FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

doc. Mgr. David Kubizňák, Ph.D.

Charles University
Institute of Theoretical Physics
Faculty of Mathematics and Physics
V Holešovičkách 2, 180 00 Prague 8, Czech Republic
david.kubiznak@matfyz.cuni.cz

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Report on Doctoral Thesis of Adam Vrátný

This is a report on Doctoral Thesis of Adam Vrátný titled *Spacetimes with black holes*, written under a supervision of Prof. Jiří Podolský at Charles University, submitted in Prague 2023.

The main objective of this Thesis is to push forward our understanding of four-dimensional black hole spacetimes encoded in the so called *Plebanski–Demianski class* of solutions. This class represents the most general (7-parametric) type D family of solutions of Einstein–Maxwell equations with or without a cosmological constant and in particular describes rotating and charged accelerating black holes possibly endowed with a gravitomagnetic (NUT) charge.

In this Thesis, a *novel simplified form* for these solutions is proposed, which allows one to identify various special solutions by simply switching off various physical parameters. The obtained metrics are further analyzed — their global structure and basic properties are discussed. Moreover, it is shown that, despite a common belief, the Plebanski–Demianski class of solutions does not admit a special case of accelerating NUT charged black hole without rotation, although such a solution is contained in this class for non-zero spin of the spacetime. The reason for this surprising result is that in the process of switching off the rotation parameter, the acceleration also disappears.

Interestingly, an accelerating black hole solution with NUT charge was found in 2006 in reference [37] of the Thesis – its properties are analyzed in detail in Chapter 3. In particular, it is shown that it is of the general algebraic type and thence goes beyond type D metrics of the Plebanski–Demianski class.

The Thesis is based on 3 published papers in *Physical Review D* and a conference proceeding. I find it well written. In particular, I really enjoyed the very nicely structured introduction which puts current investigations in historical context. The Thesis is self contained and presents interesting new results. For all these reasons I am very happy to recommend it to *be recognized* as Doctoral Thesis.

I have two sets of questions for potential discussion.

1. One of the main results of the Thesis is an analysis of the accelerating NUT charged solution obtained in reference [37] – in particular it is shown that such a solution has a more general algebraic structure than the Plebanski–Demianski class.

   (a) Is it possible to obtain this solution by some kind of (possibly singular) "$a \to 0$" limit of the Plebanski–Demianski class (which is known to contain accelerating NUT with additional rotation)?

   (b) Alternatively, is it possible that there is a more general accelerating and rotating Taub-NUT solution that goes beyond Plebanski–Demianski class, whose $a \to 0$ limit yields the solution studied in this Thesis?

   (c) Is there a way to generalize the solution [37] to the presence of the cosmological constant?

2. Turning to Chapter 3, I am a bit worried about the proper identification of rotation, acceleration, and NUT charge in the case of non-zero Λ.

   (a) I would identify acceleration with the difference between the conical deficits (cosmic string tensions) on north pole/south pole axes, and the NUT charge with the difference between the angular momenta of the north pole/south pole Misner strings.

   If this is correct, it is straightforward to check that $a \to 0$ limit of the Plebanski–Demianski metric (3.5)-(3.9) switches off the acceleration. However, in the presence of the cosmological constant, this is not the case for the $\alpha \to 0$ limit. Would this suggest that perhaps the acceleration/rotation/NUT parameters may be more complex than those identified in the metric (3.5)-(3.9)?

   (b) In fact, this problem seems to remain present also for the "standard" Kerr-NUT-AdS spacetime. There seems to be a non-trivial difference in cosmic string tensions, although supposedly the acceleration has been switched off. Is it possible that it actually has non-trivial acceleration?

   (c) More generally, is there a way to separate Misner string singularities from the conical deficits and rotation in the presence of all three of them?

In any case, I believe that the overall quality of the Thesis is *excellent* and I am very happy to recommend it to *be recognized* as Doctoral Thesis.
Kind regards,

David Kubizňák