REFEREE REPORT

Ondřej Pudich, Problém izomorfismu pro quandly odvozené z grup

The bachelor thesis presents some known and some original results concerning the isomorphism problem for principal quandles. For a group G and $f \in \mathrm{Aut}(G)$, the principal quandle $Q(G,f) = (G,*)$ is defined by $x * y = xf(x)^{-1}f(y)$. It has been shown in the literature that

(1) $Q(G,f)$ is isomorphic to $Q(G,g)$ if f and g are conjugate in $\mathrm{Aut}(G)$,

that the converse holds for simple groups (this is an unpublished note) and also for symmetric groups S_n with $n \leq 30$ (with the possible exception of $n = 15$). It is known that the converse of (1) does not hold in general.

After recalling some basic properties of quandles and principal quandles, the author finds an original characterization of the isomorphism problem based on the subgroup $M = \langle xf(x)^{-1} : x \in G\rangle$, cf. Lemma 10 and Věta 13. Upon recalling the structure of conjugacy classes of the automorphism groups of dihedral groups, it is shown that the converse of (1) holds for $G = D_{2p}$ with p prime (a new result) and an explicit check of isomorphism types of principal quandles over D_8 shows that the converse of (1) fails for $G = D_8$.

The thesis is correct as far as I can tell and it is well written. There are a few places where an experienced mathematician might forgo a detailed proof because the claim follows from general principles, but it is fine to include the details in a bachelor thesis.

Here are a few comments that the author might take into consideration in a possible future version:

page 5: It should be noted that M is generated as a subgroup of G, not as a subquandle of $Q(G,f)$.

Lemma 7: This is true but a bit silly to state, given the fact that $f = L_e \in \mathrm{LMlt}(Q(G,f))$.

Lemma 11: This follows from the general fact that all coset quandles Q are homogeneous (that is, their automorphism group acts transitively on Q) and that a principal quandle is a coset quandle.

Věta 20: A more suitable name for this results could be "Abstact description of the automorphism groups of dihedral groups".

Tvrzení 25: Indeed, every generator of M is a rotation.

Perhaps a bit more could be said about D_{16}, D_{32}, etc, to get some feel for the general behavior in the case D_{2^n}.

Overall, I recommend to accept the work as a bachelor thesis. It contains original results that might be publishable in a research journal. In addition, the presentation is very good and the organization of the thesis is excellent.

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