This thesis deals with the Tate-Shafarevich group and its relation to rational points on the curve and its rank. We first define the notion of profinite groups and characterize them as Galois groups of field extensions. Then we define the Tate-Shafarevich group using Galois cohomology and explain its relation to the rational points on the curve. Finally, we formulate the Birch-Swinnerton-Dyer conjecture, which relates the rank of an elliptic curve and the order of its Tate-Shafarevich group.