REPORT ON THE HABILITATION THESIS "Singular integral operators with rough kernels" by Petr Honzík.

Boundedness properties of singular integral operators are one of the central classical themes in Harmonic Analysis. Apart from their intrinsic interest, they have applications to the theory of Partial Differential Equations, and to a number of other areas of Mathematics. Despite an extensive literature on this field, whose modern developments are rooted in the work by Calderón and Zygmund, several important open problems are still waiting for an answer.

Honzík's thesis is focused on some questions along this direction of research. Specifically, it is concerned with problems on the boundedness of singular integral operators with rough kernels, and on parallel problems for bilinear operators.

The thesis consists of a collection of seven published papers, labeled with letters from A to G, authored or coauthored by Honzík. They are preceded by an introduction and by a chapter devoted to an overview of the papers, where the status of the art is described and the main contributions are highlighted.

The paper [A] addresses the question of the dependence on p of the boundedness in L^p of singular integral operators with integrable rough kernels and zero mean value. Subtle examples are constructed demonstrating this dependence on a parameter α appearing in an inegrability condition on the kernel introduced in an earlier paper by Grafakos and Stefanov. Their construction relies upon refined estimates on Fourier multiplier sequences.

In the paper [B], the condition for the L^p boundedness introduced in the paper by Grafakos and Stefanov is nicely weakened, in such a way that not only the size of the kernel, but also cancellations due to sign changes are taken into account. Moreover, the range of exponents p, depending on α, for which the operator is not bounded is enlarged, thus restricting the gap between positive and negative results. This is obtained by an original construction, which is quite explicit and simpler than that exploited in the paper [A].

The subtle question of the boundedness of a singular integral operator versus that of its maximal counterpart is the subject of the paper [C]. New light is shed on this question by a fine example of a kernel for which the former operator is bounded in L^2, whereas the latter is not.

An open problem concerning the validity of a weak 1–1 estimate for the so called Christ-Journé commutator operator in $\mathbb{R}^2$ is settled, with a positive answer, in the paper [D]. This is achieved thanks to a clever observation about the smoothness of the restriction of the kernel along all lines parallel to the y-axis, which enables to adapt certain techinques introduced by Christ and Rubio de Francia in their classical work on weak 1–1 estimates for singular integrals with rough operators.

Another well known open problem in this area of Harmonic Analysis amounts to decide whether a maximal singular integral operator with rough kernel is of weak type 1–1. A step in connection with the analysis of this problem is performed in the paper [E], where a (local) result in this spirit is proved on replacing L^1 by the slightly smaller space $L^1(\log\log L)^{2+\varepsilon}$ as a domain space. The proof makes use of sophisticated decomposition techniques, after which various methods and arguments from the existing literature can be remastered and adapted to the problem at hand.

The last two papers [F] and [G] deal with the even more difficult problem of the boundedness of bilinear singular integral operators with rough kernels. In [F] a fundamental advance in the theory is obtained via a new method based on a wavelet decomposition of the bilinear Fourier multiplier symbol. This is the main idea in the proof of the main results of the paper, which provide boundedness properties in L^2, and also in L^p, of the relevant bilinear operator.
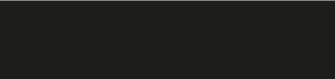
The paper [G] extends the results of [F] to the maximal operator associated with the bilinear singular integral operators with rough kernels considered in [F].

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Altogether, the material of this thesis is a remarkable contribution to its area. The results of the presented papers definitely constitute a substantial progress in the theory of singular linear and bilinear integral operators with rough kernels.

The organization of the thesis is very effective. In particular, the preliminary sections help becoming acquainted with the topic considered. The state of the art on the matter is exhaustively outlined and the difficulties to be faced in approaching the problems under consideration are clearly summarized. The main results are stated together with useful comments about novelties in the proofs, that made the advances of this habilitation thesis possible.

The work of P.Honzík demonstrates that he has an excellent knowledge of the status of the art and of the methods of the theory singular integral operators, to which he has contributed with original ideas and techniques. I expect further innovative results from Honzík's research projects. There is no doubt that he an outstanding candidate for an habilitation at the Faculty of Mathematics and Physics at Charles University, as well as at any other prestigious academic institution worldwide.

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