CHARLES UNIVERSITY
FACULTY OF SOCIAL SCIENCES
Institute of Economic Studies

Application of a Financial Agent-Based Model to the Cryptocurrency Market

Bachelor's thesis

Author: Tatiana Bielaková

Study program: Economics and Finance

Supervisor: PhDr. Jiří Kukačka, Ph.D.

Year of defense: 2023

Declaration of Authorship

The author hereby declares that he or she compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain any other academic title.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis in whole or in part and agrees with the thesis being used for study and scientific purposes.

Prague, May 2, 2023

Tatiana Bielakova

Abstract

Motivated by the occurrence of financial stylized facts (also) in the cryptocurrency markets, we study their dynamics by applying one of the most well-known financial agent-based models to them. Based on interactions between two boundedly rational types of traders, this modeling framework nests eight submodels using four attractiveness specifications and two switching mechanisms between the trading strategies. The analysis is based on three types of datasets — S&P500 to receive a benchmark to the previous research and a comparison with crypto markets, Bitcoin, and a hypothetical market-weighted Top20 cryptocurrency index. For the estimation, we utilize the simulated method of moments, a technique commonly used in complex models where analytical solutions are not feasible. Overall, the results for cryptocurrency datasets imply a very promising application of agent-based models to the analysis of crypto markets. Particularly, for Bitcoin, all submodels produce data in close agreement with the empirical data-generating process. We attribute the robust rank of results to the low level of rationality of the studied markets. However, we are unable to directly interpret the evolution of the trading groups due to the lack of the resulting group dynamics. We identify a similar problem in several other recent studies and suggest addressing this issue in further research by reevaluating the fixed parameters.

JEL Classification C13, C15, C51, C52, C53

Keywords agent-based model, Bitcoin, cryptocurrencies, simulated method of moments

Title Application of a Financial Agent-Based Model to the Cryptocurrency Market

Abstrakt

Motivovaní výskytom finančných štylizovaných faktov (aj) na trhoch s kryptomenami, študujeme ich dynamiku cez aplikáciu jedného z najznámejších modelov založených na finančných agentoch. Na základe interakcií medzi dvoma ohraničene racionálnymi typmi obchodníkov, tento modelovací rámec spája osem podmodelov pomocou štyroch špecifikácií atraktivity a dvoch mechanizmov prepínania medzi obchodnými stratégiami. Analýza je založená na troch typoch údajov — S&P500 pre získanie benchmarku s predchádzajúcim výskumom a porovanie s kryptomenami, Bitcoin dátami, a hypotetickým trhom váženým indexom kryptomien Top20. Na odhad používame simulovanú metódu momentov, techniku bežne používanú v zložitých modeloch, kde analytické riešenia nie sú možné. Celkovo výsledky naznačujú veľmi sľubnú aplikáciu modelov založených na agentoch pre analýzu kryptotrhov. Predovšetkým pre Bitcoin všetky podmodely produkujú údaje v úzkej zhode s empirickým procesom generovania údajov. Robustné hodnotenie výsledkov pripisujeme nízkej úrovni racionality skúmaných trhov. Nie sme však schopní priamo interpretovať vývoj obchodných skupín pre nedostatok výslednej skupinovej dynamiky. V niekoľkých ďalších nedávnych štúdiách identifikujeme podobný problém a navrhujeme ho riešiť v ďalšom výskume prehodnotením pevných parametrov.

Klasifikace JEL C13, C15, C51, C52, C53
Klíčová slova agentný model, Bitcoin, kryptomeny, simulovaná metóda momentov

Název práce Aplikace finančního agent-based modelu na trh kryptoměn

Acknowledgments

Firstly, I would like to express my gratitude to PhDr. Jiří Kukačka Ph.D., who provided me with patient guidance, supervision and insights throughout the whole year. His valuable comments and feedback on the drafts immensely helped create the thesis. Secondly, I thank all my friends who motivated and helped me through the semesters, especially during the exam seasons. Lastly, my deepest gratitude goes to my parents, brother and partner, who supported and encouraged me throughout my studies.

The thesis uses the IES Thesis Template. The software used was Python, Julia and Overleaf. Additionally, Grammarly and ChatGPT were used for English proofreading.

Bibliographic Record

Bielaková, Tatiana: *Application of a Financial Agent-Based Model to the Cryptocurrency Market.* Bachelor's thesis. Charles University, Faculty of Social Sciences, Institute of Economic Studies, Prague. 2023, pages 60. Advisor: PhDr. Jiří Kukačka, Ph.D.

Contents

List of Tables

List of Figures

Acronyms

DCA Discrete Choice Approach

FABM Financial Agent-Based Model

SMM Simulated Method of Moments

TPA Transitional Probability Approach

Bachelor's Thesis Proposal

Author	Tatiana Bielaková
Supervisor	PhDr. Jiří Kukačka, Ph.D.
Proposed topic	Application of a Financial Agent-Based Model to the Cryptocurrency Market

Motivation This thesis aims to provide deeper insights into the dynamics of human behaviour in the financial markets and crypto markets by reestimating the existing financial agent-based model and applying it to the Bitcoin market. Financial markets have long been characterized by so-called empirical stylized facts (Cont, 2001) that can be found across different time periods and types of markets. These facts include e.g. higher volatility, fat-tail returns, absence of autocorrelation in returns, or volatility clustering (Hommes, 2006).

One way to better understand the origin of the stylized facts is through the financial agent-based models (FABM). This approach uses numerical and computational methods to explain the complex behaviour of financial markets. In most FABM, two types of agents interact, fundamentalists and chartists. Fundamentalists base their strategies on fundamental economic factors and chartists on historical prices. The interaction between these two groups combined with the price formation mechanism of the model can replicate some of the stylized facts mentioned above.

Recent studies imply that stylized facts are also found in cryptocurrency markets (Phillip et al., 2018; Zhang et al., 2018), but as the market is relatively new, research is not yet extensive. Moreover, neither the question of the investors' behaviour is fully answered. Studies suggest that Bitcoin is mainly used as a speculative investment and could be considered a weak safe haven as it is uncorrelated with traditional assets (Baur et al., 2018; Shahzad et al., 2019). However, the evolution of Bitcoin prices during the Covid-19 pandemic indicates that Bitcoin increases the portfolio downside risk and is not acting as a safe haven (Conlon and McGee, 2021).

To answer the question of investors' behaviour in both financial and crypto markets, my thesis will focus on working with one of FABMs and will be divided into two parts. In the first part, I will concentrate on data updating and parameter reevaluation of the existing famous and widely analyzed model by Franke and Westerhoff (2012). This should offer an up-to-date view of the behaviour of the financial markets. In the second part, I will try to apply this model to the crypto market to provide new light into the relatively recent and not so researched interdisciplinary area of cryptocurrencies and FABM.

Methodology I will use daily data from the S&P 500 and Bitcoin market and apply the method of simulated moments (Franke, 2009) to the data. The method tries to minimize the difference between sample counterparts of selected moments of simulated and empirical data through a loss function. The loss function minimization is based on optimizing the model parameters. Then, after the estimation of the parameters, the quality of each group of optimized parameters will be compared. Using the Monte Carle method, I will generate several sets of daily returns based on optimized parameters to estimate the standard error of the associated loss function. By comparing the t-statistic, it will be determined whether some models are preferred over others.

Expected Contribution Overall, the results of my work should offer a comparison of the investors' representation in both types of markets and the determinants of their behavior. The original model contains data from the period 1980 to 2007. The updated model will therefore contain 15 more years of new data and will reflect the Financial Crisis, offering a more recent view of financial markets. Applying the model to crypto markets will allow comparing the representation of fundamentalist and chartist, representation-related coefficients and the intensity of switching index in both markets. This will provide information on which type of strategy is dominant in which market, what are the main drivers and how attractive one strategy is to another.

Outline

1. Introduction
2. Literature review
3. Methodology
   (a) Description of the data
   (b) Simulated Method of Moments
   (c) Methods to test the parameters results
4. Results

5. Discussion

6. Conclusion

Core bibliography

Baur, Dirk G., Kihoon Hong, and Adrian D. Lee. "Bitcoin: Medium of exchange or speculative assets?." Journal of International Financial Markets, Institutions and Money 54 (2018): 177-189.

Conlon, T., McGee, R. (2020). Safe haven or risky hazard? Bitcoin during the COVID-19 bear market. Finance Research Letters, 35, 101607.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. Quantitative finance, 1(2), 223.

Franke, R. (2009). Applying the method of simulated moments to estimate a small agent-based asset pricing model. Journal of Empirical Finance, 16(5), 804-815.

Franke, R., Westerhoff, F. (2012). Structural stochastic volatility in asset pricing dynamics: Estimation and model contest. Journal of Economic Dynamics and Control, 36(8), 1193-1211.

Hommes, C. H. (2006). Heterogeneous agent models in economics and finance. Handbook of computational economics, 2, 1109-1186.

Phillip, A., Chan, J. S., Peiris, S. (2018). A new look at cryptocurrencies. Economics Letters, 163, 6-9.

Shahzad, S. J. H., Bouri, E., Roubaud, D., Kristoufek, L., Lucey, B. (2019). Is Bitcoin a better safe-haven investment than gold and commodities?. International Review of Financial Analysis, 63, 322-330.

Zhang, W., Wang, P., Li, X., Shen, D. (2018). Some stylized facts of the cryptocurrency market. Applied Economics, 50(55), 5950-5965.

Author Supervisor

Chapter 1

Introduction

Cryptocurrency markets emerged as one of the consequences of the financial crisis. Being characterized by their lack of regulation by third parties, non-existent intrinsic value, and high volatility, they represent a completely new type of financial market. However, similar to other types of financial markets, they possess unique statistical properties called stylized facts (Cont 2001). Higher volatility, fat-tail returns, absence of autocorrelation in raw returns, or volatility clustering are one of the many well-analyzed and documented facts, regardless of whether we look at Bitcoin or other cryptocurrencies (Bariviera 2017; Urquhart 2016; Drożdż *et al.* 2018).

In the traditional financial markets, classical economic theories failed to explain these facts. As such, with the development of computational methods in the 1980s, financial agent-based models (FABMs) based on interactions of boundedly rational agents emerged (Hommes 2006). These models usually incorporate two main types of traders, fundamentalists and chartists. While fundamentalists base their strategies on fundamental economic factors, chartists base theirs on historical prices. By simulating the interaction between these two groups and combining it with the model's price formation mechanism, FABMs can replicate some of the stylized facts observed in financial markets.

However, this approach has not yet been applied to cryptocurrency markets. By applying one of the FABMs to the cryptocurrencies, this thesis aims to better understand the factors influencing its participants' behaviour and assess the overall prospects of applying these models to crypto markets. The motivation for our research is the following. The world of financial and cryptocurrency markets is a complex and dynamic system that is constantly changing, driven by the behaviour of its participants. Understanding the dynamics of these traders is essential for investors and policymakers, as it can form investment decisions and regulatory policies for economic shocks evoked by sudden market changes.

We choose to work with a well-studied model proposed by Franke & Westerhoff (2012). This model nests eight submodels based on four characteristics of relative attractiveness and two switching mechanisms, giving us enough freedom when applying it to a new type of market. To optimize our parameters, we invoke the simulated method of moments (SMM), first proposed by McFadden (1989) and Pakes & Pollard (1989). This statistical technique is often used when it is difficult to obtain closed-form solutions and is very popular among FABMs.

For benchmark purposes, we first reevaluate the model on the extended dataset of the S&P500 Index. Next, we apply the model to Bitcoin, the most studied cryptocurrency with the highest market capitalization. Furthermore, we create a hypothetical market-weighted Top20 Index to receive a more comprehensive representation of crypto markets and apply the model to this index as well. The validity of our results is then assessed through the standardly used J-test of overidentifying restrictions.

Overall, our analysis shows encouraging results in using financial agents for cryptocurrency markets, for both Bitcoin and the hypothetical Top20 index. Moreover, these findings remain consistent across various model specifications.

The thesis is sectioned following. Chapter 2 introduces the background of FABMs and research in cryptocurrency markets, highlighting their natural candidacy for applying them to FABMs. Chapter 3 provides information on the estimation methodology SMM and Chapter 4 continues with the definition of the model. In Chapter 5, we present our datasets and computing setup. Next, Chapter 6 provides the results of our analysis. Chapter 7 continues with a discussion of our research's limitations in interpreting the interactions between the two groups of traders and potential improvements for further analysis. Lastly, Chapter 8 summarizes our main findings.

Chapter 2

Literature review

This chapter introduces the literature and current research in the field of FABMs and cryptocurrencies. The main aim is to understand the development of the research as well as the essential challenges. Firstly, the background of FABMs, together with the simple classification of different types, is introduced. Next, we discuss the critical role of estimation methods. In the second section, we focus on cryptocurrencies as natural candidates for FABMs. In the first step, we consider the stylized facts identified in existing research as a practical assumption for applying FABM techniques. In the second step, we focus on the role of cryptocurrencies as a basis for understanding the context in which we will interpret our results later in the thesis.

2.1 Financial agent-based modeling

FABMs represent a computational modeling approach used to study the behaviour of individual agents in a financial market. They model interactions between agents, such as traders, investors, and market makers, to understand how they collectively influence financial markets.

FABMs emerged due to the growing recognition of the limitation of traditional paradigms, such as rational expectations and efficient market hypothesis (Fama 1970; Muth 1961). The traditional economic approach of rational markets was not able to replicate "so-called" stylized facts, which represent statistical properties typical for financial markets, such as fat-tailed returns, absence of autocorrelation in raw returns, or volatility clustering (Hommes 2006; Cont 2001). In contrast to the traditional approach, the dynamics in FABMs is based on agents who are boundedly rational and follow simple rules of thumb, which creates a more realistic and dynamic representation of traders (Tversky & Kahneman 1974; Simon 1957).

One of the first attempts at FABMs can be traced to Zeeman (1974), who studied the instability of asset prices by applying the catastrophe theory. However, the main expansion of FABMs took place in the 80s and 90s and was closely connected to the development and wider availability of new computational methods and tools. The most influential models that created the basis for further research include mainly the work of Brock & Hommes (1998), Lux & Marchesi (1999), and LeBaron *et al.* (1999).

