An instance of the Directed Steiner Network (DSN) problem consists of a directed graph G with edge costs, and k so called “terminal“ vertex pairs. The task is to find a minimum-cost subgraph of G in which all terminal pairs are connected by a path. This generalizes several NP-hard problems.

The terminal pairs induce the so called “pattern graph“, a digraph on a subset of vertices of G. We investigate the DSN problem restricted to certain classes of pattern graphs. It has been shown that the optimum may be found for certain classes in FPT time parameterized by k, and that this is impossible for all other classes of graphs, assuming FPT $\neq$ W[1].

This leads to the question whether the hard classes may be approximated in FPT time. We prove that no FPT approximation scheme may exist for any of the W[1]-hard classes, based on a stronger hypothesis, the Gap-ETH. We then give FPT algorithms with constant approximation guarantees for special classes of pattern graphs.