Thesis supervisor's report on the doctoral dissertation by RNDr. Michal Karamazov

**"Gravitational Lensing by Substructures in Dark Matter Halos"**

Dark matter halos are the fundamental building blocks in hierarchical structure formation arising from the ΛCDM cosmological model. Within them, baryonic structure forms—galaxies in the smaller halos; galaxy clusters in the larger ones. One of the main tools for observational verification of this theoretical scenario is provided by gravitational lensing. The analysis of the effect of intervening mass on the light from background objects has yielded maps of the large-scale distribution of matter at cosmological scales as well as similar maps of individual galaxy clusters.

With initial analyses confirming good agreement of theory and observations at large scales, research focus has been turning to finer spatial scales with the aim of using gravitational lensing to study the substructure of the halos. In the case of galaxy-cluster halos, some of the outstanding questions are: what is the population of subhalos; do unaccounted subhalos affect weak-lensing analyses of the clusters; are there subhalos that do not contain galaxies? In the case of galactic halos, how does the subhalo population differ from the population of satellite galaxies?

State-of-the-art mapping of the matter distribution in clusters combines weak-lensing analyses of the outer parts of the cluster with strong-lensing models of the inner parts. The latter are usually based on the fitting of simple parametric models for the individual components: the cluster halo and the subhalos of individual galaxies. While the resulting maps are impressive, they provide only limited insight into the properties of the composite lens model. The lensing properties of individual components might be known, but for the combination of two or more components our understanding of the imaging of background sources rapidly vanishes.

The overall goal of our research in this field is to build a lens model of a dark-matter halo with a population of subhalos, to systematically study its analytical lensing properties, and to detect potential systematics or degeneracies that could impact lensing analyses of observed clusters or galaxy-galaxy lenses. The work carried out by Michal Karamazov and presented in this thesis focused on two major steps toward this goal. First, he studied in detail the properties of a simple lens model with a single localized substructure represented by a spherical dark-matter halo combined with a point mass. Second, he constructed a lens model consisting of an ellipsoidal dark matter halo with a population of ellipsoidal subhalos that permits an analysis of the same range of lensing quantities studied in the simple model.

The range of critical curves and caustics of the halo + point-mass model is presented in Chapter 2 of the thesis, published as an article in The Astrophysical Journal (Karamazov, Timko, & Heyrovský 2021). Michal performed a detailed study of the lensing regimes, and presented their full mapping in the parameter space of the model. The sequence and types of different transitions as a function of separation of the point mass till the decoupling of the system into two separate lenses is unexpectedly complex, particularly in the low-mass case. Contributions of other authors to this article are limited to the following: Lukáš Timko carried out initial tests and a rough scan of the parameter space; I contributed some of the analytical results and oversaw the final write-up.

Further lensing characteristics of the simple model such as the shear and phase are explored in Chapter 3, together with the properties of images formed by such a lens. In addition, Michal studied the "weak shear" and "weak phase", quantities that we defined as the shear and phase values based on image geometry assuming the validity of the weak-lensing approximation. Detailed maps of all quantities and their mutual deviations are presented and interpreted for a representative grid of separations and masses of the point mass. This chapter was published in The Astrophysical Journal (Karamazov & Heyrovský 2022), with me contributing some of the analytical results (e.g., in Sections 3.2.1 and 3.3.1), the convergence–shear diagrams, and overseeing the write-up.

In Chapter 4, Michal describes the construction of an advanced cluster-lens model, with all properties of the cluster halo and its population of subhalos based either on observations or on properties obtained from structure-formation simulations. Maps of several lensing quantities are presented for a sample cluster, and specific features that can be explained using the simple model from the previous chapters. The constructed model provides a powerful tool for studying the properties of cluster lensing, permitting the customization of all properties including the distributions of masses, positions, and shapes of the subhalos. My contribution to this chapter is limited to the formulae for the deflection angles in Section 4.2.

In addition to the two articles mentioned above, several more will arise from the thesis: one on the cluster model described in Chapter 4, and two on analytical lens models solved as a byproduct—the ellipsoidal Navarro–Frenk–White model and its truncated version. During his studies Michal also presented his work in poster form at three international conferences: Cosmology 2018 (Dubrovnik), Tracing Cosmic Evolution with Clusters of Galaxies (Sesto 2019), Matera Oscura - Cosmology and Dark Matter within Galaxies and Clusters (Matera 2019). For his research he obtained support from the Charles University Grant Agency (project GA UK 1000218).

Supervising Michal in this research has been an exciting journey of discovery. Work on the thesis yielded multiple results with implications reaching beyond the specific studied scenario and providing new insights into gravitational lensing. These include the peculiar properties of the vanishing radial critical curves and caustics (Appendix A.2), the introduction of the convergence–shear diagram (e.g., Appendix A.3) illustrating the properties of images at any position in the lens plane, or the formulae for the shear of two combined mass distributions and its implications for the existence of zero-shear points (Section 3.3.1) in composite lens models.

On a more personal note, it has been a true pleasure for me to work with Michal. The results appearing in this thesis benefited greatly from his deep involvement in the topic, his attention to details, intellectual rigor and honesty, excellent programming skills, as well as his dependable work ethic. The submitted thesis clearly demonstrates Michal's capability of independent creative work.

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