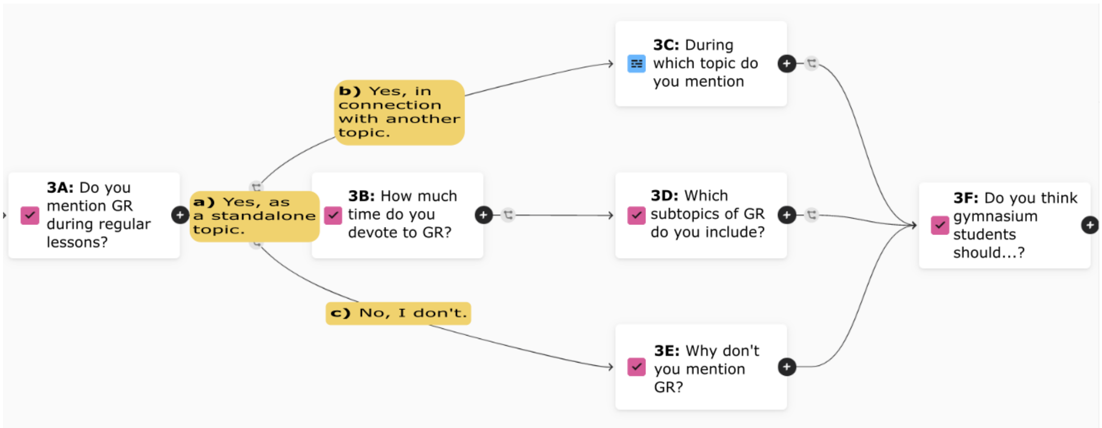


Figure A.2: Logic jumps for group 4 of the survey questions.

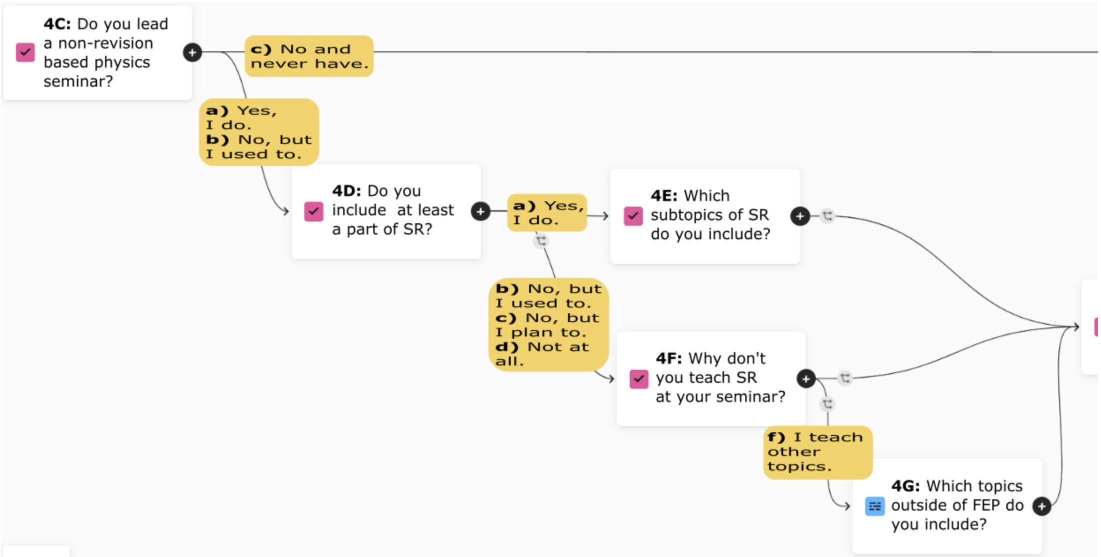


Figure A.3: Logic jumps for the first part of group 4 of the survey questions.

