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Reviewer's report on the Master's Thesis "Parameterized Approximations on Directed Steiner Problems" submitted by Martin Koreček

The main focus of the submitted thesis is the Directed Steiner Network problem, a well-known theoretical formulation of designing minimum cost networks that satisfy certain communication requirements between the terminal nodes. The thesis studies this problem from the viewpoint of parameterized approximation:

- The considered algorithms have running time parameterized by the number of terminals, that is, the running time is a polynomial function in the size of the network, times an arbitrary (typically exponential) function depending only on the number of terminals.
- The algorithms do not have to output an optimum solution, but only an approximate solution that is at most α times worse than the optimum.

Polynomial-time approximation algorithms and parameterized exact algorithms were intensively studied for Directed Steiner Network and its variants, but exploring this combination is a novel avenue of research. Besides the individual technical results, the main contribution of the thesis is putting into focus the interesting set of results that can be obtained in the intersection of these two areas.

Feldmann and Marx proved a dichotomy result characterizing those classes of patterns that admit fixed-parameter tractable (FPT) algorithms and showed that the problem is W[1]-hard for every other class of patterns. The first contribution of the thesis (Section 2) is extending this to the approximability of the problem: in the W[1]-hard cases, there is no better than 4/3-approximation in FPT time (under a suitable complexity hypothesis). In particular, this rules out the existence of a parameterized approximation scheme producing a $(1+\varepsilon)$-approximation in FPT time for any of the W[1]-hard cases. Following earlier work, the thesis first observes that the proof boils down to proving hardness for two types of obstructions: cycles and diamonds. This observation follows by checking that certain operations defined in earlier work are approximation preserving. Additionally, the thesis makes a very nice novel observation by introducing the operation of path-preserving subgraphs, which can be used to simplify arguments. An approximation lower bound for the cycle case was given by [7]. The thesis obtains the lower bound for diamond patterns by competently combining the approximation lower bound of [7] for cycles with the W[1]-hardness proof of [6] for diamonds.

Given that we cannot expect parameterized approximation schemes, the next natural question is to understand which class of patterns admit constant-factor approximation algorithms. The rest of the thesis is devoted to proving results partially answering this question. Section 3 gives a 2-approximation for the case when the pattern is a directed path. The algorithm is based on a dynamic programming approach sequentially considering the strongly connected components of the solution. While the idea is simple, the details require careful handling. The algorithm is generalized for the case when all the edges of the pattern graph can be covered by a fixed number of paths. As expected, the algorithm becomes significantly more Involved and the exponent of the running time depends on the number of paths.

Section 3 considers further more complicated patterns. Constant-factor approximations are given for the cases when the pattern is a single caterpillar, union of a constant number of caterpillars, or the union of a constant number of caterpillars and a constant number of paths. These results are exciting new steps towards a complete understanding of the constant factor approximability of this general class of problems.

The well-written thesis presents a collection of novel technical results that give new insights about the approximability of certain network design problems. The results are not trivial, deal with a well-studied family of problems, and were not known before, hence they are potentially publishable in an appropriate theoretical computer science venue. This is a high-quality work for an M.Sc. thesis; I clearly recommend the thesis for defense and giving it an excellent grade.

Given that the results of the thesis are publishable in scientific venues, it can be also considered for an award. However, the presented set of results has certain limitations and shortcomings, making them more appropriate e.g. for smaller or more specialized conferences. While the thesis provides many answers, these answers typically do not completely settle the problems: the approximation ratios are not known to be tight in any way and it is still unclear what the boundary of constant-factor approximation is. There are certain tradeoffs between approximation ratio and running time that are not discussed, for example, the 2-approximation for the path case trivially gives a 2p-approximation for the case when the pattern is the union of p paths, which is incomparable with the n^(O(p)) time 2-approximation algorithm given in the thesis. This makes the present set of algorithmic results appear a bit arbitrary: it is unclear what other approximation ratios/running times/pattern classes can be achieved and hence difficult to judge the significance of these results.

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