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Institute of Economic Studies

Bachelor's Thesis

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Introducing Stochasticity into Energy System Model Times-CZ - Reflection of War-Related Extreme Environment

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Study Programme: Economics and Finance

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Declaration

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In Prague on 1 August 2023 Šimon Otruba

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Abstract

This thesis introduces stochastic elements into the TIMES-CZ energy system model focusing on the impact of extreme events such as pandemic or recent war in Ukraine. The objective is to improve the model's precision in the face of these market uncertainties. Natural gas prices and European Union Allowance (EUA) prices, after a selection process, are represented as random variables allowing for probabilistic forecasting. These variables are derived from an analysis that combines model-based forecasts, which also include external predictions. The results of this comprehensive analysis are then integrated into the TIMES-CZ model. The correctness of these results is validated using sensitivity analysis, which evaluates the impact of results with uncertain parameters on the model's output. The findings highlight the importance of including uncertainty in energy systems modelling and could have implications for energy planning and decision-making in uncertain contexts.

Keywords

TIMES-CZ Model, Stochasticity, Energy System Modelling, Uncertainty Analysis, Sensitivity Analysis

JEL Classification

C12, C33, G21, L25, M31

Title

Introducing stochasticity into the energy system model Times-CZ - a reflection of a war-related extreme environment

Abstrakt

Tato práce zavádí stochastické prvky do energetického modelu TIMES-CZ se zaměřením na dopad extrémních událostí, jako je pandemie nebo válka na Ukrajině. Cílem je zlepšit přesnost modelu v kontextu těchto tržních nejistot. Ceny zemního plynu a ceny povolenek Evropské unie (EUA) jsou vybrány a reprezentovány jako náhodné proměnné, což umožňuje pravděpodobnostní předpovědi. Tyto proměnné jsou odvozeny z analýzy, která kombinuje předpovědi založené na modelu, do kterých jsou také začleněny externí předpovědi. Výsledky této komplexní analýzy jsou poté integrovány do modelu TIMES-CZ. Správnost těchto výsledků je ověřena pomocí citlivostní analýzy, která hodnotí dopad výsledků s nejistými parametry na výstup modelu. Zjištění zdůrazňují význam zahrnutí nejistoty do modelování energetických systémů a mohou mít dopad na plánování energií a rozhodování v nejistých kontextech.

Klíčová slova

Model TIMES-CZ, Stochastičnost, Modelování energetických systémů, Analýza nejistot, Citlivostní analýza

JEL Klasifikace

C12, C33, G21, L25, M31

Název práce

Zavedení stochastičnosti do energetického modelu Times-CZ – jako odraz extrémního válečného prostředí

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1. Introduction

The Czech Republic's National Energy and Climate Plan provides a comprehensive roadmap for the country's future energy needs, with a focus on reducing greenhouse gas emissions, increasing the share of renewable energy sources, and improving energy efficiency (Czech Ministry of Industry and Trade, 2019). Significant investments are required to achieve these goals, and accurate energy modelling is critical for making informed policy and investment decisions.

The ongoing conflict between Russia and Ukraine, as well as the sanctions imposed on Russia, have had a significant impact on the European Union's energy market. Because Russia is a significant supplier of coal, natural gas, and oil to the EU, the sanctions against Russia have disrupted the flow of energy. As a result of the conflict, less natural gas is being transported through Ukraine to Europe, resulting in higher gas prices (KPMG, 2022). In response, the EU is attempting to diversify its energy supply by looking for alternative energy sources (Pfenninger et al., 2014). In addition to programmes to increase energy cooperation and reduce reliance on fossil fuels, the EU has put in place policies to encourage the use of renewable energy and energy efficiency.

Given the inherent uncertainties in the energy market, incorporating stochastic processes into the TIMES-CZ model may improve its precision. Stochastic processes, according to Zakaria et al. (2020), can effectively represent the energy sector by modelling key parameters as random variables with established probability distributions. As a result, the model can produce probabilistic forecasts that consider the entire range of possible values for each parameter.

It is critical to emphasise that the primary goal of this research is not to develop the TIMES-CZ model, but rather to improve it through the incorporation of stochastic elements. Chapter 4.6 provides a thorough examination of the complex task at hand, with a focus on the incorporation of stochasticity into the TIMES-CZ (v02) model. This chapter provides a detailed description of the methodologies and approaches used to modify the model, as well as the results obtained. Furthermore, it specifies the precise conditions under which the newly developed stochastic scenarios outperform traditional deterministic ones in terms of reliability. This chapter investigates the potential difficulties and constraints that may arise when using stochastic scenarios. The goal of this research is to provide a comprehensive understanding of the various sources of error, as well as an assessment of the resilience of the proposed solutions in the face of these challenges.