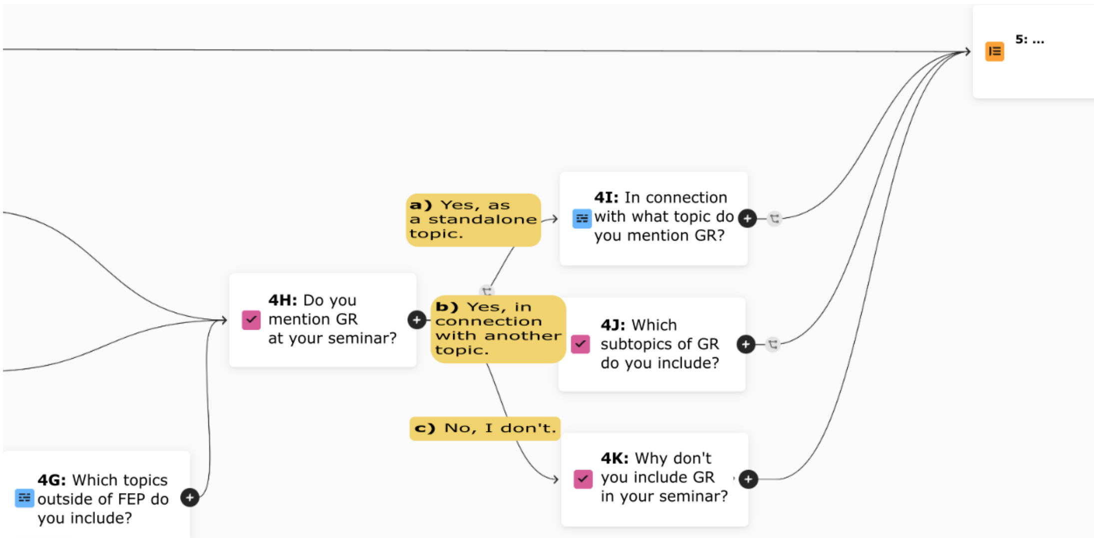


Figure A.4: Logic jumps for the second part of group 4 of the survey questions.

Questionnaire for physics teachers – theory of relativity

The purpose of this questionnaire is to map the teaching of theory of relativity at generally oriented Czech upper secondary schools. Even though you might not teach relativity directly or you have never taught it, your opinion and personal experience are valuable to us. By submitting your answer, you are helping to create a better idea about relativistic physics teaching in our country. The questionnaire takes about 10 minutes to complete.

1) **Please state the name of the school where you teach (for example Gymnasium J. K. Tyla, Hradec Králové). The name of the schools will help us map the location of respondents around the area of CR, the data assessment is completely anonymous in regards to the teacher and the particular school.** In case you teach at multiple schools eligible for this questionnaire, please fill it out for each school separately.

2) **Special Theory of Relativity (SR)**[1]

2A. Do you teach in regular lessons (i.e. not counting seminars and clubs) at least a part of the SR topic?
- a) Yes, SR is a regular part of my teaching. (**GO TO 2B**)
- b) No, but I used to teach SR before. (**GO TO 2D**)
- c) No, but I plan to include SR in my teaching. (**GO TO 2C**)
- d) No, SR is not part of my teaching, neither do I plan to include it. (**GO TO 2C**)
- e) Both B and C apply for me. (**GO TO 2D**)

2B. Which of these topics of SR do you talk about with students? (**CONTINUES TO 2F**)
- a) Michelson-Morley experiment
- b) time dilation
- c) length contraction
- d) relativistic mass/momentum
- e) chosen “paradoxes”
- f) spacetime diagrams
- g) four-vectors
- h) relativistic dynamics (equations of motion)
- i) other

2C. For what reason do you not teach SR? **(EXCEPT f) CONTINUES to 2F)**
- a) Not enough time for topics outside of the FEP.
- b) SR is too complex a topic to be taught in regular lesson in upper secondary school.
- c) Not enough quality study materials.
- d) I don’t feel confident in my knowledge of SR to teach it.
- e) I teach only the lower secondary part of gymnasium.
- f) I teach other topics that are outside the FEP. (**GO TO 2E**)
- g) Other

