We consider the additive semigroup  $\mathcal{O}_{K}^{+}(+)$  of totally positive integers in a real quadratic field  $K = \mathbb{Q}(\sqrt{D})$ . We define on  $\mathcal{O}_{K}^{+}(+)$  the partition function  $p_{K}(\alpha)$  and develop an algorithm for computing  $p_{K}(\alpha)$  for different square-free D and different  $\alpha \in \mathcal{O}_{K}^{+}$ . We then investigate the behaviour of  $p_{K}(\alpha)$ , characterizing the square-free numbers Dfor which  $p_{K}(\alpha)$  attains the numbers 1 through 5. Finally, we prove a sufficient condition for the number 6 to be attainable by  $p_{K}(\alpha)$ .