

The thesis focuses on the properties of general projective linear group $PGL_2(\mathbb{F})$ and its action on the projective line $\mathbb{P}^1(\mathbb{F})$, both for a finite and an infinite field \mathbb{F} . Only the basic knowledge from the Bachelor studies is used to prove these properties. Sharp 3-transitivity of the said group is discussed. Then, we deal with the subgroups consisting of identity and all elements whose sets of fixed points coincide. Furthermore, we show under which conditions all these subgroups have the property that all their finite subgroups are cyclic. We deduce that for a finite field \mathbb{F} , it holds that all of these groups are cyclic if and only if \mathbb{F} is equal to \mathbb{Z}_p for a prime number p . The thesis then focuses on the action of $PGL_2(\mathbb{F})$ by conjugation on the set of these subgroups. Finally, it is shown that projective special linear group $PSL_2(\mathbb{F})$ is simple.