Stochastic programming, an optimisation method that analyses uncertainties by including various possibilities with a given probability, will be used to effectively alter the TIMES-CZ model (Zakaria et al., 2020). The first step is to identify ambiguous variables such as commodity price changes, demand expansion, or regulatory changes. Then, a comprehensive analysis is conducted, which includes model-based forecasts combined with external predictions. Based on this analysis, we can define the state of the world for these variables and incorporate the results into the TIMES-CZ model (Morgan & Henrion, 1990).

To ensure the correctness of the changes, tests to validate the updated TIMES-CZ model are required. A sensitivity analysis will be conducted to determine how variations in the uncertain parameters would affect the model's output (Saltelli et al., 2008). The analysis will be performed on both the Reference Scenario and the Stochastic Scenario, with the expectation that the Stochastic Scenario will exhibit greater volatility. The results of these tests will confirm that the stochastic modifications to the TIMES-CZ model adequately account for uncertainties, thereby producing more reliable results for decision-making in energy planning and policy analysis (Loulou, & Lehtila, 2016).

This thesis holds potential importance due to its aim to enhance the TIMES-CZ model, which is used by the Environmental Center at Charles University for decision-making evaluation and strategic planning. The research introduces a degree of uncertainty into the model, which, if correctly implemented, could better account for real-world unpredictability. This could potentially lead to more accurate and robust strategic planning. However, it's important to note that the effectiveness of these enhancements will depend on their correct implementation and use. (Rečka et al., 2023).

The present thesis is organised in the following manner: In the first chapter, an introductory overview of the subject matter is presented. In this study, Chapter 2 provides a comprehensive review of the pertinent literature. Chapter 3 provides an analysis of current obstacles encountered within the energy sector, encompassing the ramifications of the COVID-19 pandemic, the ongoing conflict between Ukraine and Russia, as well as pertinent agreements at both the European Union and global levels. In Chapter 4, the methodology employed in this study is expounded upon. This includes a comprehensive description of the TIMES model generator, the TIMES-CZ Model (v02), the process of selecting variables for stochastic modelling, the acquisition and analysis of data, the establishment of predetermined time periods and corresponding values for probability computation, as well as the integration of stochasticity into the TIMES-CZ model. Chapter 5 provides an overview of the outcomes derived from the process of selecting, analysing, and incorporating stochasticity into the TIMES-CZ model. In conclusion, Chapter 6 serves as the final section of the thesis, encompassing a comprehensive summary of the research findings.

2. Literature Review

The task of modelling energy systems is complex and involves various aspects, including technological, economic, and environmental considerations. The need of transferring to sustainable energy systems and mitigating the effects of climate change has increased the importance of this task. The Integrated MARKAL-EFOM System (TIMES model) has gained significant recognition within the field of energy system planning and analysis. The current MARKAL-EFOM model incorporates stochastic programming and trade-off analysis methodologies, as previously examined in academic literature (Goldstein et al., 2021; Loulou & Lehtila, 2016). This stochastic integration allows a comprehensive analysis of energy systems, considering the intricate interplay of various variables and uncertainties. Using this framework, however, presents its own set of challenges, particularly when it comes to implementing climate change mitigation measures and transitioning to sustainable energy systems. The challenges discussed address a wide range of topics, including the proper identification and selection of variables, the control and partial mitigation of uncertainties, and the efficient integration of various data types and sources (Ioannou et al., 2019).

The selection and identification of appropriate variables for stochastic modelling in the TIMES-CZ model is a critical and complex task with a significant impact on the scenario matrix and predictability of the model's predictions. Despite significant advances and various methodologies discussed in the literature (Morgan & Henrion, 1990; Saltelli et al., 2008; Loulou & Lehtila, 2016), there are still issues in the academic literature, specifically regarding the optimal methodology for variable selection. This study aims to fill this research gap by employing a careful variable selection methodology that considers potential variables' relevance to historical data, impact on model outcomes, and interdependence with other variables. Its objective is to improve the predictability of its forecasts by conducting a thorough analysis and employing statistical techniques to identify the variables with the greatest impact while excluding those with strong interconnections (Loulou & Labriet, 2008).

Kanudia and Loulou (1998) pioneered the application of stochastic programming to the TIMES model. To evaluate the effectiveness of Quebec's climate change mitigation strategies, the researchers used the stochastic MARKAL model in their study. The use of minimax regret algorithms to analyse trade-offs and uncertainties in greenhouse gas reduction solutions demonstrates how stochastic modelling can aid in the development of dependable and robust solutions. The preceding study was later expanded upon by Loulou and Kanudia (1999), who demonstrated the incorporation of stochastic components in the development of strategies to reduce greenhouse gas emissions. The preceding studies laid the foundation for the use of stochastic programming in the domain of energy system modelling. These studies have successfully demonstrated stochastic programming's effectiveness in addressing uncertainties and providing dependable solutions.

