Reviewer report on a diploma thesis

**Title:** Random Dynamical Systems and Their Applications
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The thesis concerns (possibly non-autonomous and semilinear) differential equations

$$du(t) = [A(t)u(t) + F(u(t))]dt + Bu(t) \circ d\omega(t) \tag{1}$$

in Hilbert space that are driven by paths $\omega$ of Hölder continuity greater than $1/2$. Here, the operators $A(t), t \in [0, T]$, generate a strongly continuous evolution system, the non-linearity $F$ is assumed to be globally Lipschitz, the operator $B$ generates a strongly continuous group of operators, and the integral is understood as the generalized Riemann-Stieltjes integral in the sense of Zähle (1998, 2001).

The thesis is divided into four chapters. In the first chapter, some preliminaries on fractional integration and semigroup theory are given. The second chapter contains results on the non-autonomous equation (1); in particular, the linear problem ($F = 0$) is first shown to admit a weak solution (Theorem 17), then the non-linear problem ($F \neq 0$) is shown to admit a mild solution (Theorem 19) and a Fubini-type theorem (Lemma 21) is then given to prove that the mild solution is also the weak solution (Theorem 22). The third chapter is devoted to random dynamical systems (RDS) generated by the solutions to the autonomous problem and it is shown that the RDS admits a unique attractor and the fourth chapter features some examples.

The results contained in the thesis extend the results of Garrido et al (2016) to (possibly non-autonomous) systems driven by Hölder continuous functions and as such, they allow a pathwise treatment of a wide class of stochastic evolution equations driven by (regular) noise. The extension is definitely interesting and novel. The proof techniques, that are quite demanding from a technical point of view, had to be substantially modified. Artem Iuzbashev carried out these modifications successfully and, additionally, he addressed some omissions in the original publication along the way. As far as presentation is concerned, the thesis is well-structured, quite readable, and contains only a few misprints.

Overall, while it could be interesting to discuss the interplay of the various assumptions contained in (C.1)–(C.4) during the defense, I believe that the present thesis fulfills the requirements for a diploma thesis and I recommend that it is recognized as such.

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