2.1.1 Types and differences between FABMs

Looking at the simple classification of FABMs based on the number of agents, the most common frameworks are 2-type or 3-type designs. The 2-type design represents the most simplified version with only fundamentalists with mean-reverting and chartists with trend-following beliefs. The 3-type design includes a third type of agents such as contrarians or market makers (Chen *et al.* 2012). To illustrate this framework, Farmer & Joshi (2002) developed a 3-type model with market makers, who adjust prices based on the demand. Designs with N or an infinite number of agents are usually referred to as many many-type models. According to LeBaron (2006), the main advantage of fewer-type models compared to many-type ones is more direct connections between parameters and results which might not be seen in the more complex frameworks.

Another difference in the models is whether and how agents can change their strategies. In the early models, agents did not reevaluate their strategies. Kirman (1993) in his model introduced the herding mechanism, where the fraction of one strategy determines how likely is to change the current position. Brock *et al.* (1998) proposed an adaptive belief system where traders can change between a predetermined number of strategies. Looking at the more complex dynamics, Franke & Westerhoff (2012) compared herding, misalignment, predisposition, and wealth specifications together with two switching strategies when studying the S&P500 returns.

Other divisions can be based on differences in estimation methods, price and time dynamics, or stylized facts that they explain (Lux & Zwinkels 2018; Chen *et al.* 2012; Dieci & He 2018).

2.1.2 Estimation and evaluation of agent-based models

Following Chen *et al.* (2012), the main estimation methods are maximum likelihood estimations (MLE), least squares (LS), the simulated method of moments (SMM), and variations of these three. The basic idea of MLE is to find the values of the parameters that maximize the likelihood of observing the given set of data. Alfarano *et al.* (2005) use MLE when studying herding tendency to explain heavy tails and clustering in returns. Other examples include various continuations of this model (Alfarano *et al.* 2006; 2007). The LS method minimizes the sum of the squared differences between the observed data and the predictions of the model. Examples of the nonlinear least square estimation include Boswijk *et al.* (2007), who proposed a behavioural asset pricing model with endogenous evolutionary selection or a study of exchange rates based on the European Monetary System Crisis (de Jong *et al.* 2009; 2010). Lastly, the SMM estimates the parameters of a model by matching the set of selected moments of the model's simulated data to those of observed data. The method was originally developed by McFadden (1989) and Pakes & Pollard (1989). Gilli & Winker (2003) applied as first the SMM to the agent-based ant model developed by Kirman (1993). Other applications include Franke (2009) who calibrates six parameters of the Manzan & Westerhoff (2005) model, or Franke & Westerhoff (2012; 2016).

Following developments in computational methods from other disciplines, more complex methods of estimations have emerged in the last couple of years. Grazzini *et al.* (2017) first introduced Bayesian inference techniques to FABMs. Some other more complex developments include the non-parametric simulated maximum likelihood (Kukacka & Barunik 2017) or the sequential Monte Carlo method (Lux & Zwinkels 2018).

Despite the development in estimation techniques, one of the most commonly identified challenges of Financial Agent-Based Models remains the difficulty in accurately calibrating and validating the models (Fagiolo *et al.* 2007; Platt 2020). Fabretti (2013) identifies calibration and validation as critical issues and claims that the research has not yet given sufficient consideration to the process of calibrating. Similarly, Platt (2020) states that the current research is more focused on the introduction of new calibration methods without proper benchmarking with existing techniques.

Another critical issue related to the calibration of FABMs is the selection of moments. Typically, the moment set for a given model estimation is determined in an intuitive manner based on the expertise of the proposers following the model's dynamics (Zila & Kukacka 2023). As such, Fagiolo *et al.* (2007) point out the problem of highly parametrized models. To rely on a more advanced method of choosing the moments, Lamperti *et al.* (2018) introduced as first more advanced approach combining machine learning and intelligent iterative sampling to the model of Brock *et al.* (1998). According to their results, they were able to significantly decrease the computation time burden and receive a fairly accurate proxy of the true model.

2.2 Cryptocurrency markets

One of the consequences of the financial crisis in 2008 was the emergence of a decentralized electronic transaction system based on blockchain and digital cryptocurrencies. The system operates on peer-to-peer transactions without the need for banks or other institutions. As a result, as the trading is unregulated by higher institutions, the prices should reflect the uncertainty associated with the exchange viability (Hu *et al.* 2019). Interestingly, compared with other financial assets, the value of cryptocurrencies is not based on any tangible asset or a firm's/country's economic situation. Because of the non-existent intrinsic value, Bariviera *et al.* (2017) refer to cryptocurrencies as "synthetic" assets.

The first cryptocurrency introduced was Bitcoin by an anonymous person or group under the pseudonym Nakamoto (2008) and it started to be issued at the beginning of 2009. Bitcoin remains the most significant currency by market capitalization to this day. However, the crypto market has experienced enormous growth since its launch. Nowadays, there are more than 22 thousand cryptocurrencies with a total market cap of $1.05 trillion (CoinMarketCap 2023a).

Because of the fast growth and high volatility in both prices and volume (Ciaian *et al.* 2016; Katsiampa 2017), the academic literature about crypto markets has been quite extensive in recent years. Due to the scope of the thesis, we will mainly discuss the research around stylized facts and the role of crypto markets as an investment. However, other studies cover topics such as price dynamics, market efficiency, regulation, or diversification benefits, and we refer to the recent overviews for more information (Corbet *et al.* 2019; Ma *et al.* 2020).

2.2.1 Stylized facts of cryptocurrency markets

Due to the unique properties of cryptocurrencies compared to traditional financial assets, such as decentralization, non-existent intrinsic value and lack of regulation, a significant part of the research focuses on the comparison of crypto markets and financial markets from the point of studying their statistical properties (Bariviera 2017; Urquhart 2016; Drożdż *et al.* 2018). Based on the research done so far, the studied time horizon plays an important role, and crypto markets seem to have a clear evolution and share stylized facts similar to other financial markets.

Looking at the era before 2014, one could refer to this time as an infancy phase. Bariviera (2017) studies daily and interdaily Bitcoin returns from 2011 to 2017. He concludes that the Hurst exponent, a measure of long-term memory of time series, changes rapidly during the first studied period. Similarly, Urquhart (2016) conducts 4 tests (Ljung-Box for autocorrelation, Bartels test for independence, variance ratio test, and Hurst exponent) on Bitcoin data from 2010 to 2016 to determine its efficiency. He finds that even though the whole sample is not weakly efficient, the later subsample indicates efficiency to some extent. In line with that, Drożdż *et al.* (2018) state that after 2014, returns possess fat tails similar to well-established financial markets such as the USD/EU exchange market. As a result, they conclude that cryptocurrencies "carry the concrete potential of imminently becoming a mature market".

Although there is a growing body of literature on the properties of cryptocurrencies, the current research has some limitations, particularly in terms of exploring the properties of other cryptocurrencies. Nevertheless, despite the lack of research in this area, it appears that other major cryptocurrencies with high market capitalization also exhibit stylized facts and are significantly influenced by Bitcoin. Hu *et al.* (2019) conduct a large study of 222 cryptocurrencies and find a strong correlation between other cryptocurrencies and Bitcoin. Phillip *et al.* (2018) extend the long memory properties studied by Bariviera (2017) on 5 cryptocurrencies (Bitcoin, Ethereum, Ripples, Dash and Nem) by using the generalized long memory effect and come to the same conclusion. Other stylized facts such as fat tails, absence of autocorrelations, and volatility clustering are present in other cryptocurrencies as well (Zhang *et al.* 2018).

2.2.2 Investment and diversification benefits

Another extensive debate is about the role of cryptocurrencies in a portfolio, whether they have a safe haven or hedging properties or whether they act mainly as a speculative investment. Overall, results seem to vary depending on the studied time horizon, specific market conditions, and the studied financial assets (Zhang *et al.* 2018; Shahzad *et al.* 2019; Fang *et al.* 2019; Bouri *et al.* 2017; Baur *et al.* 2018).

Baur *et al.* (2018) find that Bitcoin is uncorrelated with traditional financial assets such as stocks, bonds, or commodities and, therefore, could act as a weak safe haven. On the other hand, Bouri *et al.* (2017) study the dynamic conditional correlation model proposed by Engle (2002) and conclude that Bitcoin is mainly suitable for diversification purposes and acts as a safe haven only for Asian Markets. Fang *et al.* (2019) find Bitcoin possesses hedging properties under specific economic uncertainty conditions. Moreover, a more recent study capturing the beginning of Covid-19 rejects Bitcoin as a possible safe haven as it increases portfolio downside risk even with a small allocation in the portfolio (Conlon & McGee 2020).

Even though the research around safe haven and hedging capabilities vary greatly, the debate on diversification benefits is more unified. One part of the research focuses on the diversification in a combination of mainly Bitcoin with traditional assets (Ma *et al.* 2020; Trimborn *et al.* 2020; Corbet *et al.* 2019), while the other compares strategies for investing in several cryptocurrencies (Liu 2019; Mensi *et al.* 2019; Platanakis & Urquhart 2019). As this thesis discusses the combination of several cryptocurrencies, we will discuss only the latter one. Liu (2019) studies ten major cryptocurrencies by market capitalization and concludes, that diversification improves overall investment results. When comparing six portfolio strategies (naive, minimum variance, risk-parity, Markowitz, maximum Sharpe ratio, and maximum utility), the naive portfolio interestingly provides the best Sharpe ratio in out-of-sample performance. Platanakis & Urquhart (2019) similarly conclude that traditional portfolio theories cannot be relied upon for out-of-sample performance in crypto markets.

To summarize, FABMs provide insights into financial markets through interactions of boundedly rational agents. Cryptocurrencies are characterized by less regulation, no intrinsic value, and high volatility. Similar to financial markets, they possess stylized facts that have been widely studied. As such, they are a natural candidate for FABMs to study the behaviour of traders.

Chapter 3

The simulated method of moments

This chapter introduces the estimation methodology. Firstly, the development and the simple intuition behind the technique are introduced in Section 3.1. Next, we propose the formal definition and discuss the weighting matrix in Section 3.2. The following Section 3.3 discusses the selected moments of Franke & Westerhoff (2012).

3.1 About the method

The SMM (also called the method of simulated moments) is a statistical technique used to estimate a model's parameters when it is difficult or impossible to obtain closed-form solutions. In such cases, traditional approaches, such as the maximum likelihood or the generalized method of moments, may not be applicable.

The method was introduced by McFadden (1989) and Pakes & Pollard (1989). McFadden (1989) provided theoretical foundations that an unbiased simulator can be used to generate a sample of moments given a set of parameters, with simulation errors independent across observation and normally distributed variance in the estimates of the moments due to the law of large numbers. The SMM was further developed by Lee & Ingram (1991) and Duffie & Singleton (1993), who extended the method to time-series data and panel data. The technique is particularly useful in complex models where analytical solutions are not available, and it has been applied in various fields, including economics, finance, and engineering.

The idea behind this method is to simulate the model repeatedly using different parameter sets and match the sample counterparts of selected moments of the simulated data to the ones of the empirical data. Optimization of the model's parameters is based on minimizing the difference between the simulated and observed moments through a loss function. Moreover, the loss function incorporates the weighting matrix, which assigns different weights to each of the moments, reflecting their relative importance or precision.

3.2 Formal definition

In the formal definition of the SMM, we follow Zila & Kukacka (2023) and Franke & Westerhoff (2012).

The aim of the agent-based model is to explain the essential stylized facts of financial markets. There are various summary statistics, also known as moments, that can be utilized to analyze these facts. As we do not know the true values of these moments, we rely on their sample counterparts.

We can start by considering a time series y and D moments of our interest that can be calculated by their sample counterparts functions

$$m_d(y),\ d = 1, ..., D.$$

Starting with the empirical part and to demonstrate it on an example, if the mean is the first moment we are interested in, we can calculate the mean of the empirical time series

$$\{y_t^{emp}\},\ t = 1, ..., T_{emp},$$

by using the arithmetic mean, which is a standardly used sample counterpart

$$m_1(y_t^{emp}) = \frac{1}{T_{emp}} \sum_{t=1}^{T_{emp}} y_t^{emp}\ .$$

Next, we can store these moment results of the empirical time series into a $D \times 1$ vector

$$m^{emp} = [m_1(y^{emp}), ..., m_D(y^{emp})]^T,$$

where ^{T} denotes transposition.

As a following step, we assume we possess a well-specified stochastic model that accurately depicts a certain aspect of the real world, with the empirical time series being one of the many possible realizations. As such, we can use the model to generate a simulated time series

$$\{y_t^{sim}(\theta)\},\ t = 1, ..., T_{sim},\ T_{sim} \geq T_{emp},$$

where θ represents a vector of true parameters we are trying to estimate. By simulating the model N times, we mitigate the effect of randomness. Next, for each simulated time series, we can calculate the sample counterpart of the D moments of our interest and take the average to receive the mirror version of the empirical moment. The corresponding functions calculating the averages of simulated moments can be defined following:

$$m_d^{sim}(\theta) = \frac{1}{N}\sum_{n=1}^{N} m_d(y_n^{sim}(\theta)),\ d = 1, ..., D$$

We can organize these moments into a $D \times 1$ simulated vector of moments

$$m^{sim}(\theta) = [m_1^{sim}(\theta), ..., m_D^{sim}(\theta)]^T.$$

Then the loss function is defined as

$$J = h(\theta)^T W h(\theta),$$

where $h(\theta) = m^{emp} - m^{sim}$ and W is a positive $D \times D$ weighting matrix. The optimization means finding parameters that minimize the loss function, i.e the optimized estimated parameter set is

$$\hat{\theta} = \arg\min_{\theta \in \Theta}(J(\theta)),$$

where Θ denotes the admissible space. Lastly, we utilize the J test statistic to assess the compatibility of the moment conditions (Franke 2009; Jang 2015):

$$\bar{J} = J(\hat{\theta}) \xrightarrow{T \to \infty} \chi^2_{D-L},$$

where L denotes the total number of parameters that we are estimating. A valid model is one that has a J function value less than the χ^2 value with $D - L$ degrees of freedom corresponding to a certain significance level. If the model surpasses the critical value, it means that the model fails to replicate the observed data in at least one dimension.