2D. For what reason do you not teach SR anymore? **(EXCEPT e) CONTINUES to 2F)**
- a) Not enough time for topics outside of the FEP.
- b) SR is too complex a topic to be taught in regular lesson in upper secondary school.
- c) Not enough quality study materials.
- d) I don’t feel confident in my knowledge of SR to teach it.
- e) I teach other topics that are outside the FEP. (**GO TO 2E**)
- f) Other

[1] We used STR (Special Theory of Relativity) instead of SR and similarly GTR instead of GR in the original Czech version because those abbreviations, especially the first one, are more common in Czech.

2E. Which topics outside of the FEP do you include? (open question) **(CONTINUES to 2F)**

2F. Do you think that the basics of SR should be taught in regular lessons at gymnasium?
   a) Definitely yes.
   b) Probably yes.
   c) Probably not.
   d) Definitely not.

3) General theory of relativity (GR)

In this section you will be asked about topics connected with GR, such as **curvature, spacetime near bodies, gravitational time dilation, black holes, gravitational waves, Big Bang, gravitational red shift,** etc.

3A. Do you mention in your regular teaching, at least briefly, GR?
   a) Yes, I talk about GR as a standalone topic. (**GO TO 3B**)
   b) Yes, I occasionally mention GR in connection with another topic. (**GO TO 3C**)
   c) No, I don't mention GR. (**GO TO 3E**)

3B. How much time do you devote to GR? **(CONTINUES to 3D)**
   a) 10 minutes at most.
   b) 30 minutes at most.
   c) A whole lesson (45 minutes).
   d) 2 lessons at most.
   e) More than 2 lessons.

3C. In connection with what topic do you mention GR? (open question) **(CONTINUES to 3F)**

3D. Which of these topics of GR do you talk about with students? **(CONTINUES to 3F)**
   a) spacetime
   b) surface and spatial curvature
   c) gravitation = curvature of spacetime
   d) gravitational time dilation (e.g. GPS)
   e) astronomical consequences of GR (e.g. black holes, perihelion shift, …)
   f) cosmological consequences of GR (e.g. Big Bang, expansion of universe, …)
   g) gravitational waves
   h) other

3E. For what reason do you not mention GR? You can select more than one. **(CONTINUES to 3F)**
   a) Not enough time for topics outside of the FEP.
   b) GR is too complex a topic to be taught in regular lesson in upper secondary school.
   c) Not enough quality study materials.
   d) I don't feel confident in my knowledge of GR to teach it.
   e) I teach only the lower secondary part of gymnasium.
   f) Other

3F. Do you think gymnasium students should be introduced to basic ideas of GR in school?
   a) Yes, it is an interesting part of physics.
   b) Yes, but not in regular lessons (i.e. include it in seminars).
   c) Even basics of GR are too complicated and abstract a topic for secondary school, those interested can find information on their own.
   d) Other

4) Physics seminars

4A. Do you think the basics of SR should be taught as part of a physics seminar?

a) Definitely yes.
b) Probably yes.
c) Probably not.
d) Definitely not.
e) No, because it should be part of regular lessons.

4B. Do you think the basics of GR should be taught as part of a physics seminar?

a) Definitely yes.
b) Probably yes.
c) Probably not.
d) Definitely not.
e) No, because it should be part of regular lessons.

