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Bachelor Thesis Report, Dávid Kubek

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To whom it may concern,

it is my pleasure to write this report on the bachelor thesis titled “Experimental Analysis of the Simplex Method for the Multicommodity Flow Problem” at the request of Dr. Martin Koutecky, the supervisor for the thesis.

Multicommodity Min-Cost Flow (MCF) Problems are a class of network flow problems where multiple types of goods (commodities) share the same network and make use of the same resources. They capture many applications on traffic, production, and communication networks. Unlike network flow problems for a single commodity, to the best of my knowledge, there are no combinatorial algorithms for these problems. This is largely due to the loss of total unimodularity of the constraint matrix, which in turn means that vertices of the underlying polyhedron can be fractional even for integer input.

The constraint matrix of multicommodity flow matrices is known to have highly-fractional circuits, exponential in the input. This implies that there should exist instances of multicommodity flow with highly-fractional vertices, which would pose a significant challenge to the design of a combinatorial algorithm. The extent of this fractionality is a difficult analytical question, and so it is natural to approach it computationally first for some observations that can guide a study. This is the topic of the thesis. It is a well-justified and natural contribution to our understanding of MCF problems.

The first main contribution is the design of software capable of solving a Simplex run using exact arithmetic, i.e., representing rational numbers as fractions and not in floating point arithmetic, while also being able to trace the solutions in each iteration of the algorithm. Section 2.1 of the thesis describes the challenges in this development and the different avenues that were explored. Ultimately, the decision fell on modifying the GLPK library, which includes an exact simplex algorithm, to include a tracing functionality. In Section 2.2, details of this implementation are provided.

In Section 3, the software is used on a range of standard libraries. The observed fractionality throughout a run of the Simplex method is displayed in many plots and tables. Fractionality in the Dantzig rule of largest coefficient is observed to be higher than for the other methods for the MMCF dataset. This difference is statistically validated. For the netlib dataset, the pivot rules behave similarly. While this difference is not further explored in the thesis, I believe the reason is that the MMCF dataset contains many instances with randomly generated information.

Fractionality is also observed to increase with a larger number of iterations and dimension; these effects are as expected. With a small computation, it is validated that fractionality grew exponentially in the dimension in the experiments. The main body of the thesis concludes with some natural directions for future work. In Appendix A, a well-written summary of some simplex and revised simplex details is given. Appendix B provides some instructions on using the new software package.

Overall, I was very impressed by the software work and there is merit in its public availability in its own right. I was equally impressed by the computational experiments and analysis, which have been successful in validating the original hypothesis. The thesis is well-written, and quite enjoyable to read. It is a very good computational and mathematical contribution that I enthusiastically recommend to be accepted.

I would like to share a discussion point for the defense. I believe there is merit in specifically investigating the fractionality for vertices that can appear throughout a Simplex run for different pivot rules, as performed in this thesis. One of the reasons is the existence of a strong starting point in form of the GLPK library. Are you also interested in the overall fractionality of all vertices of the polyhedron? For that, an exact-arithmetic variant of a vertex enumeration algorithm such as Avis-Fukuda or Double Description would be highly desirable. Are you aware of a possible starting point for a software implementation based on these algorithms?

Best regards,

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