3.2.1 Weighting matrix

The idea of the weighting matrix is to assign more importance to stable moments while considering possible correlations between individual moments. The standardly considered candidate is inverse to the variance-covariance matrix

$$W = \Sigma^{-1},$$

however, we first need to estimate Σ. To do so, we follow Franke & Westerhoff (2012) and use a block bootstrap approach. We consider B block samples from the empirical time series with a length of 250 days. However, for the moments capturing the long memory properties, we consider longer blocks with a length of 750 days. Next, we calculate the bootstrapped moments through the counterpart functions mentioned above.

$$m^b = [m_1^b, ..., m_D^b],\ b = 1, ..., B$$

to obtain the frequency distributions. Next, we calculate $\overline{m} = \frac{1}{B}\sum_{b=1}^{B} m^b$ and define the estimate of the variance-covariance matrix as

$$\hat{\Sigma} = \frac{1}{B}\sum_{b=1}^{B}(m^b - \overline{m})(m^b - \overline{m})^T$$

Then we calculate the inverse of $\hat{\Sigma}$ to obtain the weighting matrix

$$W = \hat{\Sigma}^{-1}$$

3.3 The selected moments

The model of Franke & Westerhoff (2012), fully specified in Chapter 4, matches the nine moments of the empirical time series to explain four stylized facts. These four properties of financial data are the absence of autocorrelations in the raw returns, fat tails, volatility clustering, and the long memory feature.

The first selected moment is the first-order autocorrelation coefficient of the raw returns. For the volatility clustering, the model matches the mean value of the absolute returns to scale the volatility. Next, the fat tail characteristic is evaluated using the Hill estimator to calculate the tail index of the absolute returns. Lastly, six lags $i \in \{1, 5, 10, 25, 50, 100\}$ are considered to capture the long memory property.

Chapter 4

Franke and Westerhoff (2012) model

This chapter introduces the model by Franke & Westerhoff (2012), starting with a brief overview in Section 4.1 and continuing with the formal definition in Section 4.2. In Subsection 4.2.1, we discuss price and demand formation, while Subsection 4.2.2 presents two approaches to the switching strategy. Finally, Subsection 4.2.3 considers principles and combinations that impact strategy attractiveness.

4.1 About the model

The preliminary versions of the model can be traced back to Franke (2008) and Franke & Westerhoff (2011). Franke (2008) explores a framework in which individual agents shift between two types of the sentiment using particular transition probabilities and the following work examines the structural stochastic volatility. Franke & Westerhoff (2012) then further develops the possible variations of switching mechanisms and determinants of relative attractiveness by considering seven different models and assessing them. Due to the complex analysis of the optimized results, the winning model was set up as a benchmark version for future research. In the follow-up study, Franke & Westerhoff (2016) study the phase plane of the price and the majority index of the winning model in more detail. For the benchmark assessment, Barde (2016) tests the model and compares it with the herding model and its asymmetric version using various calibration setups and Platt (2020) uses the model when comparing different calibration methods.

4.2 Formal definition

4.2.1 Demand and price impact functions

The model consists of two groups of traders, chartists and fundamentalists, together with a market maker who determines the price of an asset for each period. The market maker is assumed to maintain an inventory, which he uses to serve any excess demand and adds any excess supply. He then reacts to this imbalance by adjusting each period's price with a constant positive factor μ in the direction of excess demand. As such, the log price p_t at the start of the period t is based on the equation

$$p_t = p_{t-1} + \mu(n^f_{t-1}d^f_{t-1} + n^c_{t-1}d^c_{t-1}),$$

where $n^f_{t-1}, n^c_{t-1} = 1 - n^f_{t-1}$ denotes the market fractions of the fundamentalists and chartists and d^f_{t-1}, d^c_{t-1} indicates the corresponding demand from an average trader of both groups from the last period.

Regarding the formation of demands, the model follows two simple deterministic rules. Firstly, an average fundamentalist trader believes the market is inversely tied to how far the price deviates from its fundamental value. As such, if we assume an asset's exogenous constant fundamental price $p^\star$, the demand is proportionate to $(p^\star - p_t)$. Secondly, the demand of an average chartist trader is proportional to the returns he observed in the previous period, i.e. $(p_t - p_{t-1})$. Moreover, the demands of both groups involve noise terms that reflect a certain within-group heterogeneity. Combining all, the demand functions for fundamentalists and chartists for the period t are

$$\begin{aligned} d^f_t &= \phi(p^\star - p_t) + \varepsilon^f_t,\ \varepsilon^f_t \sim N(0, \sigma^2_f) \\ d^c_t &= \chi(p_t - p_{t-1}) + \varepsilon^c_t,\ \varepsilon^c_t \sim N(0, \sigma^2_c), \end{aligned}$$

where ϕ, χ represent constant non-negative parameters, and σ^2_f, σ^2_c denotes the volatilities of the noise terms ε^f_t, ε^c_t for the fundamentalists and chartists.

4.2.2 Switching mechanisms

The model considers two types of switching between the groups, the transition probability approach (TPA) and the discrete choice approach (DCA). Both techniques incorporate index a_{t-1} that stands for the relative attractiveness of fundamentalism versus chartism behavior at the end of period $t-1$.

Starting with the TPA approach, the idea is that increasing values of the index increase the probability π^{cf} of a chartist becoming a fundamentalist and decrease the probability π^{fc} of a fundamentalist switching to a chartist. In the case of a decreasing index, the situation works vice versa. If we have a large population and assume that the relative changes in the switching probabilities are both linear and symmetrical, it can be shown that at a macro level, the probabilistic factors become insignificant and the following deterministic equation will determine the market fractions

$$\begin{aligned}
n_t^f &= n_{t-1}^f + n_{t-1}^c \pi^{cf}(a_{t-1}) - n_{t-1}^f \pi^{fc}(a_{t-1}) \\
n_t^c &= 1 - n_t^f \\
\pi^{cf}(a_{t-1}) &= \min[1, \nu \exp(a_{t-1})] \\
\pi^{fc}(a_{t-1}) &= \min[1, \nu \exp(-a_{t-1})]
\end{aligned}$$

where $\nu > 0$ denotes the flexibility parameter.

The second approach DCA involves payoff indices u^f and u^c based on past capital gains. The market shares then follow the formula

$$n_t^s = \frac{\exp(\beta u_{t-1}^s)}{\exp(\beta u_{t-1}^f) + \exp(\beta u_{t-1}^c)}, \qquad s = c, f$$

where β is referred to as the intensity of choice. Economically, the fraction of traders following a particular strategy depends positively on the payoffs from the previous periods. The intensity of choice then represents how sensitive the traders are to differences in the two trading strategies. With respect to the fundamentalists, we can rewrite the market share as

$$n_t^f = \frac{1}{1 + \exp[-\beta(u_{t-1}^f - u_{t-1}^c)]}$$

In the expression above, the term $(u_{t-1}^f - u_{t-1}^c)$ measures the payoff difference between fundamentalist and chartist trading. Due to the similar meaning, the term can be substituted by the relative attractiveness of fundamentalism strategy a_{t-1}. Then the DCA takes the following form

$$n_t^f = \frac{1}{1 + \exp(-\beta a_{t-1})}, \quad n_t^c = 1 - n_t^f$$

From the description of the two switching approaches, it can be seen that the DCA impacts the market shares directly on the level n_t^f, via the term $\frac{1}{1+\exp(-\beta a_{t-1})}$, and the TPA through impact the change $(n_t^f - n_t^c)$, via the term $n_{t-1}^c \pi^{cf}(a_{t-1}) - n_{t-1}^f \pi^{fc}(a_{t-1})$.

4.2.3 Determining the relative attractiveness

The model considers four possible components of the relative attractiveness index — herding (**H**), wealth (**W**), predisposition (**P**), and misalignment (**M**).

Looking at the first component, herding reflects that the more adherents one group has, the more attractive it becomes. The effect on the index a_t is proportionate to the difference between the market fractions of fundamentalists and chartists $n_t^f - n_t^c$ and is measured by the term α_n.

The second principle deals with the wealth difference between the two strategies, and the definition is more challenging. Firstly, we start by considering both strategies' short-term capital gains at day t. If we assume an average trader, his gains can be derived as a demand formulated at the day $t-2$ and executed the next day at the price p_{t-1}

$$g_t^s = [\exp(p_t) - \exp(p_{t-1})] d_{t-2}^s, \qquad s = c, f$$

Next, let η be a memory coefficient between one and zero. Then we consider the wealth of an average trader at day t to be calculated as a weighted average between capital gains earned that day and wealth from the previous day.

$$w_t^s = \eta w_{t-1}^s + (1-\eta) g_t^s, \qquad s = c, f$$

This definition of wealth can be then rewritten as

$$w_t^s = (1-\eta) \sum_{k=0}^{\infty} \eta^k g_{t-k}^s, \qquad s = c, f$$

which allows the interpretation of wealth as accumulated profits of a strategy s over the infinite time discounted by $(1-\eta)$. Accordingly, the next principle we are interested in is the difference between wealth $(w_t^f - w_t^c)$ and the proportionate impact on a_t is measured by α_w.

Next, the predisposition term α_0 reflects a certain a priori preference towards one of the strategies. Regarding the tendency towards fundamentalism, the term is positive, and vice versa for chartism.

Lastly, the misalignment indicates that the further the asset price from the fundamental value is, the riskier the chartism strategy becomes. It is calculated as the squared deviations of p_t from the $p^\star$, and the proportionate effect on the index a_t is expressed through α_p.

There are several ways of combining these components to define the relative attractiveness index; however, after some preliminary results, the original paper narrows the focus on the following four combinations

W $\quad a_t = \alpha_w(w_t^f - w_t^c),$

WP $\quad a_t = \alpha_w(w_t^f - w_t^c) + \alpha_0,$

WHP $\quad a_t = \alpha_w(w_t^f - w_t^c) + \alpha_n(n_t^f - n_t^c) + \alpha_0,$

HPM $\quad a_t = \alpha_n(n_t^f - n_t^c) + \alpha_0 + \alpha_p(p_t - p^\star)^2,$

where $\alpha_n, \alpha_p, \alpha_w$ are positive and α_0 may take any value.

We obtain eight unique models by combining all types of relative attractiveness with two switching mechanisms. The original study neglected the WHP-TPA model due to the irrelevance of the results, so the authors studied only seven models in detail. Additionally, four parameters are fixed for all seven models. For clarity, Table 4.1 summarizes all the estimated parameters and the values they can attain together with the fixed parameters proposed by Franke & Westerhoff (2012).

Table 4.1: The overview of estimated and fixed parameters

	Coefficient	**Possible values**
ϕ	aggressiveness of fundamentalists	non-negative
χ	aggressiveness of chartists	non-negative
η	memory	(0,1)
σ_f	volatility of the fundamentalists' noise term	non-negative
σ_c	volatility of the chartists' noise term	non-negative
α_0	predisposition	any
α_n	herding	strictly positive
α_p	misalignment	strictly positive
α_w	wealth	strictly positive
$p^\star$	fundamental value of the market asset	fixed at 0.0
μ	market impact factor of demand	fixed at 0.01
ν	flexibility parameter (TPA)	fixed at 0.05
β	intensity of choice (DCA)	fixed at 1.0

Chapter 5

Data and methodology

The chapter introduces the financial datasets and setups for the optimization. Section 5.1 presents the datasets we work with together with the logic behind creating a hypothetical cryptocurrency index. Next, we discuss the setup for the preliminary and final optimization setups in Section 5.2.

5.1 Description of data

For the analysis, we use log returns, so with respect to the log price p_t, we can write

$$r_t = p_t - p_{t-1}$$

Based on the thesis's original scope, we work with two types of datasets: S&P500 and Bitcoin. Starting with the traditional financial markets, for the extended version of the S&P500 Index, we include 10882 observations from 1980-01-02 to 2023-02-28 based on the closing prices from Yahoo Finance (2023) [database accessed 2023-03-03]. The S&P500 index allows us to reevaluate the original study, including major economic events such as the Financial Crisis or the Covid-19 pandemic, and receive a more recent view of financial markets. Moreover, it gives us a benchmark for the primary purpose of this thesis — applying the model to crypto markets.

For the crypto analysis, we first maximize the observed period for the Bitcoin data and cover 3594 observations from 2013-04-29 to 2023-02-28 using prices from CoinGecko (2023) [database accessed 2023-03-06]. Secondly, we create a hypothetical market-weighted crypto index. Based on the current week, the index includes the top 20 cryptocurrencies by market cap, and we reevaluate the weights in the index daily. For historical information on the most significant currencies by market capitalization, we gather information from the weekly tables available at CoinMarketCap (2023b). For the daily prices of these currencies, we use the CoinGecko (2023) database due to the convenient use of pycoingecko library. We cover 2985 observations from 2014-12-28 to 2023-02-28 [database accessed 2023-03-17]. Unfortunately, the use of two sources and relatively large datasets of 20 currencies leads to some discrepancies in the availability of the data, especially in the early years. However, the weights are always reevaluated only for available currencies, so even though the term Top20 can be in this meaning misleading, the index itself should provide a relatively realistic picture of the cryptocurrency market.

