Thesis Review

Faculty of Mathematics and Physics
Charles University, Prague

❑ Supervisor's Review ☑ Referee's Review **
❑ BSc. Thesis ☑ MSc. Thesis

Author:	Matus Papajcik
Thesis title:	Exact spacetimes in 2+1 gravity
Study program:	Physics - Theoretical physics
Submitted:	2023

Supervisor/Referee: Prof. Hideki Maeda
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Thesis quality (technical field and expertise):
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

Objective accuracy (error appearance):
☑ nearly perfect ❑ standard ❑ frequent, but minor ❑ serious errors

Thesis results:
☑ original ❑ both original and compiled ❑ productive compilation ❑ copied

Thesis size:
❑ large ☑ standard ❑ just acceptable ❑ insufficient

Thesis quality (style and grammar, and graphic arrangement):
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

Misprints:
☑ negligible ❑ acceptable number ❑ very frequent

Overall thesis quality:
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

** Copy and paste this check box, ☑, if it is applicable.

Supervisor's/Referee's Comments:

The main topic of this master thesis is exact solution to the Einstein equations in 2+1 dimensions and its properties in the presence of a cosmological constant and a Maxwell field as a matter field. In 2+1 dimensions, the number of independent components of the Riemann tensor and Ricci tensor are equal, so that there is no gravitational wave and the local spacetime geometry is completely determined by the Ricci tensor which is governed by the Einstein equations. Due to this property, 2+1-dimensional gravity has been actively studied in order to gain insight into the quantum theory of gravity, which is still incomplete.

Since the quantum state of spacetime should be a superposition of classical solutions to the Einstein equations, a complete clarification of the classical solution space would be a major contribution to the study of quantum gravity. In the vacuum case (without a Maxwell field) in 2+1 dimensions, a general solution to the Einstein equations has been obtained in the reference [13] under few assumptions. In this case, the spacetime is locally maximally symmetric, i.e., Minkowski, de Sitter (dS), or anti-de Sitter (AdS) spacetime. Nevertheless, spatial identification of the AdS spacetime allows to describe a black hole, and this BTZ black hole has been playing a central role in the study of quantum gravity.

In the presence of a Maxwell field, the spacetime is no longer maximally symmetric locally, and it is not at all obvious what classical solutions are possible. Also, as the author shows in Section 1.2, a Maxwell field in 2+1 dimensions is equivalent to a massless scalar field because the dual of the 2-form field of the Maxwell field is a 1-form field. Therefore, exact solutions with a Maxwell field can be applied as exact solutions with a massless scalar field. For these reasons, it is a very challenging and nontrivial question whether a general solution can be obtained in the Einstein-Maxwell system in the presence of a cosmological constant in 2+1 dimensions, which is a natural generalization of the result in the reference [13].

This master thesis gives a partial but very advanced answer to this question. Based on the results in the literature [13], the 2+1-dimensional solutions are classified into the Kundt class and the Robinson-Trautman class depending on whether the expansion scalar of a null geodesic vector field is vanishing or not. In Chapter 2, the author derived the general solution for the Kundt class. In Chapter 3, the general solution for the Robinson-Trautman class was derived when the Faraday tensor is aligned with the privileged geometrical direction. In Chapter 4, on the other hand, the author solved most of the field equations in the non-aligned case, but did not succeed in solving all of them. Nevertheless, the author still finds a special solution using a separation of variables and discussed whether exact solutions exist in the non-aligned case. Lastly, in Chapter 5, the author developed an algebraic classification of 2+1-dimensional spacetime. Such a classification has been done by Garcia-Diaz in reference [12] using the Cotton tensor corresponding to the Weyl tensor in higher dimensions, or equivalently the Cotton-York tensor. In this master thesis, the author performed a spacetime classification using the Newman-Penrose-type scalars constructed from the Cotton tensor and showed that this Penrose-type classification is equivalent to the classification using the Cotton(-York) tensor. At the end of the chapter, as a demonstration, this Penrose classification was applied to the solution of the aligned Robinson-Trautman class.

This master thesis is written in a very reader-friendly manner and has an appropriate bibliography. The derivation of the solutions in Chapters 2 through 4 are carefully explained step by step, and nothing is omitted. The geometric quantities required for the calculations are summarized in Appendix A, which allows the reader to follow all the steps completely. This philosophy is continued in the explanations in Chapters 1 and 5, which also provide a good review of the 2+1-dimensional spacetime and its algebraic classification. In summary, this master thesis produces very strong results and they are carefully and cleanly presented. The standard of this master thesis is very high and entirely sufficient to award a master degree.

Questions raised (and to be answered by the author during the Thesis Defence):

I think that the charged rotating BTZ solution discussed in reference [9] is included in the Kundt class. Does it correspond to the case where the \alpha is non-zero in Eq. (3.69)?

Incidentally, I found the following typos:

- In (1.69), 1/\sqrt{-g} in the bracket should be \sqrt{-g}.
- In the second line in the first paragraph in Section 5.4, "(5.81)" should be (5.75).

Supervisor's/Reviewer's recommendation on Thesis rating:

☑ excellent ❑ very good ❑ standard ❑ reject

Done in ....Sapporo, Japan........

Date ..12th January, 2023.....

Name .... Hideki Maeda.........

Signature ... 前田秀基 ...