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Review of the Bachelor Thesis of Dávid Kubek

The thesis at hand experimentally examines the fractionality of linear programming problems. The *fractionality* of a rational number is the denominator in its reduced (co-prime) rational representation, the fractionality of a vector is the maximum of the fractionalities of its entries, and the fractionality of a polyhedron is the maximum of the fractionalities of its vertices. The notion of fractionality implicitly comes up e.g. in a typical exposition of interior point methods for linear programming, where one shows that if a solution has sufficiently small optimality gap, then it must be very close to a vertex, and one can "jump" to this final solution. However, this generic bound is exponential in the dimension, and it is of interest to examine when this exponential behavior is attained, or, on the other hand, when better bounds could be proven. For example, if it could be shown that a problem has polynomial fractionality, then an FPTAS solving this problem could actually be used to find the optimal value in polynomial time. (Notably, it still might not be possible to find the optimal *solution*, but that is another story.) A particular problem of interest is min-cost multi-commodity flow (MMCF), for which Hladík in his bachelor thesis showed the existence of large "circuits" which strongly hint at the existence of high fractionality instances.

The present work focuses on an experimental (as opposed to theoretical) evaluation of fractionality of two types of linear programming (LP) instances: MMCF instances, and general LP instances from the Netlib benchmark suite. Rather than attempting to enumerate all vertices, the chosen approach is to simply trace a run of the simplex algorithm on an LP instance and record the encountered fractionalities. While this may not really characterize the fractionality of a polytope, it still provides a lower bound, and it also characterizes vertices which are in some sense "relevant", because the simplex method passes through them.

Before getting to the result, I wish to highlight that the implementation of this experiment was a massive endeavor. While there are several mature LP solvers, for obvious reasons they all work in floating point arithmetic and are thus unsuitable for the type of analysis we wished to undertake. For this reason, the author of the thesis attempted multiple approaches before finally modifying the GLPK library to be able to get the data. This was a valuable lesson to us both that the simplex algorithm in its practical form is all kinds of things but *simple*.

As for the data which were obtained: On the one hand, the MMCF instances indeed exhibit a high fractionality trend, as hinted at by the work of Hladík, as do the Netlib instances. On the other hand, perhaps somewhat surprisingly, Figure 3.14 shows that the vast majority of *optimal* vertices had very small fractionality. This means that while the simplex method (particularly with Dantzig's pivoting rule) passes through vertices of high fractionality, it nevertheless ends up in a vertex of small fractionality. Note that for the claim about an FPTAS discovering the optimal value, one is content with at least one small-fractionality optimum; it does not matter if other vertices (even optima) have high fractionality. In this sense, the results suggest an interesting further research direction: exploring the optimal face and finding out the smallest possible fractionality.

On the formal side, the thesis is well written and the experimental results are presented with helpful plots.

Overall, this thesis constitutes a solid contribution to questions about fundamental topics such as linear programming, theory of polyhedra, or multi-commodity flow. I suggest the grade of “1” without hesitation.

Sincerely,
Martin Koutecký