Figure 5.1: Comparison of log returns: Bitcoin and Top20 Index

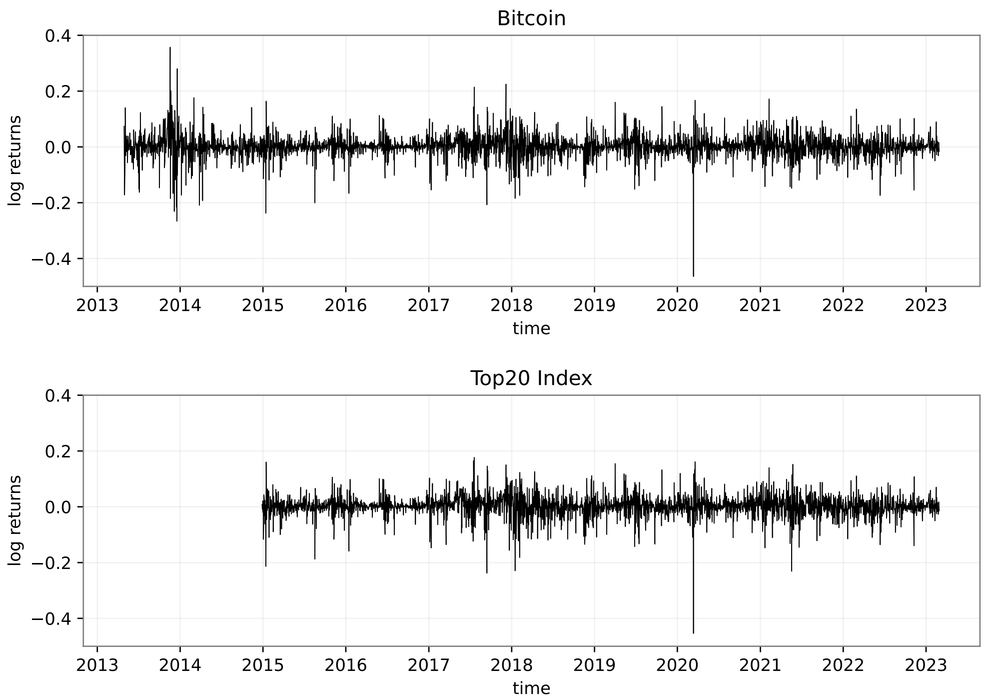


Figure 5.1 presents a comparison of the historical log returns of Bitcoin and the Top20 Index. It can be easily seen that the overall shape is similar. The most dominant extreme value is at the beginning of March 2020, representing the start of Covid, where the log returns dropped by almost 50% on 2020-03-12. We present Table 5.1 summarizing the standard descriptive statistics for all covered datasets.

Table 5.1: The overview of descriptive statistics of log returns of empirical datasets

	S&P500	Bitcoin	Top20
Mean ($\times 10^{-4}$)	3.31	14.33	18.69
Median ($\times 10^{-4}$)	5.63	15.59	23.53
Min ($\times 10^{-2}$)	-22.89	-46.47	-45.41
Max ($\times 10^{-2}$)	10.96	35.75	17.69
SD ($\times 10^{-2}$)	1.14	4.13	3.84
Skewness	-1.09	-0.53	-1.06
Kurtosis	24.34	10.91	10.68

5.2 Setup description

In the preliminary analysis, we analyze constraints covering all submodels' behaviour. As the models consist of different parameter sets, the behaviour changes considerably, especially for parameters such as fundamentalists' aggressiveness or the predisposition effect. As such, we relax the constraints that are either unstable within the model itself or between different models and, at the same time, tighten the stable ones. Moreover, the preliminary analysis of the WHP-TPA model, combining wealth, herding and predisposition components, suggested relevant results. Therefore, we decide to include it in the final estimation and estimate a total of eight models instead of seven as in the original study.

Looking at the technical details, during the first stage, we work with 100 repetitions, each consisting of 30 independent simulations with 2000 iteration steps. For the weighting matrix, we create 5000 samples of non-overlapping blocks from the empirical time series of length 250 days for the first five moments of our interest. For the moments capturing the long memory properties (autocorrelation of absolute returns at lags $i \in \{10, 25, 50, 100\}$), we use blocks with a length of 750 days.

Due to the heterogeneity between the models' behaviour, we obtain relatively wide parameters' constraints, so for the final analysis, we increase the iteration steps to 6000. Simply speaking, the increase in iterations gives the optimization a higher chance of reaching the optimum or at least getting reasonably close to it. Next, we increase the number of independent simulations to 100 to mitigate the effect of randomness.

Chapter 6

Results

The chapter displays and discusses the optimized parameters and the loss function values. Section 6.1 presents the results for the S&P500 dataset. Section 6.2 and Section 6.3 continue with the results of the application to the cryptocurrency market, discussing the Bitcoin and Top20 Index datasets, respectively.

6.1 S&P500

Before going into details, let us again enhance the differences between the models. The relative attractiveness specifications are based on combinations of four components - wealth (W), predisposition (P), herding (H), and price misalignment (M). In the TPA approach, the relative attractiveness determines the probability of changing the strategy, and in the DCA, the effect of the index is directly on the level of fractions.

Table 6.1 presents the optimized parameter estimated for the empirical S&P500 dataset. Overall, we receive approximately three times lower average J-values than the original study for all models, which we attribute mainly to a better optimization method and setup. Firstly, the optimization methods have developed in the last ten years, and we likely use a more advanced optimization package than the original study. Secondly, the wider constraints give the optimization more freedom in reaching or getting close to the optimum. Moreover, we also use a larger dataset with more observations that reveals more information on the behaviour in the financial markets. However, all models are still rejected as the true models by the J-test of overidentification at the 5% significance level.

Table 6.1: Franke and Westerhoff (2012): Results for the S&P500 data

Par./ Model L	DCA W 6	DCA WP 7	DCA WHP 8	DCA HPM 7	TPA W 6	TPA WP 7	TPA WHP 8	TPA HPM 7
$\hat{\phi}$ $\langle 0,10 \rangle$	7.007 (4.46,9.81)	3.322 (0.03,9.86)	3.345 (0.13,9.12)	0.257 (0.19,0.33)	7.049 (4.48,9.19)	4.462 (0.11,9.83)	5.434 (0.04,9.91)	0.266 (0.17,0.35)
$\hat{\chi}$ $\langle 0,5 \rangle$	3.859 (0.208,4.989)	1.375 (0.075,4.658)	1.197 (0.039,3.620)	0.125 (0.001,0.413)	4.134 (2.251,4.970)	1.107 (0.006,4.741)	1.578 (0.117,4.174)	0.106 (0.003,0.382)
$\hat{\eta}$ $\langle 0.8,1 \rangle$	0.996 (0.994,0.998)	0.996 (0.992,0.998)	0.995 (0.988,0.998)	-	0.995 (0.993,0.997)	0.996 (0.990,0.998)	0.970 (0.823,0.998)	-
$\hat{\sigma}_f$ $\langle 0,3.5 \rangle$	0.768 (0.50,1.23)	1.607 (0.71,2.12)	1.753 (0.86,2.17)	1.870 (1.33,1.99)	0.859 (0.78,0.90)	1.705 (0.88,2.12)	1.524 (0.88,2.06)	1.834 (1.33,1.98)
$\hat{\sigma}_c$ $\langle 0,4 \rangle$	1.645 (0.84,2.25)	0.967 (0.69,1.98)	0.963 (0.79,1.91)	0.891 (0.83,0.90)	1.723 (1.42,2.03)	1.031 (0.82,2.01)	0.944 (0.83,1.86)	0.889 (0.84,0.90)
$\hat{\alpha}_0$ $\langle -12,6 \rangle$	-	-3.751 $(-9.85,3.95)$	-4.652 $(-10.89,3.38)$	-10.138 $(-11.85,-4.01)$	-	-5.059 $(-8.85,3.88)$	-2.956 $(-10.45,4.40)$	-10.993 $(-11.99,-4.68)$
$\hat{\alpha}_n$ $\langle 0,8 \rangle$	-		2.434 (0.38,6.80)	3.258 (2.67,4.40)	-		3.816 (0.16,7.94)	0.628 (0.02,1.56)
$\hat{\alpha}_p$ $\langle 40,200 \rangle$	-	-	-	56.02 (42.16,172.52)	-	-	-	67.90 (51.40,198.34)
$\hat{\alpha}_w$ $\langle 200,3200 \rangle$	2065.80 (854.28,3131.23)	2424.54 (911.52,3178.77)	2557.68 (1315.12,3152.05)	-	2561.65 (1049.64,3191.86)	2751.91 (1893.87,3195.47)	2098.84 (415.56,3180.15)	-
$\bar{J} = J(\hat{\theta})$	9.588 (7.93,13.14)	9.098 (7.75,10.49)	8.619 (6.85,10.89)	6.646 (6.25,7.32)	8.556 (7.21,10.17)	8.617 (7.51,9.95)	9.563 (7.92,10.88)	6.838 (6.46,7.33)
χ^2_{9-L}	7.81	5.99	3.84	5.99	7.81	5.99	3.84	5.99
p-value	0.022	0.011	0.003	0.036	0.036	0.013	0.002	0.033
J-test at 5%	rej.H_0	rej.H_0	rej.H_0	rej.H_0	rej.H_0	rej.H_0	rej.H_0	rej.H_0

Note: The optimized parameters are based on 100 repetitions, each averaged over 100 independent simulations with 6000 iterations. The constraints for the parameters are denoted in $\langle\ \rangle$ brackets, and 95% sample confidence intervals are in ( ) parentheses.

The best-performing models with respect to p-values are W-TPA and HPM-DCA, with a value of 0.036. A similar p-value of 0.033 also has the HPM-TPA specification. Since the HPM models have lower J-values in both switching approaches and p-values are relatively similar for W and HPM, the HPM specification seems to be the winner regardless of the switching approach. This suggests that the combination of herding, predisposition towards one of the strategies, and price misalignment appears to be the best when capturing the dynamics in the financial market represented by the S&P500 Index. Additionally, in the case of HPM, the estimated parameter $\hat{\alpha}_n$ in both the DCA a TPA approach pushes to the lower bound, meaning that the J-value could be likely improved with looser bounds. Looking at the interpretation of the HPM models, the negative predisposition parameter suggests an a priori preference for chartism. Next, the high values of the price misalignment suggest stronger tendencies to change strategy to fundamentalism when the price of an asset gets far away from its fundamental value, and the herding is of similar magnitude to the original study.

Concerning the switching, the DCA and TPA approaches have no clear overall winner. W and WP specifications receive better results in the TPA setting, and WHP and WP perform better in the DCA.

Regarding the estimated parameters, we can see considerable heterogeneity between the models. Firstly, the wealth component leads to a higher magnitude of the aggressiveness parameters $\hat{\phi}$ and $\hat{\chi}$ compared to the HPM specification, which aligns with the original study. The memory coefficient is very close to the upper bound, suggesting the strong influence of past wealth on the wealth component. Next, the noticeable difference compared to the original study is that the fundamentalists have roughly two times higher volatility in the noise term than the chartists in the WP, WHP, and HPM models. This could be attributed to adding more than a decade of new observations, including major economic events such as the Financial Crisis, the Great Recession, and Covid-19.

Generally, the confidence intervals for the estimated parameters are wide, with the exception of the memory coefficient $\hat{\eta}$, while the confidence intervals for the J-values are relatively narrow. This suggests that many parameter sets with different estimates lead to similar loss function values. Additionally, the loose bounds compared to the previous research capture the majority of parameters behaviour, except for herding where the HPM strongly pushes to the lower bound and misalignment in the HPM-TPA is close to the upper bound. For the estimated parameters determining the relative attractiveness, the wealth $\hat{\alpha}_w$ and herding $\hat{\alpha}_n$ component is of similar magnitude to the original study. On the other hand, the price misalignment is several times higher, meaning a stronger tendency to switch to a fundamentalist strategy when the price gets far from the fundamental value. The predisposition parameter is significantly negative for WP and HPM models, suggesting a priori preference for chartism.

6.2 Bitcoin

Looking at the results from Bitcoin data, the main finding is that all submodels are valid based on the J-test at a 5% significance level (see Table 6.2). Not rejecting the null hypothesis means that they offer a close approximation of the data-generating process. This result suggests that using models based on interactions between boundedly rational agents to explain the behaviour of cryptomarkets looks promising.

The model with the highest p-value at 0.57 is W-TPA, closely followed by the W-DCA at 0.54. The results of the models with W specifications, both in terms of the highest p-values and smallest J-values, mean that the model based on the wealth differences between the two strategies captures the behaviour at the Bitcoin market the best.

The second-best models are WP, combining the differences in wealth and certain predispositions towards one of the strategies. A direct comparison can be made with the HPM models, which are based on the same number of parameters. The J-values are approximately lower by 60%, and the p-values are more than doubled, indicating a clear superiority of the WP specification.

Comparing the TPA and DCA switching approach, the TPA performs better with a lower average J-value in all models. This contrasts with the S&P500 results, where the switching approaches performed similarly. However, as the J-values between models with the same specifications but different switching differ only slightly, the essential factor when comparing the quality of the models is the specification, not the switching mechanism.