The primary aim of this research is to enhance the current body of knowledge by utilising the TIMES-CZ model to analyse the energy system of the Czech Republic. Stochastic programming will be utilised to effectively model and represent uncertainties associated with the implementation of diverse energy and climate policies. The approach under consideration is consistent with the methodologies described in the documentation of the TIMES model (Goldstein et al., 2021; Loulou et al., 2016). The application of this methodology will enable the reproduction of the outcomes resulting from various energy and climate strategies on the energy infrastructure of the Czech Republic. As a result, this will offer significant perspectives for policymakers and other relevant stakeholders engaged in the pursuit of making informed decisions. This study will additionally examine the difficulties that have been identified in prior research, with a particular focus on the process of selecting variables and the incorporation of data.

Similarly, Korkmaz, Schmid, and Fahl (2021) used a stochastic methodology to investigate the potential risks associated with Europe's transition to a sustainable energy system. The authors' research, like that of Kanudia and Loulou (1998), emphasises the effectiveness of stochastic modelling in evaluating the uncertainties and risks associated with the energy system transition. However, it is critical to recognise the challenges that arise when using stochastic modelling in complex and large-scale energy systems. The computational resources and data prerequisites pose significant challenges, which are consistent with the concerns we hope to investigate in our research.

The study conducted by Rečka, Máca, and Ščasný (2023) involved a thorough examination of the capacities of the TIMES-CZ model in assessing the Green Deal and Carbon Neutrality initiatives in Czechia. The objective of their research was to conduct a thorough assessment of the energy system by examining different policy scenarios. The authors' methodology for incorporating diverse policy scenarios into the TIMES-CZ model serves as a valuable point of reference for our own research.

In her study, Barberán (2020) examined the integration of exogenous factors into the prediction of time series data, specifically focusing on its applicability to the identification of variables in energy system modelling. The research emphasised the significance of considering the impact of external variables on the predictability of model results. The chosen approach is consistent with our research objective of utilising a rigorous variable selection methodology that considers the significance of potential variables in relation to past data, their influence on model results, and their interconnections with other variables.

In their study, Babonneau et al. (2012) investigated the effects of various sources of uncertainties on the evaluation of energy and climate policies. This was achieved through the utilisation of stochastic programming in a comprehensive bottom-up model, as well as employing Monte-Carlo simulation in a comprehensive top-down model. This is an important problem in our own research efforts.

Ioannou et al. (2019) proposed a framework for power generation system planning that includes hybrid uncertainty modelling via a multi-stage stochastic optimisation approach. The authors' research, such as that of Goldstein et al. (2021), makes a valuable contribution to the academic discipline by providing a thorough examination of multistage stochastic optimisation, a concept with significant relevance in the realm of energy system modelling. The authors' methodology for dealing with uncertainties in energy system modelling, particularly in the area of long-term planning under uncertain conditions, is consistent with the goals and objectives of our research.

Data analysis is a critical component of this research. As evidenced by the works of Schwarz (1978), Engle (1982) and Razali & Wah, 2011). the literature has extensively examined the methodologies used for data analysis. These methodologies provide a comprehensive framework for data analysis and determining the most influential variables. The methodologies used in this study also consider the relationships between the variables and their impact on the model's outcomes. The data analysis methodologies used are specifically designed to improve the accuracy of the model's predictions and to provide a comprehensive understanding of the energy system. Pflug and Pichler (2016) made a significant contribution to this field by providing a comprehensive treatment of multistage stochastic optimisation, a concept that is highly relevant to energy system modelling. Their research covers the mathematical foundations of approximation theory as well as practical algorithms for the generation and manipulation of scenario trees. They focus on estimating and bounding the modelling error using novel distance concepts, time consistency, and the role of model ambiguity in the decision process. Their work's methodologies and concepts provide valuable insights for dealing with uncertainties in energy system modelling, particularly in the context of long-term planning under uncertainty.

In conclusion, the existing body of literature highlights the considerable potential of stochastic programming as a valuable tool for modelling energy systems. The use of this methodology provides significant benefits in effectively addressing the inherent uncertainties and risks associated with climate change and the ongoing transition to sustainable energy systems. Nevertheless, academic literature also emphasises the difficulties and intricacies linked to this undertaking, specifically in relation to the identification of relevant variables, the integration of data from various sources, and the computational resources required. The principal objective of our research is to tackle these challenges and make a significant contribution to the ongoing endeavours in the progression of energy system models that demonstrate improved resilience and reliability.

