Fourier series are an important tool of mathematical analysis with many applications. This thesis focuses on their use in several specific mathematical problems. The first application is the proof of the isoperimetric inequality, according to which the circle has the greatest area among closed curves of a given length. Next topic is the sequence of fractional parts of numbers of the form $n\gamma$, where n runs through natural numbers and γ is an irrational number. The so-called equidistribution theorem holds for this sequence describing how this sequence fills the interval (0, 1). Fourier series are then also used to obtain a formula for calculating the sum of even powers of the reciprocals of natural numbers. The last chapter is devoted to the Gauss circle problem, which investigates the estimation of the number of lattice points inside a circle of a given radius.