4C. Do you teach a broadening (i.e. not strictly revision based) physics seminar at upper secondary school?

a) Yes, I do.
b) No, but I used to.
c) No, and I have never taught it before. (**GO TO 5A**)

4D. Do you teach at least part of SR at the seminar? (If you have answered that you no longer teach a seminar in the last question, please assume that the following questions are about your past teaching of the seminar).

a) Yes, I normally include SR among the seminar topics.
b) No, but I used to teach SR. (**GO TO 4F**)
c) No, but I plan to add SR into my teaching. (**GO TO 4F**)
d) No, SR is not part of my teaching and I do not plan to add it. (**GO TO 4F**)

4E. Which subtopics of SR do you talk about with students? (**CONTINUES TO 4H**)

a) Michelson-Morley experiment
b) time dilation
c) length contraction
d) relativistic mass/momentum
e) chosen “paradoxes”
f) spacetime diagrams
g) four-vectors
h) relativistic dynamics (equations of motion)
i) other

4F. For what reason do you not teach SR at your seminar? You can select more than one. (**EXCEPT f) CONTINUES TO 4H**)

a) Students are not interested / more interested in other topics.
b) SR is too complex a topic to be taught at a upper secondary school seminar.
c) Not enough quality study materials.
d) I don’t feel confident in my knowledge of SR to teach it.
e) I teach SR in regular lessons.
f) I prefer including other interesting topics. (**GO TO 4G**)
g) Other

4G. What topic (or topics) do you include instead of SR? (open question)

4H. Do you mention, at least briefly, GR at your seminar?

a) Yes, I talk about GR as a standalone topic. (**GO TO 4J**)
b) Yes, I occasionally mention GR in connection with another topic. (**GO TO 4I**)
c) No, I don't mention GR. (**GO TO 4K**)

4I. In connection with what topic do you mention GR? (open question) (**CONTINUES TO 5A**)

4J. Which subtopics of GR do you talk about with students? (**CONTINUES TO 5A**)

a) Spacetime
b) surface and spatial curvature
c) gravitation = curvature of spacetime
d) gravitational dime dilation (e.g. GPS)
e) astronomical consequences of GR (e.g. black holes, perihelion shift, …)
f) cosmological consequences of GR (e.g. Big Bang, expansion of universe, …)
g) gravitational waves
h) other

4K. For what reason do you not teach GR at your seminar? You can select more than one. (**CONTINUES TO 5A**)

a) Students are not interested / more interested in other topics.
b) GR is too complex a topic to be taught at an upper secondary school seminar.
c) Not enough quality study materials.
d) I don't feel confident in my knowledge of GR to teach it.
e) Other

5) Below please select how often and in what way do you yourself refer students to interesting physics topics outside of regular teaching.

5A. Popular physics books:

a) Fairly regularly (e.g. a few times per month).
b) Sometimes (not more than once a month).
c) Rarely (at most a few times per school year).
d) Never.

5B. Television and radio shows:

a) Fairly regularly (e.g. a few times per month).
b) Sometimes (not more than once a month).
c) Rarely (at most a few times per school year).
d) Never.

5C. Internet videos (Youtube, etc.):

a) Fairly regularly (e.g. a few times per month).
b) Sometimes (not more than once a month).
c) Rarely (at most a few times per school year).
d) Never.

5D. Internet websites:

a) Fairly regularly (e.g. a few times per month).
b) Sometimes (not more than once a month).
c) Rarely (at most a few times per school year).
d) Never.

5E. Physics-themed events:
   a) Fairly regularly (e.g. a few times per month).
   b) Sometimes (not more than once a month).
   c) Rarely (at most a few times per school year).
   d) Never.

5F. Or perhaps any other, if something not included comes to your mind: (open question)

6) General questions:

6A. Do you get questions from your students regarding physical topics that don't belong into your regular teaching (e.g. relativity, quantum mechanics, etc.)?
   a) Yes, often.
   b) Yes, sometimes.
   c) Rarely.
   d) Practically never.

6B. If you do get such a question outside of the scope of teaching and you don't immediately know the answer, how do you solve that situation?
   a) I refer the student to appropriate literature.
   b) I refer the student to an internet source of information.
   c) I refer the student to a colleague or a specific expert in the field.
   d) I look for necessary information myself and the go over the topic with the student.
   e) Other

6C. How often do you use internet sources (e.g. images, videos, applets, websites) directly during your teaching?
   a) Fairly often.
   b) Sometimes.
   c) Rarely.
   d) Never.

