Abstract: In order to generate observable electromagnetic signatures, astrophysical black holes have to interact with matter. Arround the black hole, matter typically forms into a symmetric disc through which it gradually inspirals towards the black hole. If the disc is dense enough, it can significantly perturb the motion of free test particles. The perturbation makes the originally completely integrable dynamical system prone to chaos. In this thesis, we focus on finding the homoclinic orbits which are the 'seeds of chaos' in the geodesic motion around black holes. Specifically, we find the homoclinic orbits in the Schwarzschild and in the extreme Reissner-Nordström space-times, and analyse how they behave under perturbation by a Kuzmin-Toomre disc and by a Majumdar-Papapetrou ring, respectively.