Generally, the confidence intervals for most estimates are narrower compared to the previous case and do not hit bounds. The exceptions are, again, the lower bounds of the herding component for the HPM specifications and the wealth boundaries, especially for W-TPA. Moreover, the confidence intervals of chartists' aggressiveness hit zero for all models, suggesting a questionable significance of this parameter. On the other hand, the fundamentalists' aggres-

Table 6.2: Franke and Westerhoff (2012): Results for the Bitcoin data

Par./ Model	DCA				TPA			
L	W 6	WP 7	WHP 8	HPM 7	W 6	WP 7	WHP 8	HPM 7
$\hat{\phi}$ $\langle 0,10 \rangle$	7.222 (6.95,7.76)	7.946 (5.69,9.64)	7.371 (1.97,9.72)	1.220 (1.12,1.27)	7.229 (6.96,7.84)	7.505 (4.89,9.34)	7.626 (5.38,9.45)	1.151 (1.05,1.27)
$\hat{\chi}$ $\langle 0,5 \rangle$	0.252 (0.003,1.192)	0.858 (0.010,2.805)	0.740 (0.007,3.086)	0.061 (0.002,0.216)	0.287 (0.001,1.152)	0.489 (0.009,1.609)	0.606 (0.005,2.196)	0.074 (0.003,0.273)
$\hat{\eta}$ $\langle 0.8,1 \rangle$	0.987 (0.986,0.987)	0.986 (0.984,0.988)	0.986 (0.983,0.987)	-	0.987 (0.986,0.988)	0.986 (0.985,0.988)	0.986 (0.982,0.988)	-
$\hat{\sigma}_f$ $\langle 0,3.5 \rangle$	1.668 (1.65,1.69)	1.701 (1.63,2.54)	1.733 (1.63,2.58)	2.516 (2.49,2.54)	1.671 (1.66,1.69)	1.717 (1.63,2.73)	1.684 (1.63,1.72)	2.505 (2.47,2.53)
$\hat{\sigma}_c$ $\langle 0,4 \rangle$	2.797 (2.76,2.85)	2.751 (1.65,2.98)	2.692 (1.65,2.90)	1.618 (1.61,1.63)	2.805 (2.76,2.84)	2.696 (1.68,2.85)	2.740 (2.51,2.95)	1.616 (1.60,1.63)
$\hat{\alpha}_0$ $\langle -12,6 \rangle$	-	-2.773 (-11.02,5.14)	-2.456 (-11.24,5.04)	-11.045 (-11.98,-9.15)	-	-3.124 (-10.66,5.40)	-2.431 (-11.31,4.95)	-11.078 (-11.96,-9.50)
$\hat{\alpha}_n$ $\langle 0,8 \rangle$	-	-	2.285 (0.12,5.01)	3.077 (2.39,3.80)	-	-	1.603 (0.12,4.83)	1.088 (0.17,1.82)
$\hat{\alpha}_p$ $\langle 40,200 \rangle$	-	-	-	149.633 (123.99,176.13)	-	-	-	156.707 (129.35,185.23)
$\hat{\alpha}_w$ $\langle 200,3200 \rangle$	2827.15 (2099.35,3188.98)	2380.34 (863.68,3171.47)	2444.09 (904.06,3139.26)	-	2750.32 (1993.70,3185.04)	2336.98 (822.45,3165.33)	2625.21 (1767.86,3167.80)	-
$\bar{J} = J(\hat{\theta})$	2.158 (1.91,2.59)	2.738 (2.17,4.62)	2.785 (2.06,5.19)	4.614 (4.38,4.93)	2.005 (1.76,2.37)	2.584 (1.88,4.71)	2.637 (1.97,4.59)	4.450 (4.12,4.69)
χ^2_{9-L}	7.81	5.99	3.84	5.99	7.81	5.99	3.84	5.99
p-value	0.540	0.254	0.095	0.100	0.571	0.275	0.104	0.108
J-test (5%)	not rej. H_0	not rej. H_0	not rej. H_0	not rej. H_0	not rej. H_0	not rej. H_0	not rej. H_0	not rej. H_0

Note: The optimized parameters are based on 100 repetitions, each averaged over 100 independent simulations with 6000 iterations. The constraints for the parameters are denoted in $\langle \rangle$ brackets, and 95% sample confidence intervals are in ( ) parentheses.

siveness reaches higher values compared to the S&P500 data. The volatilities of the noise terms $\hat{\sigma}_f$ and $\hat{\sigma}_c$ are also higher in all models, which is in line with the more volatile nature of Bitcoin itself. Lastly, the estimations of other parameters determining the relative attractiveness are similar to the results of S&P500 data, with the exception of the misalignment parameter that is more than doubled. This suggests that traders are more sensitive to the deviation of the price from its fundamental value and are more likely to change their strategy to fundamentalism.

6.3 Top20

Finally, after receiving promising results for Bitcoin, we take a closer look at the more realistic representation of the whole crypto market, an index consisting of Top20 cryptocurrencies. Compared to the previous cases, we had to extend the upper bound for the aggressiveness of fundamentalists from 10 to 16 as all models including the wealth component reached the upper bound in the preliminary analysis.

Looking at the results in Table 6.3, we immediately notice that we get higher J-values than in the case of Bitcoin itself but lower than in the S&P500 Index. An intuitive explanation for this robust rank is that the values correspond to how rational or irrational the market is. S&P500 is the most commonly used benchmark for financial markets, the more mature and stable form of market that we study. As such, we can expect more reasonable participant behaviour. Cryptomarkets are, on the other hand, much more volatile and less regulated, attracting the riskier type of investors who are more prone to irrational actions. At the same, the index consisting of the Top20 cryptocurrency is more resilient towards fluctuations of individual currencies, includes a larger pool of participants and therefore could be considered more rational than Bitcoin.

Looking at the results of the Top20 Index, only W-DCA and W-TPA cannot be rejected at the 5% significance level, and WP models are rejected only weekly. These findings, consistent with those from the Bitcoin market, indicate that wealth specifications that rely on the differences in wealth between the two strategies are the most effective in capturing the behaviour in crypto markets. Compared to the financial markets represented by the S&P500 Index, the HPM specifications perform poorly and are strongly rejected. As such, among the models with seven parameters, WP specification is preferred. Both DCA and TPA switching approaches perform similarly in terms of J-value.

Table 6.3: Franke and Westerhoff (2012): Results for the Top20 data

	DCA				TPA			
Par./ Model L	W 6	WP 7	WHP 8	HPM 7	W 6	WP 7	WHP 8	HPM 7
$\hat{\phi}$ $\langle 0,16 \rangle$	12.447 (11.35,13.87)	13.21 (11.22,15.80)	13.135 (10.97,15.68)	1.467 (1.39,1.54)	12.953 (11.41,14.77)	13.147 (10.94,15.48)	13.175 (11.13,15.37)	1.499 (1.44,1.70)
$\hat{\chi}$ $\langle 0,5 \rangle$	1.845 (0.174,4.040)	1.174 (0.035,3.957)	1.499 (0.015,4.394)	0.135 (0.006,0.381)	1.867 (0.020,4.138)	1.640 (0.003,4.749)	1.741 (0.028,3.976)	0.133 (0.002,0.477)
$\hat{\eta}$ $\langle 0.8,1 \rangle$	0.982 (0.979,0.984)	0.982 (0.977,0.984)	0.980 (0.973,0.984)	-	0.980 (0.978,0.983)	0.981 (0.975,0.985)	0.980 (0.975,0.983)	-
$\hat{\sigma}_f$ $\langle 0,3.5 \rangle$	1.649 (1.62,1.67)	1.638 (1.58,1.68)	1.676 (1.60,2.36)	2.395 (2.36,2.43)	1.649 (1.62,1.67)	1.655 (1.59,2.03)	1.659 (1.60,2.03)	2.397 (2.37,2.43)
$\hat{\sigma}_c$ $\langle 0,4 \rangle$	3.272 (3.18,3.36)	3.064 (2.79,3.36)	3.084 (1.54,3.42)	1.525 (1.50,1.55)	3.258 (3.09,3.41)	3.123 (2.09,3.43)	3.143 (2.02,3.40)	1.529 (1.51,1.55)
$\hat{\alpha}_0$ $\langle -12,6 \rangle$	-	-6.335 ($-11.57.4.46$)	-4.125 (-11.44,5.63)	-6.523 ($-7.90,-4.96$)	-	-4.843 (-11.30,4.89)	-2.750 (-11.40,5.20)	-6.513 ($-7.26,-5.76$)
$\hat{\alpha}_n$ $\langle 0,8 \rangle$	-	-	3.137 (0.14,7.66)	3.911 (3.23,4.86)	-	-	3.418 (0.16,7.70)	1.379 (0.81,2.03)
$\hat{\alpha}_p$ $\langle 40,200 \rangle$	-	-	-	183.538 (144.37,199.07)	-	-	-	187.86 (168.74,199.78)
$\hat{\alpha}_w$ $\langle 200,3200 \rangle$	2647.36 (1782.37,3185.24)	2042.30 (935.26,3121.52)	2132.48 (864.83,3122.58)	-	2386.02 (731.38,3173.02)	2254.59 (881.48,3160.94)	2268.69 (953.27,3155.90)	-
$\bar{J} = J(\hat{\theta})$	5.922 (5.58,6.35)	6.096 (5.50,7.71)	6.489 (5.75,9.80)	11.012 (10.76,11.34)	5.958 (5.48,6.42)	6.409 (5.64,9.96)	6.453 (5.73,10.18)	11.075 (10.77,11.48)
χ^2_{9-L}	7.81	5.99	3.84	5.99	7.81	5.99	3.84	5.99
p-value	0.115	0.047	0.011	0.004	0.114	0.041	0.011	0.004
J-test (5%)	not rej. H_0	rej. H_0	rej. H_0	rej. H_0	not rej. H_0	rej. H_0	rej. H_0	rej. H_0

Note: The optimized parameters are based on 100 repetitions, each averaged over 100 independent simulations with 6000 iterations. The constraints for the parameters are denoted in $\langle\ \rangle$ brackets, and 95% sample confidence intervals are in ( ) parentheses.

In all models, fundamentalists display higher levels of aggressiveness compared to other markets, exceeding Bitcoin by approximately 60%. Chartists, on the other hand, exhibit even greater levels of aggressiveness, with values several times higher, especially for HPM models. While the fundamentalists' volatility of the noise term is comparable to that of Bitcoin, the chartists' volatility is roughly 15% higher, with the exception of HPM models. The higher value of $\hat{\alpha}_n$ and $\hat{\alpha}_p$ suggests more pronounced herding behaviour compared to previous markets and a stronger influence of price misalignment.

To sum up the central message of our results, we demonstrated a very promising application of financial agents to cryptocurrency markets, both on the level of Bitcoin and a hypothetical Top20 Index. These findings, represented by small values of the average loss function and tested by the J-test of overidentification, are consistent between different model specifications.

Chapter 7

Discussion

The chapter discusses the limitations of our analysis and potential improvements for further research. Firstly, we examine the limitations of our setup and constraints in Section 7.1. In Section 7.2, we discuss our inability to interpret the meaningful evolution of the two groups. Lastly, we talk about alternative evaluation metrics in Section 7.3.

7.1 Constraints

To start, let us dive into the constraints setup and its limitations. In our preliminary analysis of the S&P500 dataset, we checked the behaviour of all models with a relatively modest setup (100 replications, each averaged on 30 independent simulations with 2000 iteration steps). The heterogeneity between the models, especially for the parameters as the aggressiveness of fundamentalists $\hat{\phi}$ and the predisposition component $\hat{\alpha_n}$, led to setting rather loose constraints. This aimed to give the optimization enough space to find the minimum and have the same constraints for different empirical datasets when possible. In the extended versions, we then encountered two issues.

Firstly, despite setting relatively wide constraints for the estimated parameters, the final runs with more iteration steps hit the bounds. The explanation for encountering this problem is that with a lower number of iterations, the optimization did not reach the boundary, and thus such behaviour could not be detected. Due to this, we could likely get lower numbers for the HPM models by extending the boundaries. Secondly, the relatively wide constraints often lead to wide distributions of the parameters. This means relatively many combinations of parameters lead to similar results, undermining the final value of the parameters calculated as an average.

In the future, several measures could be taken to improve the results in this matter. First, conducting more replications would reveal more information about the distributions of the parameters and J-values. Currently, with 100 replications, it is uncertain whether the wide distributions of certain parameters are a property of the model or whether we did not have enough values to reveal the true shape of the parameters' distribution. However, increasing replications linearly increases the computational time, so we would need a more powerful computing budget.[1]

Similarly, widening the constraints or increasing the number of iteration steps would be computationally expensive. As such, we could bypass this issue, at least to some extent, by having multiple constraint sets for different models. From all models, it can be seen that the behaviour between the two switching approaches is relatively similar, and we get much heterogeneous behaviour from the different relative attractiveness specifications. Therefore, we could create constraint sets for different model specifications. However, we would lose the ability to compare the models themselves directly with each other, so having a more powerful computational setup for future estimations would be a preferable option.

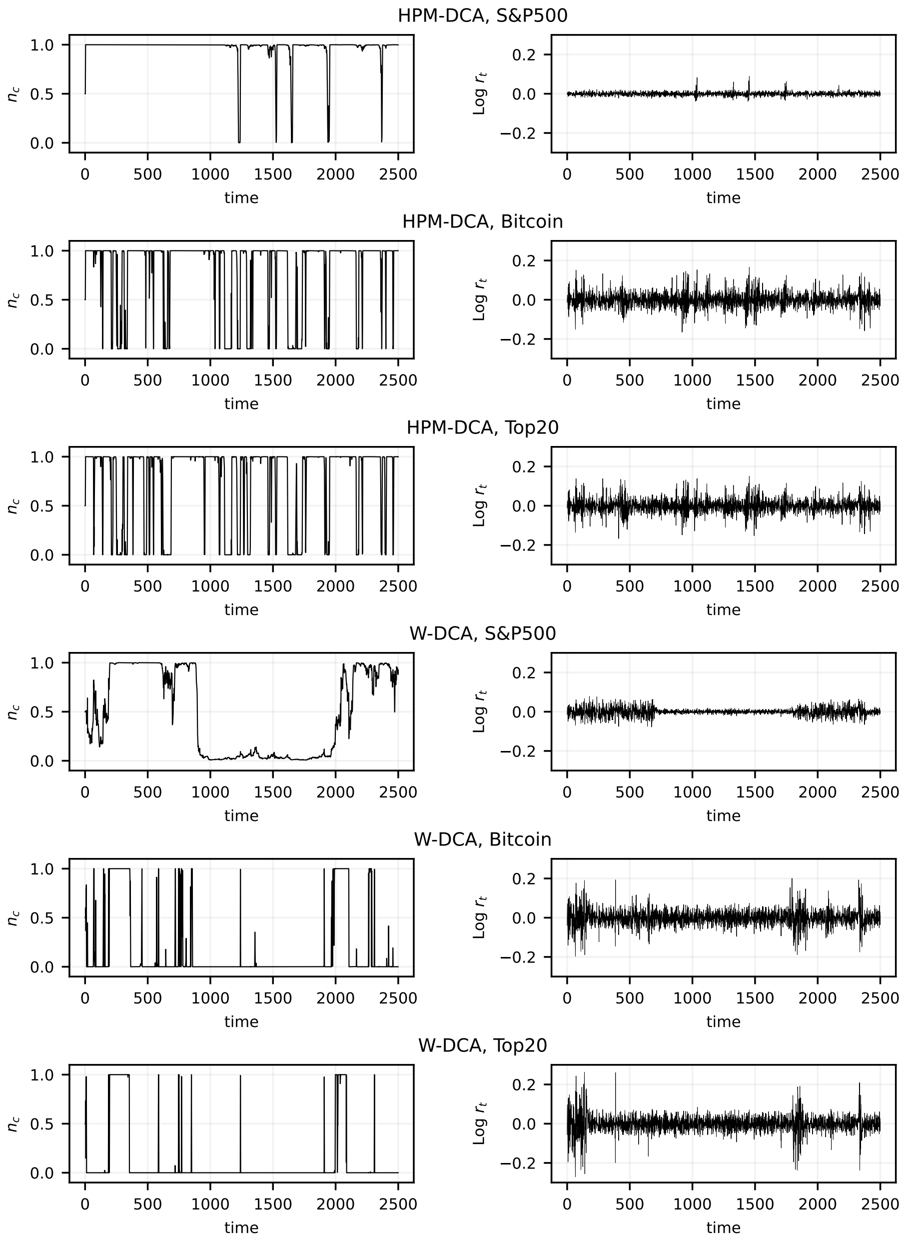
7.2 Interpreting the results

Up to this point, we have intentionally postponed the discussion on the evolution of the trading groups. We receive two opposite extremes, either absolute dominance of chartists or fundamentalists, which is very unlikely in real life. Moreover, there is almost no gradual dynamic in the switching. Figure 7.1 illustrates these issues on the evolution of chartists and log returns based on the optimized sets of parameters for all datasets for the HPM-DCA and W-DCA models. Figures for chartists' evolution for all models are then available in Appendix A (see Figure A.1 for S&P500, Figure A.2 for Bitcoin, and Figure A.3

