The Advisor's Assessment of Diploma Thesis

A. Iuzbashev: Random Dynamical Systems and Their Applications

The theory of random dynamical systems (RDS) and random attractors has been developed over the past few decades in a general setting and applied mostly to linear and bilinear SDE's and SPDE's with noise terms white in time (i.e. to equations where the driving noise is a Wiener process). In recent years, similar programme has been adopted for various „noises with memory", typically fractional Brownian motions (fBm).

While for linear equations it is easy to mimick the well known methods for a very large class of noise terms, the bilinear equation is more complex and volnurable to choice of the type of stochastic integral. For fBm the asymptotic properties were studied in several papers but it is known that such equations do not define an RDS.

Some results in this direction have been obtained, however, by using pathwise integrals if the driving process is a fBm.

The student's task was to summarize and, possibly, extend these results for larger classes of noise and to figure out which conditions are really essential.

I my opinion, Artem Iuzbashev has successfully accomplished this task. The Thesis contains a number of results which are (at least technically) extensions of earlier papers and in some cases also fill small gaps in the proofs thereof (for example, the stochastic Fubini theorem in Section 2). Section 2 also contains existence and uniqueness theorems which novelty concerns not only the type of noise but also time-dependent generators. This requires an extension of semigroup theory to two parameter evolution operators, which is fairly hard. In particular, I would mention Theorem 17 on existence of weak solution to a bilinear problém, where construction of the evolution operator was intrinsically used in the proof – this part is completely original in the Thesis. In Section 3, existence and uniquness of the random attractor is proved for semilinear equation. The main novelty is implementation of the condition (E) which replaces the law of iterated logarithm from the standard theory.

Some of the proofs in the Thesis are technically quite hard and require a good knowledge of fractional calculus. On the other hand, some nontrivial results form the semigroup and evolution operators theory have been used as well. As mentioned above, most of results from Section 2 and 3 are in fact Iuzbashev's own results though most (but not all) of them follow some standard procedures.

The first version of the Thesis contained some mistakes or incomplete proofs, which

have been corrected in the present version. This in particular concerns the proof of stochastic Fubini theorem and a mistake in an estimate in the proof of Theorem 17. Note that in the latter case, an analogous mistake has appeared already in the original paper (Garrido at all), the results of which have been extended here.

I recommend to accept the submitted Thesis for defence.

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