A Report on
"Minion Cores of Clones"
by Maryia Kapytka.

The author studies multi-sorted clones on a 2-element domain, that is, clones whose elements are k-tuples of Boolean operations of the same arity and composition is applied component-wise. Such clones play an important role in Clone Theory, where some clones on a larger domain can be viewed as multi-sorted Boolean clones, and in the Computational Complexity of the Constraint Satisfaction Problem, where multi-sorted clones appear naturally if we have several sorts of variables on different domains.

Already on a 3-element domain the set of all clones seems to be too complicated to be classified, and we may consider clones up to a minion homomorphism, that is a mapping that preserves arities and compositions with projections. This notion comes from the notion of a minion, which is a set of operations containing projections and is closed under composition with projections (but not any composition). The main goal of the thesis is to classify all k-sorted Boolean clones definable by binary relations up to a minion homomorphism and find a minion core for each of them, where a minion core is a minion such that every minion homomorphism to itself is an automorphism. Notice that a minion core is not necessary a clone.

The author proves 4 main results. Firstly, she characterizes all k-sorted Boolean clones definable by binary k-sorted relations. Secondly, the author defines 7 families of multi-sorted minions and proves that every such a k-sorted Boolean clone is equivalent up to a minion homomorphism to one of them. Then, it is proved that each of the defined minion is a minion core. Finally, for every pair of the minion cores (and therefore for every pair of multi-sorted Boolean clones) it is showed whether there exists a minion homomorphism from one to another. As a result, the author presents a diagram illustrating the preodering coming from the existence of such a homomorphism.

The thesis is very well written. For instance, the thesis is 73 pages long and I found only about 10 misprints. I was impressed how the paper is structured, how precise the definitions are, and how complete the proofs are. Sometimes it is a problem to explain facts that are almost obvious and many authors just ignore such proofs. On the contrary, the author of this thesis always provides a formal explanation, which makes the thesis a bit longer but shows their high mathematical level.

I have only several remarks that could make the thesis even better. Many proofs in Section 2.4 are too long I strongly believe that they can be simplified. Additionally, in Section 2.4 the author often refers to Remark 2.28 in the argument. It is a bit confusing because usually the claim follows immediately from the definition and extracting the argument from such a long remark is not an easy task. Moreover, sometimes I could not find a way how to use this remark at all. Finally, some references are not to the original results or missing.

Nevertheless, the results obtained in the thesis are important and highly nontrivial. They give us a better understanding of Boolean clones and Boolean Minions, and should be published in a journal. To prepare a proper publication the author just needs to shorten known results and some proofs, and work on the bibliography. Thus, I recommend the highest grade for the thesis.