Report on bachelor thesis of David Stern: Partitions on totally positive elements in real quadratic fields

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This bachelor thesis consists in a study of the partition function p_K for real quadratic fields $K = \mathbb{Q}(\sqrt{D})$, analogous to the usual partition function $p(n)$ for the integer numbers. Namely, p_K is defined as follows: for a totally positive algebraic integer $\alpha \in K$, $p_K(\alpha)$ is the number of ways to write α as a sum of totally positive algebraic integers in K. In Chapter 2, the author introduces an effective algorithm to compute the values of p_K at any algebraic integer $\alpha = a + b\omega_D$, $a, b \in \mathbb{Z}$, and uses it to compute $p_K(\alpha)$ for small values of a, b and $D = 2, 5, 13$. This inspires some general results concerning the function p_K, which are proved in Chapter 3. Concretely, for each positive integer n there is some critical value D_n such that $p_K(n) = p(n)$ for every $D > D_n$ (Theorem 13). Moreover, the question whether $2, 3, 4, 5, 6$ lie in the image of p_K is considered, for which necessary and/or sufficient conditions are obtained.

First and foremost, I find impressive the amount and the extent of the original results in this work, which is unusual at this level. These provide a clear picture of the considered questions, and it might stimulate the study of analogous ones for higher degree number fields. In most of the proofs, the strategies and arguments that have been followed are often quite clever. However, I consider that some details and explanations should have been added at some points, where the proofs become rather technical and harder to follow. In any case, the amount of details included throughout the thesis is fair, including the preliminaries. In this sense, it seems to me that the organization of the thesis is quite logical. The amount of typos is also standard, and in general they do not complicate the understanding of the main ideas. Among the most noticeable ones, in the statement of Theorem 13, the inequality $D \geq D_n$ should be strict.

In the overall, I consider that this is an excellent bachelor thesis, which I recommend for the grade 1.