6D. According to your experience, how many percent of your students are interested in physics enough to engage in it (e.g. look for information, read books, do their own physics projects, etc.) outside school in their free time?
   a) 0 %
   b) 1-2 %
   c) 3-5 %
   d) 6-10 %
   e) 11-15 %
   f) 16-20 %
   g) 21-25 %
   h) Over 25 %

6E. Please state the length of your teaching practice rounded to whole years: (open question)

6F. This space is devoted to you. If you want to say anything regarding the current state of teaching relativity at Czech upper secondary schools, or if you have a favourite source of information about relativity, we will be happy if you let us now. (open question)

6G. Thank you for answering our questions [etc.]…

A.3 Statements of Physics Teachers Regarding the Role of Relativity in Gymnasiums

The following is an expanded list of answers that some respondents gave in the survey when presented with the opportunity to express their opinion regarding teaching of relativity. The answers were originally in Czech and later translated into English.

Answers included in Chapter 2:

- *Usually it is not possible to discuss everything in depth, because SR is included as the last topic of teaching in a very packed year. It is discussed in depth in the physics seminar.*
- *Personally, I would prefer to reduce some chapters so that there is more time left for SR (GR), but at the same time I am often not able to skip the optional chapters of some areas preceding physics. Therefore, we most often encounter the issues of relativity through students' questions.*
- *Due to the number of lessons devoted to physics in compulsory education and also the interest in physics, I consider the teaching of SR to be unnecessary. Seminars also offer more useful topics for further study at universities. I personally teach in a seminar the use of derivatives and integrals in physics.*
- *I don't know how it is in other high schools, but we all teach relativity. Most students are more interested in this topic and enjoy it more than previous "classic" topics. They also come to a chapter for the first time where it is clear that they will only look at the edge and that the real depth of the problem is much greater.*
- *In my judgment, the material does not belong to a general gymnasium at all. It confuses students who have difficulty with high school physics. Students gain the feeling that physics is not only difficult but even absurd.*
- *This topic is interesting, unfortunately it is taught in the fourth year of upper gymnasium, when students have their heads full of maturita and it is very difficult for teachers to excite them for physics in this period.*
- *Not enough time.*
- *I teach SR during labs.*
- *I did not teach SR in a regular class for the first time this year when one physics lesson was removed from the schedule. Colleagues haven't taught SR in a long time. In the next classes, I plan to include SR again and sacrifice something else (probably Electrostatics). I consider the introduction to SR and quantum physics to be essential, because of the difference from the world that the students know from their experience.*

- *The topic is quite interesting for students, but unfortunately due to lack of time considered marginal, so it is not possible to show students the application of SR, for example, in terms of astronomy, astrophysics, quantum physics, and particle physics.*
- *It depends on the "space-time" that the teacher has available to teach SR. The students should leave high school with at least the following three pieces of information on this topic: that things are "a little different" than in everyday life, when they are "different", and why they are "different". They can find out how things are at any time and study it later, it is essential to understand the causes and accept the "otherness".*
- *It would be nice to have some high school ideas for GR reasonably written - and, for example, reasonably linked to astronomy.*
- *I haven't taught SR yet, but I hope to one day. It is also a challenging topic for me, so I will have to study it diligently.*
- *I think the topic is interesting, everyone should at least have an idea about it. Unfortunately, I don't understand enough on my own to be able to answer all the questions.*

Additional answers:

