Posudek diplomové práce

Matematicko-fyzikální fakulta Univerzity Karlovy

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Thesis title	Parameterized Approximations of Directed Steiner Networks
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Program	Computer Science **Specialization** Theoretical Computer Science

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Review text:

The thesis considers the Directed Steiner Network (DSN) problem, for which an edge-weighted directed graph is given together with a demand graph between terminal vertices, and the aim is to compute the cheapest subgraph that satisfies all the demands, i.e., there is a directed path from s to t if (s, t) is an edge in the demand graph. In its most general form this problem is known to be hard to approximate in FPT time, where the parameter is the number of terminals. However some special cases, where the demand graph has a certain structure, are known to be tractable. Among them are Directed Steiner Tree (DST), for which an optimum solution can be computed in FPT time, and Strongly Connected Steiner Subgraph (SCSS), for which a 2-approximation can be computed in FPT time. The thesis sets out to explore the complexity landscape of parameterized approximations for the DSN problem with respect to the structure of the demand graphs.

A similar approach was taken in previous work for exact computations and a complete dichotomy was found that identifies those classes of demand graphs, which lead to DSN subproblems that are FPT, and those classes that lead to DSN subproblems that are W[1]-hard. The thesis studies the W[1]-hard cases in order to prove both upper and lower bounds on the obtainable approximation ratios in FPT time for different classes of demand graphs. The results of the thesis do not characterize the whole complexity landscape of parameterized approximations, but presents some interesting cases:

- it is shown that none of the W[1]-hard cases admits a parameterized approximation scheme, by proving that no parameterized $(4/3 - \varepsilon)$-approximation is possible, under Gap-ETH,
- if the condensation of the demand graph forms an arbitrarily long path, then a 2-approximation can be computed in FPT time,
- the previous result can be generalized to demand graphs for which the condensation can be covered by a constant number of arbitrarily long paths,

- for demand graphs that form an arbitrarily long caterpillar after removing redundant demands, a 3-approximation can be computed in FPT time,
- the previous result can be generalized to a $3p$-approximation if the demand graph is the union of p caterpillars, and it follows from a known lower bound for DSN that no $o(p)$-approximation is possible,
- for demand graphs that can be covered by p caterpillars and q paths, a $(2+p)$-approximation can be computed in FPT time if p and q are constants.

In addition to this, some interesting open problems are discussed, namely demand graphs that form d-diamonds and grids. For the former it is easy to obtain a d-approximation, and, based on the known hardness results for DSN, it is conjectured that this is essentially best possible in FPT time, even if d is constant.

While the results presented in this thesis are quite strong overall, there are a couple of remarks to be made:

- on page 16 it is claimed that known lower bounds for Travelling Salesman carry over to Path-DSN, where the demand graph is a path. However this is not correct. Instead, known lower bounds for SCSS should have been used to establish the hardness for Path-DSN.
- known results from the literature are not always cited according to the standards of the research area. Especially in the introduction, some known results are stated without giving credit to the respective authors.
- the exposition of known and new results and how they relate to one another could be more concise. Especially some of the polynomial-time approximation lower bounds are not mentioned in the introduction, where the context of the obtained results should be set. Furthermore, the discussion regarding conjectured lower bounds for d-diamonds and how they relate to the overall picture of the complexity landscape of parameterized approximations is missing from the introduction.

However, none of the above points belies the fact that the obtained results are of high quality. The topic of this thesis is quite subtle in regards to all the dependencies between the many parameters. Regardless of this, the student has shown a very good grasp of the complexity of the topic, both in the thesis but also during the many meetings we had while the project was ongoing. He has also demonstrated the ability to conduct research at a very high level and was able to contribute original ideas to the discussions.

Overall I think this is an excellent thesis and I recommend a grade of 1.

I recommend the thesis for defense.

I suggest to consider the thesis for the annual award.

The results of this thesis are quite strong for a master thesis and can easily be published in an internationally competitive conference of the field, such as WG or SWAT.

date August 9, 2023

Signature: