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Report on the doctoral thesis of

Proof Systems: A Study on Form and Complexity
by **Raheleh Jalali Keshavarz**

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The thesis consists of an Abstract, Acknowledgements, Table of Contents, Introduction, Bibliography, and three chapters describing the following research.

(A) Semi-analytic Rules and Interpolation: Iemhoff [22,23] recently established that every sufficiently strong terminating sequent calculus with rules of a certain form has uniform interpolation. The key result in this chapter of the thesis is the extension of those arguments and results to obtain Craig interpolation and uniform interpolation for (i) sequent calculi comprising of more general rule forms called semi-analytic rules, and (ii) substructural logics (including with modalities). The crucial point is that the form of the semi-analytic rules are amenable to the inductive interpolation argument. Especially in the case of uniform interpolation, there are a lot of details to care for, and R. J. Keshavarz handles this admirably well. The definition of the semi-analytic rules is also interesting: the subformula property is replaced by the occurrence-preserving property which states that every variable occurring in an active formula in the premise occurs in the single active formula in the conclusion. It would have been nice to have seen more concrete examples of these rules to fix their form in the reader's mind and to illustrate their expressivity. Aside from providing a methodology for obtaining (uniform) interpolation, it also follows from the contrapositive that those logics that do not have (uniform) interpolation cannot have a (terminating) sequent calculus comprising of semi-analytic rules. This argument—in the spirit of Iemhoff—yields negative expressivity results for proof systems, something that is rather rare.

(B) Proof Complexity of Focussed Calculi: R. J. Keshavarz shows that there is a sequence of classical/intuitionistic theorems with short proofs (i.e. polynomial in the size of the theorem) in the Hilbert calculus such that the short proofs cannot be achieved using a sequent calculus with rules restricted to a certain form. Specifically, in the classical case the sequent calculi comprise of rule forms called polarity preserving focused (PPF) rules, and in the intuitionistic case the sequent calculi comprise of rule forms called monotonicity preserving focused (MPF) rules. A noteworthy aspect of this work is the discovery of these two classes of rule forms, although in the intuitionistic case the possibility is left open that it is not that the proofs are exponential in length but that the rules are simply not expressive enough to attain intuitionistic logic (Thm 3.3.18).

This work has already been reported in the following publication:

Raheleh Jalali: An Exponential Lower Bound for Proofs in Focused Calculi.

26th International Workshop on Logic, Language, Information and Computation (WoLLIC 2019): 342–355

This is good venue and an appropriate venue for the publication of this work.

(C) Proof Complexity of Substructural Calculi: While (A) and (B) were concerned with the expressivity of sequent calculi built from certain rule forms, this part in concerned with the propositional proof complexity of substructural logics. R. J. Keshavarz establishes exponential lower bounds for the length of proofs in Frege and extended Frege systems for logics between the basic substructural logic FL or Visser's basic logic BPC and a super-intuitionistic infinite branching logic.

All three parts require a high degree of technical maturity. For parts (A) and (B) this is primarily in the area of structural proof theory and substructural logics. Part (C) is a contribution to the propositional proof complexity in the less-well-studied context of substructural logics. In this way R. J. Keshavarz has been able to make a tangible contribution to close but distinct areas of logic and has developed a familiarity with the literature and techniques in these areas. This is a noteworthy aspect of this thesis.

There are some aspects of the thesis that could have been improved (apart from several typographical errors). In general, there were not enough citations and discussion of important and related works in the literature. For example, how do your negative results compare with the limitative expressivity results for structural rule extensions of FL in [10]? Moreover, it was not clearly stated in (A) which, if any, of the interpolation results were new. It would also be advisable to accompany uniformly those citations of high relevance with the author(s) names. I detected some inconsistencies in certain formal statements. For example, on page 9 a sequent calculus is defined as comprising of a finite set of rules and axioms, but in Definition 2.2.7 the sequent calculus H contains all of the provable sequents from G as axioms (an infinite set in general). A hypothesis of Corollary 3.3.11 seems to be that G consists of focused atoms and PPF rules and is complete for CPC and yet its conclusion is that G is either incomplete or not feasibly complete for CPC. Fortunately I was able to understand what was intended in every case.

The terminology of a "focused rule" in (B) is unfortunate because this term is used frequently in the context of ordering of rules in linear logic dating back to the 1990s. Athough Iemhoff uses this term in [22], in the subsequent [23] this is changed to "centred rule" for this very reason.

At the start of Chapter 3 it is stated: "In the field of proof theory, proof systems, as the main players of the game, deserve to be considered as the objects of the study themselves." R. J. Keshavarz rightly observes that this is not the status quo: "[Proof systems] are designed and used based on their expected applications and not their inherent mathematical values. They are just the second rank citizens, far from the independent mathematical objects that they could have been." (start of Chapter 2). I am sympathetic to the author's view that proof systems could be considered as primary citizens but what I missed was an argument justifying why this should be so. Moving past the question of ‘why?’, I observe that a research programme titled “universal proof theory” tackling the question of ‘how?’ is proposed at the beginning of Chapter 2 and this looks to be a promising continuation of the work in (A) and (B).

Conclusion. The above critical remarks do not alter my positive impression of this thesis. In particular, I was able in every case to discern what was intended and I was able to verify the proofs. The interesting results, the technical skill, and the contribution to multiple research areas combine to make this a nice and interesting thesis. Therefore I have no hesitation in recommending that this work be accepted as a PhD thesis.

Questions. Here are some questions that I would propose to the candidate:

1. You stated that proof systems deserve to be studied as objects of interest (primary citizens) rather than only as a tool. What reasons would you give to justify this statement?

2. You establish that the logics that do not have uniform interpolation cannot have a semi-analytic sequent calculus. Do you know of any logics (in the language under consideration) that have uniform interpolation but do not have a semi-analytic sequent calculus?

3. An informal notion of a terminating proof calculus could be that proof search terminates under all repeated rule applications from conclusion to premises. This could be formalised as a well-founded order on sequents such that in every rule instance, the premise is less than the conclusion. However your definition of termination (Def 2.2.3) also requires that proper subsequents of the conclusion are less than the conclusion. Can you comment on the need for these additional requirements?

4. Do you have any plans for extending the research described in this thesis? If so, what do you plan on doing next?