[1]In our analysis, the majority of simulations were executed on a server with an Intel Xeon W 3.00 GHz processor with 10 cores and 32 GB RAM. With the setting of 100 replications, each averaged over 100 simulations with 6000 iteration steps, the average computational time is around 16 hours for the most extended S&P500 dataset and the most complex WHP-TPA model. For the shorter Bitcoin and Top20 datasets, the computational time decreases to a third and takes approximately 5 hours. In the case of less complex specifications with six or seven estimating parameters, the optimization declines by 10-20%.

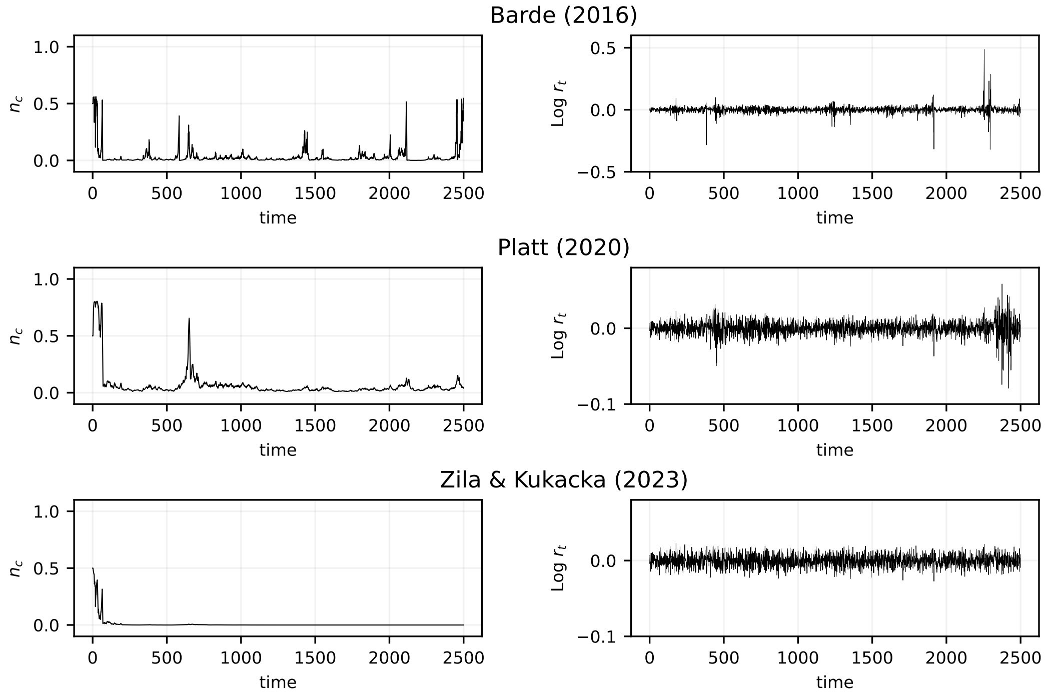
for Top20 Index). The behaviour observed in both cases is highly improbable and contradictory, indicating that chartists have a dominant influence in the HPM-DCA model, while fundamentalists have a stronger impact in the W-DCA model. The only behaviour that persists in both types is stronger switching tendencies in crypto markets. The only evolution that does not seem completely unrealistic is the W-DCA for S&P500, but the log returns show an unlikely behaviour.

Figure 7.1: Evolution of chartists and log returns for HPM-DCA and W-DCA models



However, the issue of unlikely optimization results is not unique to our study. Figure 7.2 displays the evolution of chartists based on optimized results from the three recent studies by Barde (2016), Platt (2020), and Zila & Kukacka (2023). As can be seen, optimized parameters from all studies encounter similar unlikeliness of real representation, however, from the opposite bank. Looking at the graphs, there is almost no interaction between chartists and fundamentalists, with the chartists representing less than 4% of all participants for the majority of the time.

Figure 7.2: Evolution of chartists — HPM-DCA



Note: The graphs are based on optimized parameters sets for HPM-DCA model for S&P500 Index (Platt 2020; Zila & Kukacka 2023) and USP/GBP exchange rates (Barde 2016).

Zila & Kukacka (2023) use a very similar number of observations and only a slightly different setup with two main differences, narrower constraints and 500 replications. Interestingly, when we compare the final loss functions, they receive just a slightly smaller average J-value of 6.32 compared to our value of 6.65. Despite having similar J-values, the final economic interpretation would be entirely different, with one study being entirely dominated by chartists and the other by fundamentalists. This suggests some inconsistencies that should be addressed in future research. Looking at what could be improved in our work, compared to other studies, we use much wider constraints. Even though the model itself does not have upper limits for the parameters with the exception of the memory coefficient, some upper boundaries may be needed to capture the dynamics meaningfully.

Another option is to scrutinize and reevaluate the fixed parameters introduced in Franke & Westerhoff (2012). The original study does not specify reasons for choosing particular values of these parameters. However, they were likely determined based on information up to 2007, not capturing events such as the Financial Crisis, Covid-19, or the Russian invasion of Ukraine, which significantly shook the markets. Therefore, we believe that reevaluating the values of the fixed parameters could encourage more dynamic interactions between chartists and fundamentalists.

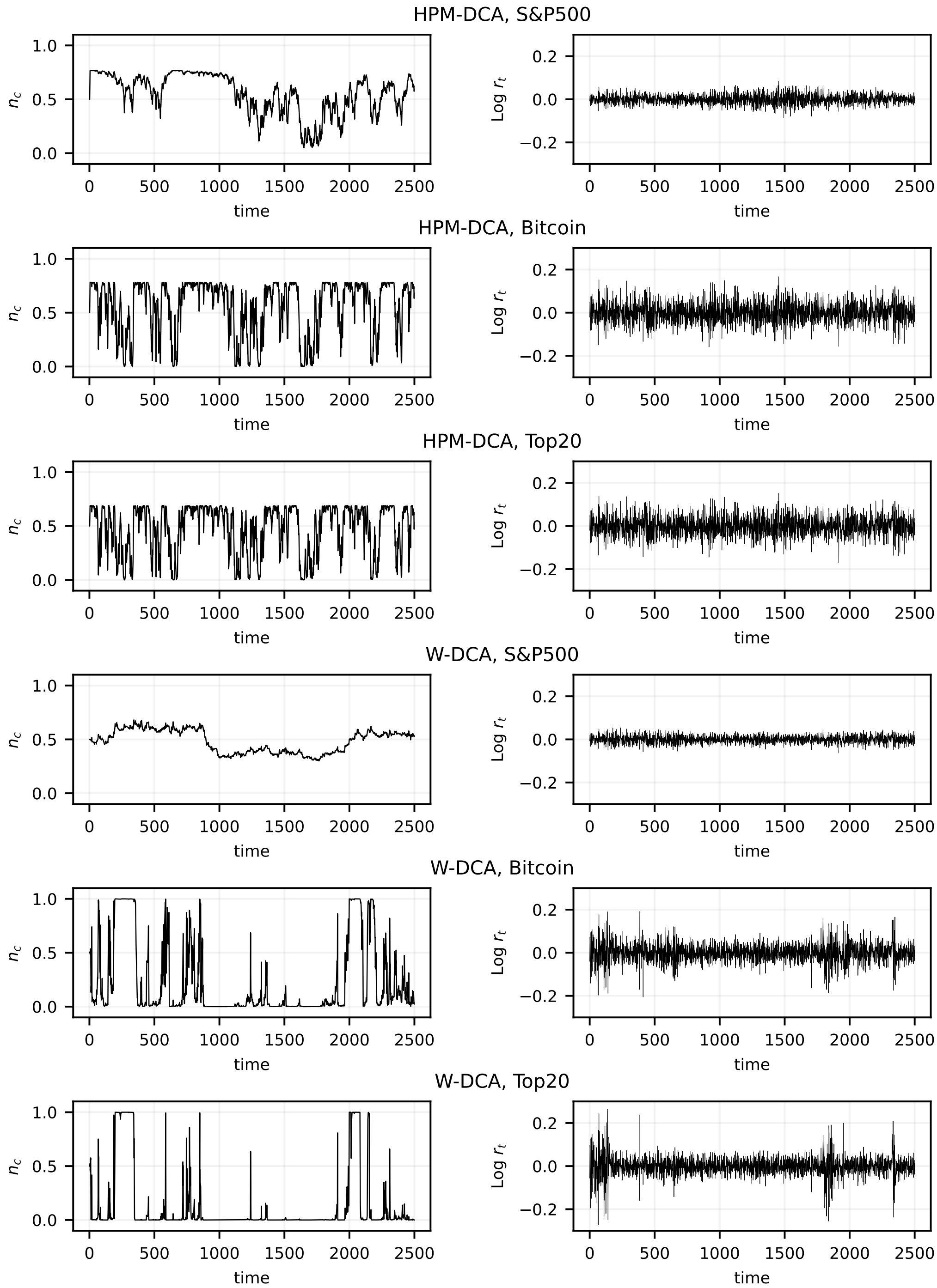
7.2.1 Potential reevaluation of fixed parameters

Although a more detailed analysis of the fixed parameters would be needed for further research, we highlight several observations we noticed while working with the model. For additional analysis, we used our optimized results and tried different settings. We scaled the market impact factor of demand μ, the flexibility parameter ν for the TPA switching, and the intensity of choice β for the DCA switching. Moreover, we changed the intensity of noise terms.

The only change that did not drastically increase the J-value and encourage dynamics was changing the parameter β for the DCA switching from 1 to 0.1. The J-values, in this case, roughly doubled. Figure 7.3 presents the evolution of chartists and log returns with the same parameter sets as in Figure 7.1, but with the scaled β to 0.1. The switching is boosted in all cases, and we decreased the occurrence of extreme instances of pure dominance of one of the groups. Especially the W-DCA specification in the crypto markets suggests behaviour and log returns that are much closer to what we could expect. However, when looking at Figure 7.3, we must keep in mind that we rescaled β after the optimization. If we had rescaled β prior to the optimization, the optimization would have likely arrived at a different set of parameters, which could have shown different dynamics.

Unfortunately, for the TPA switching, we could not find any setting that would stimulate the interactions, and we leave this task to further research.

Figure 7.3: Evolution of chartists and log returns for HPM-DCA and W-DCA models with $\beta = 0.1$

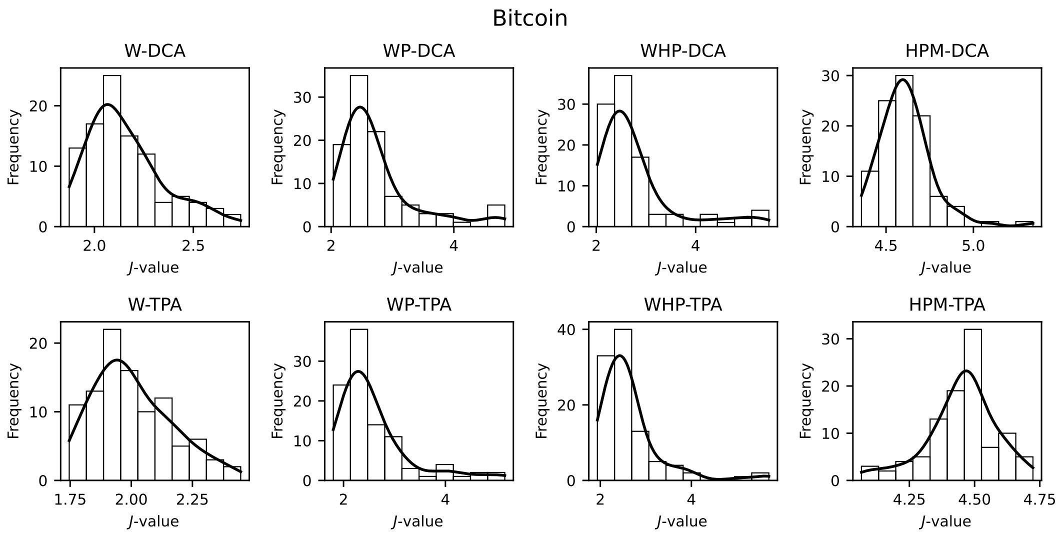


7.3 Evaluation metrics

Lastly, we briefly discuss the value metrics and possible alternatives to evaluations of our results. We evaluate the results using the standardly used J-test of overidentifying restrictions. For each replication, we take an average of 100 simulations. However, outlying values may shift the average, especially if we work with a small sample, which is our case. As such, an alternative measure more persistent towards outlying values would be using the median.

