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Review of the Bachelor Thesis of Aleš Prokop

The thesis gives a theoretical exploration of a deterministic dynamical process on weighted graphs, and provides new and valuable insights. The dynamical process is motivated by modeling opinion diffusion in societies, and can be viewed as the description of the mean behavior of a certain stochastic discrete process; this motivation is however not important for the thesis.

The thesis begins by an implementation of an experimental framework for exploration of the properties of the process. This framework allows one to analyze both random instances, as well as "adversarial" instances using the black box optimization tool Nevergrad. This was used in the thesis to look for and find counterexamples to various hypotheses.

The bulk of the thesis is theoretical, which speaks to the insights gained using experiments. The main goal of the thesis was to prove that the diffusion process converges from any initial state. This goal was not attained, but several partial results were.

First, the author shows that the process always converges on cliques and stars. To be able to obtain more results, the author turns to a simplification of the process. In the so-called "threshold diffusion process" we first set an arbitrarily small threshold $\delta > 0$, and then the process "zeroes out" any vertex which would drop below this threshold. With this property at hand, the author is able to prove much more: that all trees converge.

Morever, the author also explores the directed analogue of the process, and he shows a sort of dichotomy: if a graph is a DAG, then the threshold diffusion process always converges, while if the graph contains a cycle, then there always exists an initial state from which the process does not converge but instead periodically repeats the same set of states.

Another interesting contribution is disproving the contractivity of the diffusin process. To show that it converges, one might hope that the distance between consecutive states always decreases by at least some multiplicative constant. Not only is this not true, but the author even shows a construction of an instance which is "expanding" (there are three consecutive states and the distance between them grows) arbitrarily late in the process. Similarly, one could hope that the process would be "vertex monotone", that is, each vertex is either non-increasing or non-decreasing throughout the process. This, too, is false. Similarly, one could hope that the process is "edge monotone", but this is also false.

Besides directed graphs, the author also considers other generalizations. First, each vertex may be assigned a "stubbornness". The author shows that there are inputs which do not converge in this regime. Second, each edge may be assigned an "influence" weight. The author shows that the "leaf lemma", a key lemma from the previous parts, does not hold in this setting.

Lastly, the thesis provides a certain polyhedral characterization of the set of fixed points of the diffusion process.

On the formal side, the thesis is well written and structured, and contains helpful figures.

Overall, even though these results are partial, they already begin painting a picture of this diffusion process, and show specific questions to which attention should be given in the future. It is quite a valuable and original contribution to the state of the art, and I suggest the grade of "1" without hesitation.

Sincerely yours,
Martin Koutecký