



Prof. Dr. M. Bartelmann • ITP • Philosophenweg 16 • 69120 Heidelberg

Dagmar Zádřapová  
Studijní oddělení  
Ke Karlovu 2027/3  
121 16 Praha 2  
The Czech Republic

[dagmar.zadrapova@matfyz.cuni.cz](mailto:dagmar.zadrapova@matfyz.cuni.cz)

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**Referee's report on the doctoral thesis of  
RNDr. Michal Karamazov**

Prof. Dr. Matthias Bartelmann  
Phone: +49-6221-54-4817  
Fax: +49-6221-54-4221  
[bartelmann@uni-heidelberg.de](mailto:bartelmann@uni-heidelberg.de)

Dear Dr. Rokyta,  
Dear Dr. Kulich,  
Dear Colleagues:

RNDr. Karamazov's thesis deals primarily with gravitational lensing by galaxy clusters. Following a concise introduction to the gravitational lensing effect and dark-matter haloes, it discusses three main questions in one chapter each. The first two of these (chapters 2 and 3) discuss spherically symmetric lens models perturbed by point masses, while the third (chapter 4) approaches realistic mass distributions by taking ellipsoidal lensing bodies into account.

The first question addressed concerns critical structures of large-scale lenses perturbed by point masses. Since gravitational lensing is a map from one two-dimensional space to another generated by a curl free vector field, its Jacobian matrix is a symmetric, real, two-by-two matrix. It therefore has two real eigenvalues which are generally functions of position. Critical structures are defined as sets of those points where any of these eigenvalues vanishes. Since the Jacobian matrix at these points is not invertible, they mark locations where the lens map changes qualitatively. As has been shown in catastrophe theory, such critical structures have universal properties because they can be characterized in a way independent of the specific lens model. Fundamental parts of critical curves are folds and cusps, which can be combined to form more complicated catastrophes, and which change in characteristic metamorphoses. Analyzing critical structures of gravitational lenses is therefore important for drawing conclusions from observations of gravitational lensing independent of details of the lensing mass distributions.

Even though the assumption of a spherically symmetric lens perturbed by point masses is idealistic, the critical structures produced by such models are therefore relevant also for more realistic or complicated mass distributions. The first main chapter (chapter 2) of this thesis reveals the rich morphology of critical structures dependent on essentially two parameters, one of them characterizing the strength of the large-scale lens, the other characterizing the relative strength of the point-mass perturbation. Even though the individual morphological types appearing here are well known, this chapter presents two main new

results. The first of these is the analytic treatment of critical structures of the perturbed lens model. The second concerns the disappearance of the so-called radial critical structure if a point mass located at the center of the lens exceeds a certain mass limit.

In the second main chapter (chapter 3) of this thesis, the same type of perturbed lens model is used to study image properties, mainly of point-like sources. This is done mainly by considering image magnifications, distortions, and orientations. New aspects presented here are first of all an innovative type of diagram, showing the Jacobian determinant of the lens map in the plane spanned by the two optical scalars (shear and convergence), and second of all a detailed and richly illustrated investigation of the change in image orientations caused by the perturbing point masses. A third new aspect is the emphasis of magnified images with zero shear, which should constitute an object class expected regularly in strongly gravitationally lensing systems, but which has not been observed so far and only been mentioned in passing in the previous literature.

In the third main chapter (chapter 4), the lens model is extended to becoming much more realistic than in the previous chapters. Giving up idealizations also implies that the analysis of critical structures and image configurations needs to become much more qualitative than before. Nonetheless, important aspects of the previous discussions, such as the richness of critical structures, their morphological types and metamorphoses, image orientations, magnified images with zero shear and so on apply here as well and are being thoroughly discussed.

Galaxy clusters are an important object class for cosmological research because they can be considered as a kind of index fossil for astrophysics and cosmology. As a population of cosmic objects, clusters are highly sensitive to details of the cosmological model. As discussed in this thesis, the level of substructure in clusters should be indicative of the type of dark matter, and gravitational lensing can substantially magnify relevant effects. Since the shapes of galaxy clusters and their substructures are expected to be highly irregular, it is centrally important to understand which of the observed features of gravitational lensing are universal in the sense of being independent of model details, and which are not.

I see the main merit of this thesis in moving several steps ahead of the existing literature towards this goal. Furthermore, since strong lensing effects are independent of absolute scales, the results presented here can be applied also to completely different classes of astrophysical situations, among which are galaxy-scale lenses perturbed by satellite galaxies or black holes, or even to microlenses embedded into larger-scale, smooth lensing objects. I therefore consider this thesis to be an original and innovative contribution to our understanding of gravitational lensing with rich possible implications for astrophysics and cosmology. In addition, the thesis is well and concisely written, clearly pointing out assumptions and limitations of the applied approaches, relating its results to neighbouring areas of astrophysics and cosmology, illustrating its main aspects in intuitive ways, and thoroughly discussing its implications. I thus arrive at the firm conclusion that this thesis proves RNDr. Karamazov's ability for creative scientific work.

Yours sincerely,

Matthias Bartelmann