- *Abolition of physics in the 4th year of the gymnasium requires a reduction of the curriculum. For years I had 10-12 lessons of SR in the 4th year. All fundamental relationships were derived, paradoxes was analyzed... In recent years, at least in the seminar I supplemented what did not fit into the time schedule of regular physics lessons.*
- *Not enough time, small allotment of lessons.*
- *At a time when the hourly allowance for physics is generally very small, I consider it reasonable to include those parts that are necessary for understanding other topics (atomic physics).*
- *Due to the quality of the students, it is difficult to include such topics.*
- *The teaching of SR is only a very marginal problem of all the problems in the current teaching of physics (lesson allotment, teaching aids, laboratory equipment, teacher education).*
- *Today's students are quite difficult to get interested in something...they find what they are interested in on the internet and often have so much different information in their heads that teachers have a problem squeezing something new into them.*
- *Due to the low lesson allotment for physics at gymnasiums, in the case of SR and even more so GR, there is a problem of lack of time. However, due to my astrophysics background, I always include SR and especially GR in my teaching.*

- *I stated that I teach SR in regular classes, i.e. in physics classes in the final year. I want to specify that this subject is not compulsory for all students here, but is chosen by roughly half of the students (within the science block). Due to the fact that there are fewer and fewer physics lessons (e.g. thanks to mandatory state maturita exams or state entrance exams, more and more days in which regular lessons take place are lost) and due to the low willingness of students to devote themselves to more difficult topics, we are still thinking about which part of physics to drop . SR is among the first that come to mind. We don't want to leave it out yet, but it's possible that we won't be able to avoid it in the future. Because I think the basics of SR are beautiful, in the 3rd year, in the last lesson of physics, I read excerpts from the book Mr. Tomkins in Wonderland to the students (all of them, i.e. even those for whom it is the very last lesson of physics at secondary school). With those students who continue with physics in the 4th year, we discuss the basics of SR in 4 lessons.*

- *The fundamental problem of lack of interest in physics and natural sciences is that physics is usually not taught at elementary schools by qualified teachers, and those who lead it then degrade it to the use of PowerPoint. Disinterest is also due to little knowledge of mathematics. Pupils learn something, but cannot use the learned in practice. Many can't even learn effectively - just memorize by heart without understanding the problem. He is not practicing. Pressure from the public and parents is seldom in favor of the natural sciences. Most people interested in universities apply to humanities (they consider them easier).*

- *Even SR is no longer taught by us at the gymnasium during the compulsory lessons of physics for everyone, but E equals mc2 is known nearly by everyone...*

- *The study of physics in our country faces a small interest of students, but also "effective" use of funds (if at least 10 students do not register for the seminar, they are "unlucky" and have to choose, for example, geography, history, art education, etc.). A PITY...*

- *The hourly allowance of physics is so low that it is a problem to discuss what is in the FEP; in the FEP we do not even have astronomy and astrophysics; I don't teach physics in language classes, but "tell stories about physics".*

- *At our school, half of the classes are focused on living languages and some students show a lack of interest in science. In all classes (but especially in the language classes) we are bothered by a lack of time.*

- *From my point of view, special relativity is definitely an interesting topic. However, I have noticed that SR is one of the marginal topics and is usually put off until the end of the school year, where there are not enough hours left. In practice, therefore, the scope of its teaching is determined mainly by time possibilities.*

- *I think that the problem of teaching SR will be topical when students first master the basic mechanics satisfactorily (or at least expand a fraction).*

- *A new textbook would be good, because the current booklet on SR is completely useless (it does not contain a single derivation, but many quotes by A. Einstein, often completely out of the question). The concept of relativistic mass has also been abandoned by professionals long ago.*

- *It is a pity that due to the small lesson allotment of physics, it is not possible to discuss this topic at least partially in class.*

- *The theory of relativity also interests tired 4th year students, especially those who have not shown much interest in physics.*

- *SR is a wonderful part of physics that is different from the others, it talks about time and space around us, and at the same time the students are not interested in this part of physics either. So I don't know what's more interesting to teach them...*

- *SR is one of the high school maturita topics at our school.*

- *SR definitely does not belong to ordinary teaching, it is difficult to manage even "standard" topics. I personally feel that at our school, for example, optics is neglected. And to include, for example, time dilation at the expense of explaining the origin of the rainbow is not at all desirable. Therefore, SR clearly belongs to the seminar.*