We present the frequency distributions of J-values for Bitcoin Figure 7.4. The level of symmetry varies between models, with most distributions skewed to the right, meaning that, indeed, we would benefit from using the median. For the frequency distributions for other datasets, we refer to Figure A.4 for S&P500 and Figure A.5 for Top20 Index in Appendix A.

Figure 7.4: Distributions of J-values based on optimized sets of parameters: Bitcoin



Chapter 8

Conclusion

This thesis is concerned with one of the first applications of a financial agent-based model to the cryptocurrency markets. To do so, we used a widely analyzed model of Franke & Westerhoff (2012), and for the estimation, we implied the simulated method of moments. The model is based on interactions between two types of traders, fundamentalists and chartists. It uses two switching approaches and four combinations of parameters to determine the attractiveness of the other trading strategy. We invoked the J-test of overidentifying restrictions to evaluate the results.

Firstly, we reevaluated the original study on S&P500 Index to receive a more up-to-date view of financial market behaviour. The extended time period included observations throughout major economic events such as the Financial Crisis or Covid-19. Next, we applied the model to Bitcoin. After receiving promising results from the Bitcoin analysis, we created a third type of data set, a hypothetical market-weighted Top20 cryptocurrency index. The index includes the top 20 cryptocurrencies by market cap on a weekly basis and reevaluates the weights daily.

We considerably widened the constraints for the parameters compared to previous research. We received roughly three times lower J-values for the final analysis than the original study for the S&P500. The best-performing specification with the lowest J-value is the HPM combining herding, predisposition and price misalignment. The overall winning model is the HPM-DCA with the p-value of 3.6%. Although still rejected by the J-test alongside all other specifications at the 5% significance level, the preference for this model when capturing behaviour at financial markets is consistent with the original study.

For the Bitcoin data, all models could not be rejected as true models at the 5% significance level. Among them, the ones with the highest p-value of 57.1% and 54% were W-TPA and W-DCA specifications, respectively.

Thirdly, the optimization of the hypothetical Top20 Index produced smaller J-values compared to the S&P500 Index and higher compared to Bitcoin, except for the HPM specifications. We believe that this robust rank can be attributed to how irrational or rational the market is. The S&P500 represents a stable and mature financial market benchmark with more reasonable participant behaviour, while crypto markets are volatile, less regulated, and attract riskier investors prone to irrational actions. At the same time, the Top20 Index is more resilient to individual currency fluctuations, includes a larger pool of participants, and could be therefore considered more rational than Bitcoin.

The model with the highest p-value of 11.5% and 11.4% were W-DCA and W-TPA, similar to Bitcoin. This suggests that models concerning the wealth differences between the two strategies capture the behaviour in crypto markets the best. The switching mechanism plays only a marginal role for the J-test, as the values usually differed only slightly for all studied markets.

To summarise the main message from the results, we demonstrated a very promising application of financial agents to cryptocurrency markets, both on the level of Bitcoin and a hypothetical Top20 Index.

Additionally, we identified a research gap when interpreting the evolution of chartists in the market. Evolutions based on optimized parameter sets in several recent studies suggest the absolute dominance of one group of agents and a lack of dynamics between the groups. For the optimized parameters in our study, we were able to evoke some level of dynamics in the DCA switching by scaling the parameter β from 1 to 0.1. To encourage more interactions, we believe that further research needs to focus on scrutinizing the fixed parameters.

Bibliography

ALFARANO, S., T. LUX, & F. WAGNER (2005): "Estimation of agent-based models: The case of an asymmetric herding model." *Computational Economics* **26**: pp. 19–49.

ALFARANO, S., T. LUX, & F. WAGNER (2006): "Estimation of a simple agent-based model of financial markets: An application to australian stock and foreign exchange data." *Physica A: Statistical Mechanics and its Applications* **370(1)**: pp. 38–42. Econophysics Colloquium.

ALFARANO, S., T. LUX, & F. WAGNER (2007): "Empirical validation of stochastic models of interacting agents a "maximally skewed" noise trader model." *Eur. Phys. J. B* **55**: pp. 183–187.

BARDE, S. (2016): "Direct comparison of agent-based models of herding in financial markets." *Journal of Economic Dynamics and Control* **73**: pp. 329–353.

BARIVIERA, A. F. (2017): "The inefficiency of Bitcoin revisited: A dynamic approach." *Economics Letters* **161**: pp. 1–4.

BARIVIERA, A. F., M. J. BASGALL, W. HASPERUE, & M. NAIOUF (2017): "Some stylized facts of the bitcoin market." *Physica A: Statistical Mechanics and its Applications* **484**: pp. 82–90.

BAUR, D. G., K. H. HONG, & A. D. LEE (2018): "Bitcoin: Medium of exchange or speculative assets?" *Journal of International Financial Markets, Institutions and Money* **54**: pp. 177–189.

BOSWIJK, H. P., C. H. HOMMES, & S. MANZAN (2007): "Behavioral heterogeneity in stock prices." *Journal of Economic Dynamics and Control* **31**: pp. 1938–1970.

Bouri, E., P. Molnar, G. Azzi, D. Roubaud, & L. I. Hagfors (2017): "On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier?" *Finance Research Letters* **20**: pp. 192–198.

Brock, W., C. Hommes, W. Brock, & C. Hommes (1998): "Heterogeneous beliefs and routes to chaos in a simple asset pricing model." *Journal of Economic Dynamics and Control* **22**: pp. 1235–1274.

Brock, W. A. & C. H. Hommes (1998): "Heterogeneous beliefs and routes to chaos in a simple asset pricing model." *Journal of Economic Dynamics and Control* **22(8)**: pp. 1235–1274.

Chen, S. H., C. L. Chang, & Y. R. Du (2012): "Agent-based economic models and econometrics." *The Knowledge Engineering Review* **27**: pp. 187–219.

Ciaian, P., M. Rajcaniova, & d'Artis Kancs (2016): "The economics of bitcoin price formation." *Applied Economics* **48**: pp. 1799–1815.

CoinGecko (2023): "Cryptocurrency prices by market cap." `https://www.coingecko.com/` [Accessed on 2023-03-06].

CoinMarketCap (2023a): "Bitcoin price today, BTC to USD live, marketcap and chart." `https://coinmarketcap.com/currencies/bitcoin/` [Accessed on 2023-03-03].

CoinMarketCap (2023b): "Check cryptocurrency price history for the top coins." `https://coinmarketcap.com/historical/` [Accessed on 2023-03-14].

Conlon, T. & R. McGee (2020): "Safe haven or risky hazard? Bitcoin during the Covid-19 bear market." *Finance Research Letters* **35**: p. 101607.

Cont, R. (2001): "Empirical properties of asset returns: Stylized facts and statistical issues." *Quantitative Finance* **1**: pp. 223–236.

Corbet, S., B. Lucey, A. Urquhart, & L. Yarovaya (2019): "Cryptocurrencies as a financial asset: A systematic analysis." *International Review of Financial Analysis* **62**: pp. 182–199.

Dieci, R. & X. Z. He (2018): "Heterogeneous agent models in finance." *Handbook of Computational Economics* **4**: pp. 257–328.

Drożdż, S., R. Gebarowski, L. Minati, P. Oswiecimka, & M. Watorek (2018): "Bitcoin market route to maturity? Evidence from return fluctuations, temporal correlations and multiscaling effects." *Chaos: An Interdisciplinary Journal of Nonlinear Science* **28**: p. 071101.

Duffie, D. & K. J. Singleton (1993): "Simulated moments estimation of markov models of asset prices." *Econometrica* **61**: p. 929.

Engle, R. (2002): "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models." *Journal of Business & Economic Statistics* **20(3)**: pp. 339–350.

Fabretti, A. (2013): "On the problem of calibrating an agent based model for financial markets." *Journal of Economic Interaction and Coordination* **8**: pp. 277–293.

Fagiolo, G., A. Moneta, & P. Windrum (2007): "A critical guide to empirical validation of agent-based models in economics: Methodologies, procedures, and open problems." *Computational Economics* **30**: pp. 195–226.

Fama, E. F. (1970): "Efficient capital markets: A review of theory and empirical work." *The Journal of Finance* **25**: p. 383.

Fang, L., E. Bouri, R. Gupta, & D. Roubaud (2019): "Does global economic uncertainty matter for the volatility and hedging effectiveness of Bitcoin?" *International Review of Financial Analysis* **61**: pp. 29–36.

Farmer, J. D. & S. Joshi (2002): "The price dynamics of common trading strategies." *Journal of Economic Behavior Organization* **49**: pp. 149–171.

Yahoo Finance (2023): "S&P 500 charts, data news." `https://finance.yahoo.com/quote/%5EGSPC/` [Accessed on 2023-02-05].

Franke, R. (2008): "A microfounded herding model and its estimation on german survey expectations." *European Journal of Economics and Economic Policies: Intervention* **5**: pp. 301–328.

Franke, R. (2009): "Applying the method of simulated moments to estimate a small agent-based asset pricing model." *Journal of Empirical Finance* **16**: pp. 804–815.

Franke, R. & F. Westerhoff (2011): "Estimation of a structural stochastic volatility model of asset pricing." *Computational Economics* **38**: pp. 53–83.

Franke, R. & F. Westerhoff (2012): "Structural stochastic volatility in asset pricing dynamics: Estimation and model contest." *Journal of Economic Dynamics and Control* **36**: pp. 1193–1211.

Franke, R. & F. Westerhoff (2016): "Why a simple herding model may generate the stylized facts of daily returns: Explanation and estimation." *Journal of Economic Interaction and Coordination* **11**: pp. 1–34.

Gilli, M. & P. Winker (2003): "A global optimization heuristic for estimating agent based models." *Computational Statistics Data Analysis* **42**: pp. 299–312.

Grazzini, J., M. G. Richiardi, & M. Tsionas (2017): "Bayesian estimation of agent-based models." *Journal of Economic Dynamics and Control* **77**: pp. 26–47.

Hommes, C. H. (2006): "Chapter 23 heterogeneous agent models in economics and finance." *Handbook of Computational Economics* **2**: pp. 1109–1186.

Hu, A. S., C. A. Parlour, & U. Rajan (2019): "Cryptocurrencies: Stylized facts on a new investible instrument." *Financial Management* **48**: pp. 1049–1068.

Jang, T. S. (2015): "Identification of social interaction effects in financial data." *Computational Economics* **45**: pp. 207–238.

de Jong, E., W. F. Verschoor, & R. C. Zwinkels (2009): "A heterogeneous route to the european monetary system crisis." *Applied Economics Letters* **16**: pp. 929–932.

de Jong, E., W. F. Verschoor, & R. C. Zwinkels (2010): "Heterogeneity of agents and exchange rate dynamics: Evidence from the ems." .

Katsiampa, P. (2017): "Volatility estimation for Bitcoin: A comparison of garch models." *Economics Letters* **158**: pp. 3–6.

Kirman, A. (1993): "Ants, rationality, and recruitment." *The Quarterly Journal of Economics* **108**: pp. 137–156.

Kukacka, J. & J. Barunik (2017): "Estimation of financial agent-based models with simulated maximum likelihood." *Journal of Economic Dynamics and Control* **85**: pp. 21–45.

Lamperti, F., A. Roventini, & A. Sani (2018): "Agent-based model calibration using machine learning surrogates." *Journal of Economic Dynamics and Control* **90**: pp. 366–389.

LeBaron, B. (2006): "Chapter 24 Agent-based computational finance." *Handbook of Computational Economics* **2**: pp. 1187–1233.

LeBaron, B., W. B. Arthur, & R. Palmer (1999): "Time series properties of an artificial stock market." *Journal of Economic Dynamics and Control* **23**: pp. 1487–1516.

Lee, B. S. & B. F. Ingram (1991): "Simulation estimation of time-series models." *Journal of Econometrics* **47**: pp. 197–205.

Liu, W. (2019): "Portfolio diversification across cryptocurrencies." *Finance Research Letters* **29**: pp. 200–205.

Lux, T. & M. Marchesi (1999): "Scaling and criticality in a stochastic multi-agent model of a financial market." *Nature 1999 397:6719* **397**: pp. 498–500.

Lux, T. & R. C. Zwinkels (2018): "Empirical validation of agent-based models." *Handbook of Computational Economics* **4**: pp. 437–488.

Ma, Y., F. Ahmad, M. Liu, & Z. Wang (2020): "Portfolio optimization in the era of digital financialization using cryptocurrencies." *Technological Forecasting and Social Change* **161**: p. 120265.

Manzan, S. & F. Westerhoff (2005): "Representativeness of news and exchange rate dynamics." *Journal of Economic Dynamics and Control* **29**: pp. 677–689.

McFadden, D. (1989): "A method of simulated moments for estimation of discrete response models without numerical integration." *Econometrica* **57**: p. 995.

Mensi, W., M. U. Rehman, K. H. Al-Yahyaee, I. M. W. Al-Jarrah, & S. H. Kang (2019): "Time frequency analysis of the commonalities between bitcoin and major cryptocurrencies: Portfolio risk management implications." *The North American Journal of Economics and Finance* **48**: pp. 283–294.

MUTH, J. F. (1961): "Rational expectations and the theory of price movements." *Econometrica* **29**: p. 315.

NAKAMOTO, S. (2008): "Bitcoin: A peer-to-peer electronic cash system." .

PAKES, A. & D. POLLARD (1989): "Simulation and the asymptotics of optimization estimators." *Econometrica* **57**: p. 1027.

PHILLIP, A., J. CHAN, & S. PEIRIS (2018): "A new look at cryptocurrencies." *Economics Letters* **163**: pp. 6–9.

PLATANAKIS, E. & A. URQUHART (2019): "Portfolio management with cryptocurrencies: The role of estimation risk." *Economics Letters* **177**: pp. 76–80.

PLATT, D. (2020): "A comparison of economic agent-based model calibration methods." *Journal of Economic Dynamics and Control* **113**: p. 103859.

