Gravitational lensing serves as an invaluable tool for studying the distribution of matter in the universe. This matter is predominantly dark and clumped into centrally concentrated hierarchically structured halos of galaxies and galaxy clusters. Early comparisons suggested discrepancies between the high population of substructures predicted by cosmological simulations and the lack of corresponding observational data. More recently, a major discrepancy in the opposite sense was reported from analyses of lensing galaxy clusters: the lensing efficiency of their substructures was found to be much higher than predictions based on cosmological simulations. In this thesis, we examine gravitational lensing by substructures embedded in dark-matter halos with Navarro—Frenk—White (NFW) density profiles. We start by a detailed investigation of a simple model with a single point mass perturbing the halo. Using analytical methods, we study its critical curves, caustics and their transitions. Next, we explore the geometry of lensed images and the weak-lensing characteristics of the same simple model, developing novel ways of their visualization. Finally, we construct a more realistic lens model of a galaxy cluster consisting of an ellipsoidal cluster halo combined with a population of truncated ellipsoidal galaxy halos. For both types of halos we present analytical deflection-angle formulae. We choose parameters of the galaxy cluster based on observations and cosmological simulations. We generate source-plane and image-plane maps of the shear, phase and other lensing quantities and discuss them, applying insights derived from our study of the simpler model. The developed code can be used for advanced modeling and detailed studies of cluster lensing, which may possibly indicate routes toward resolving the substructure lensing-efficiency discrepancy.