SHAHZAD, S. J. H., E. BOURI, D. ROUBAUD, L. KRISTOUFEK, & B. LUCEY (2019): "Is Bitcoin a better safe-haven investment than gold and commodities?" *International Review of Financial Analysis* **63**: pp. 322–330.

SIMON (1957): *Models of man; social and rational.* Oxford, England: Wiley.

TRIMBORN, S., M. LI, & W. K. HARDLE (2020): "Investing with cryptocurrencies: A liquidity constrained investment approach." *Journal of Financial Econometrics* **18**: pp. 280–306.

TVERSKY, A. & D. KAHNEMAN (1974): "Judgment under uncertainty: Heuristics and biases." *Science* **185**: pp. 1124–1131.

URQUHART, A. (2016): "The inefficiency of Bitcoin." *Economics Letters* **148**: pp. 80–82.

ZEEMAN, E. C. (1974): "On the unstable behaviour of stock exchanges." *Journal of Mathematical Economics* **1**: pp. 39–49.

ZHANG, W., P. WANG, X. LI, & D. SHEN (2018): "Some stylized facts of the cryptocurrency market." *Applied Economics* **50(55)**: pp. 5950–5965.

ZILA, E. & J. KUKACKA (2023): "Moment set selection for the smm using simple machine learning." *SSRN Electronic Journal* .

Appendix A

Supporting Figures

Figure A.1: Evolution of chartists based on optimized sets of parameters: S&P500

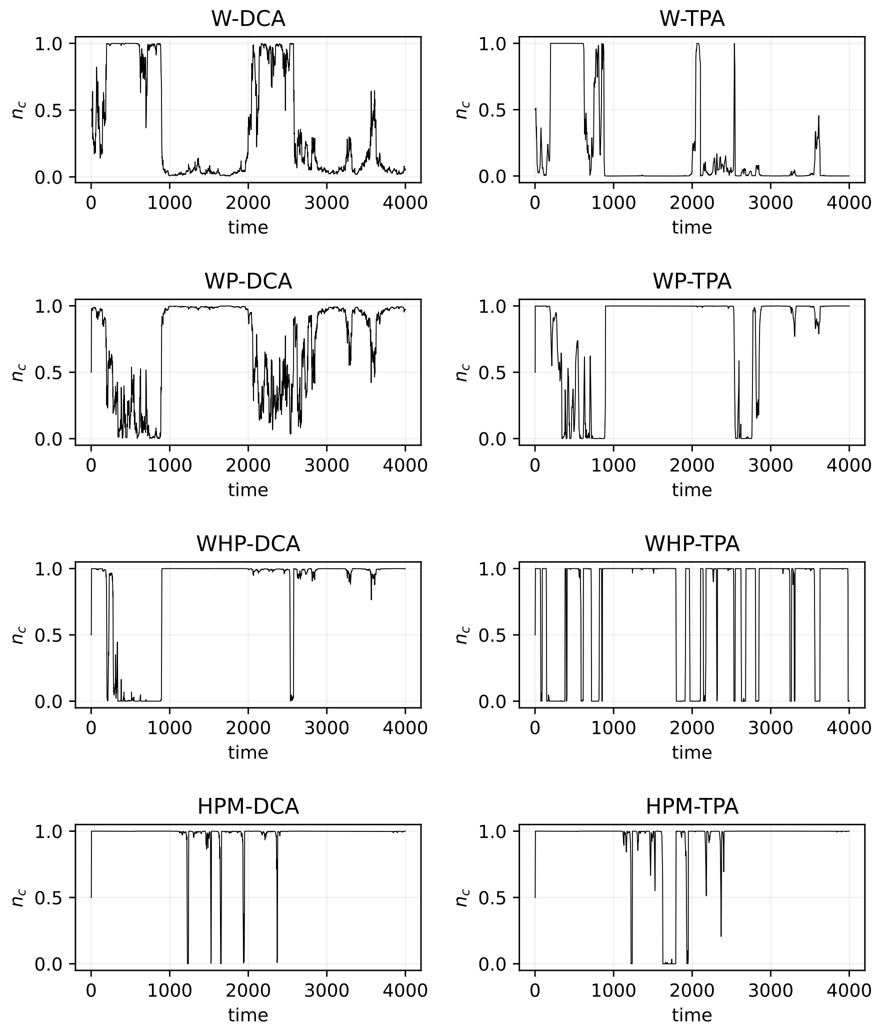


Figure A.2: Evolution of chartists based on optimized sets of parameters: Bitcoin

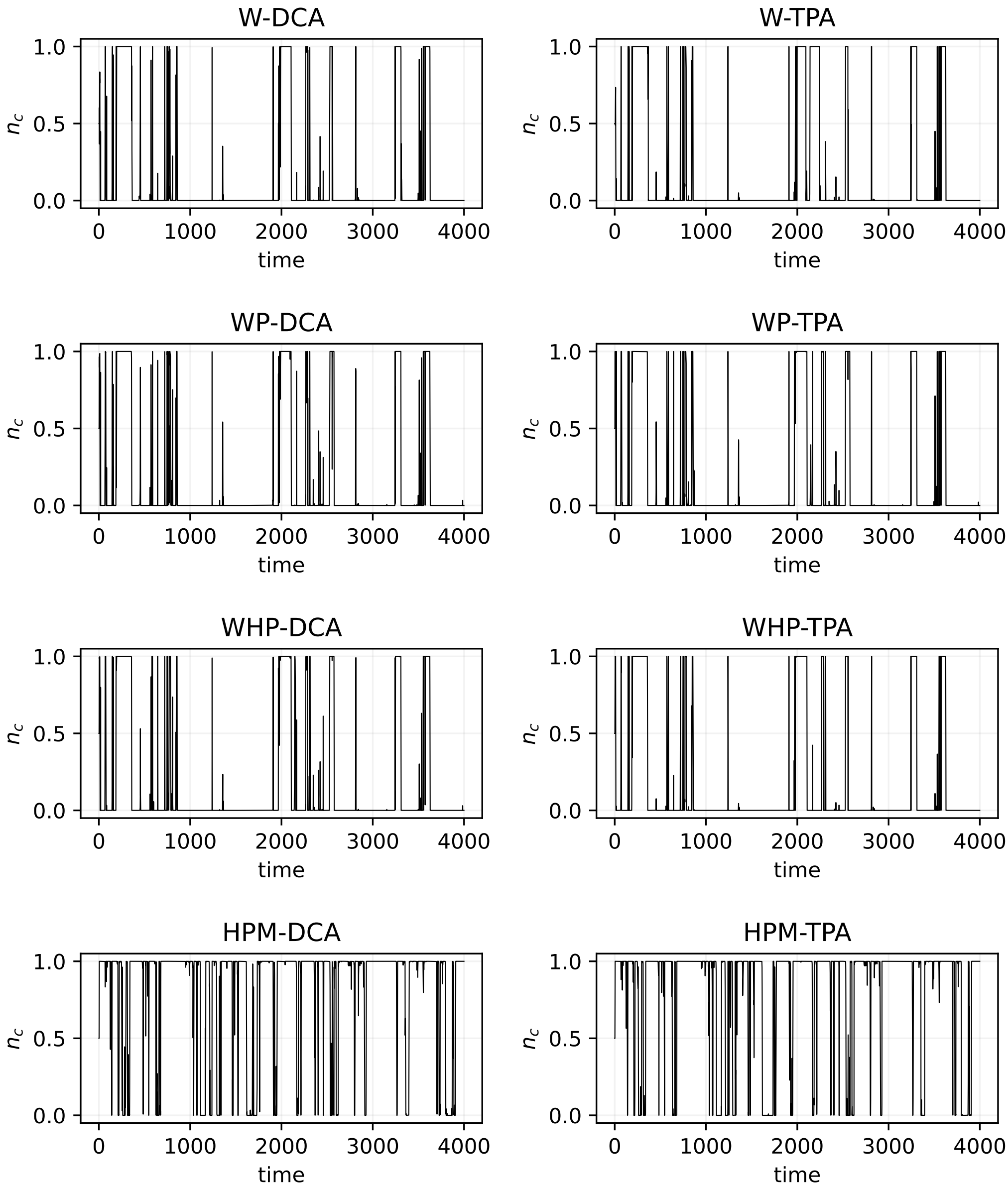


Figure A.3: Evolution of chartists based on optimized sets of parameters: Top20 Index

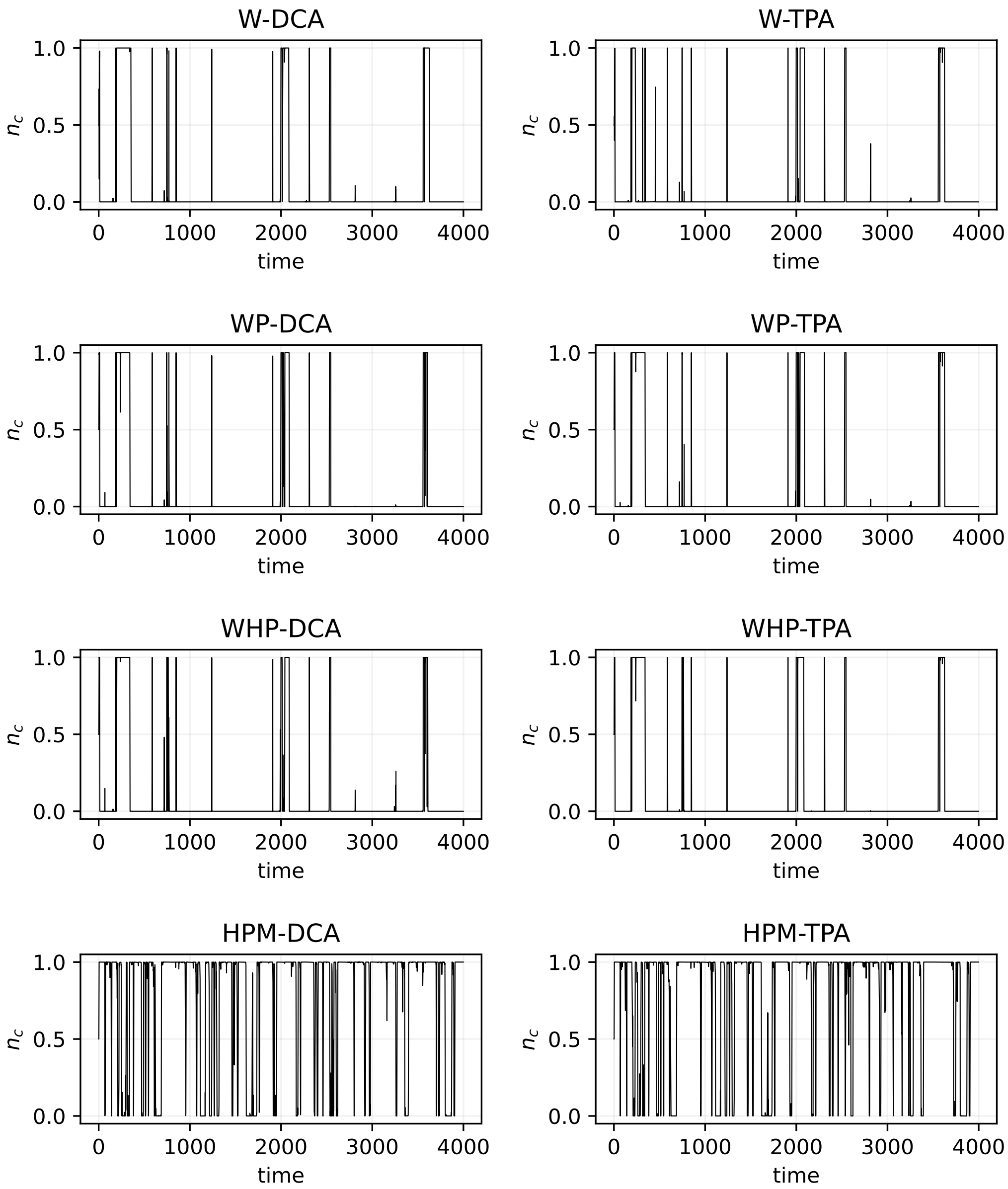


Figure A.4: Distributions of J-values based on optimized sets of parameters: S&P500

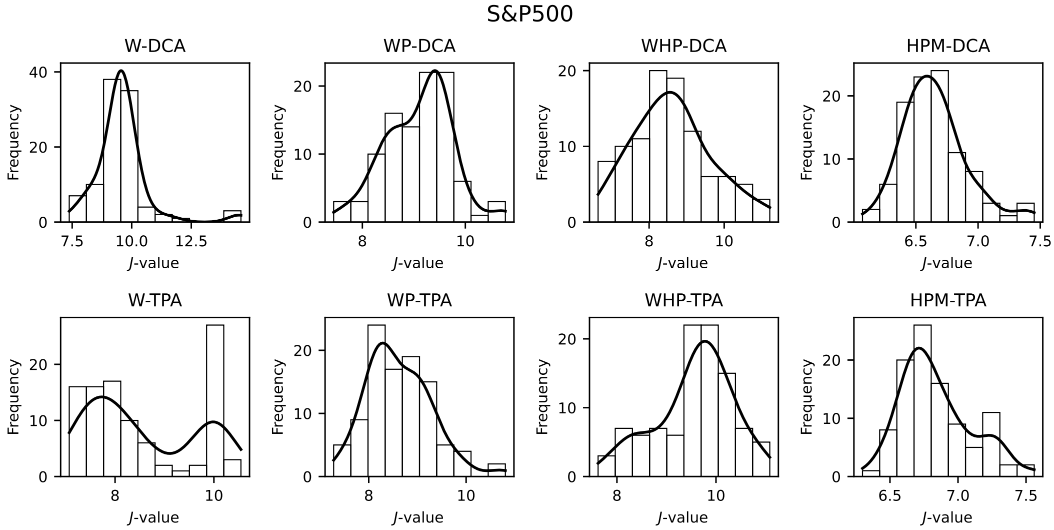


Figure A.5: Distributions of J-values based on optimized sets of parameters: Top20 Index