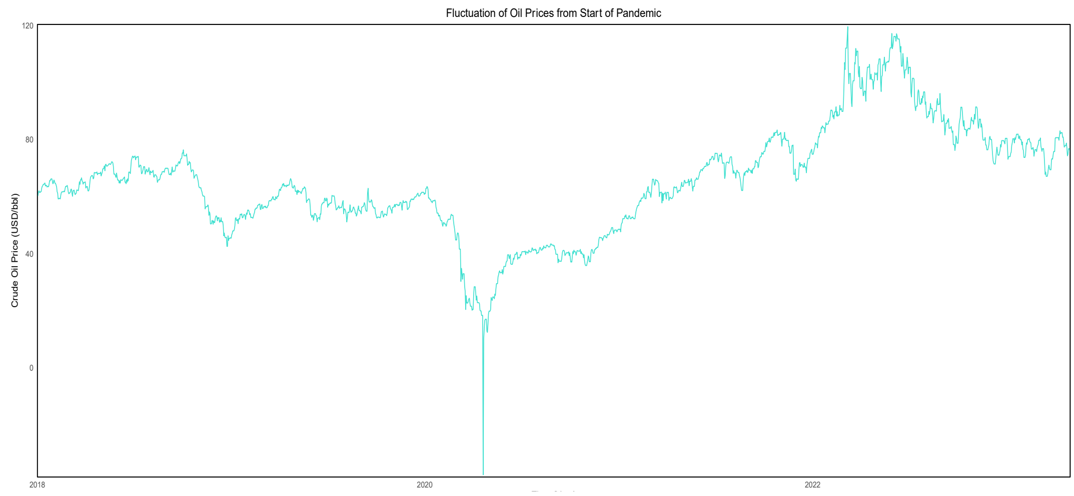
3. Contemporary Challenges in Energy Sector

This chapter explores into the significant concerns confronting the energy sector today. It examines in depth the enormous issues posed by the COVID-19 epidemic, the Russian-Ukraine war and EU and global accords, which have substantially altered global energy markets. The objective of this section is to explain the effects of these disruptions and the subsequent reactions, providing insights into the sector's resilience and adaptation in the face of such crises.

3.1. COVID- 19

The COVID-19 pandemic led to a significant decline in energy demand, resulting in a historic drop in oil prices. In April 2020, oil futures experienced their first-ever negative value due to the collapse in energy demand and excess oil supply in the market. The market saw a modest recovery when countries began to emerge from lockdowns and OPEC agreed to production cuts. However, major oil producers' attempts to control the fall in energy prices were unsuccessful. This improvement was short-lived, as a new wave of COVID-19 cases in August 2020 caused a further drop in energy demand (Koutroulis & Sarno, 2021).

Figure 1: Impact of COVID-19 Pandemic on Global Energy Demand and Oil Prices



Source: Trading Economics

The epidemic has had an unprecedented influence on the global energy industry. Travel restrictions and lockdowns reduced demand for oil and gas significantly, notably in the aviation and transportation sectors, which account for 60% of total oil demand. This resulted in a drop in energy prices. The problem was compounded further by an oversupply of oil caused by a conflict between major oil companies. The epidemic created numerous interruptions in the energy sector. Energy prices fell sharply in the early months of the pandemic due to a combination of lower demand and oversupply. In March 2020, energy costs plummeted by 50%, aggravating the situation (IEA, 2020).

In November 2020, the energy market experienced its largest decline since March as new, stricter lockdown measures were implemented in response to the pandemic. These measures jeopardized the fragile recovery in demand. However, positive signs for the energy industry have emerged from the major economies' economic recovery and the success of vaccination campaigns. Energy prices have risen to $60 per barrel, and demand is expected to increase (EIA, 2021).

The natural gas sector was also significantly affected by the Covid-19 pandemic. Although the impact of the COVID-19 pandemic on the demand for gas was relatively less pronounced compared to other fossil fuels in 2020, it is anticipated that gas will be significantly affected by the pandemic in the forthcoming decade or two. The observation is evident in the projected decrease of over 9% in the estimated global demand for gas by the year 2030. The challenge arises from the historical impact of gas on global greenhouse gas (GHG) emissions and the increasing commercial and policy-driven incentives to bypass or expedite the role of gas as a transitional energy source (Dmytrów, Landmesser, & Bieszk-Stolorz, 2021).

The ongoing pandemic has intensified existing difficulties faced by gas producers, such as limited access to financial resources and capital markets. Moreover, the decreased demand for domestic gas has resulted in the postponement or cancellation of specific U.S. LNG export shipments. In the foreseeable future, expediting the shift towards renewable energy sources would lead to a decrease in domestic gas consumption and potentially impede the progress of constructing new liquefied natural gas (LNG) export facilities. According to Dmytrów, Landmesser, and Bieszk-Stolorz (2021), the proposed action has the potential to worsen the issue of oversupply within the United States, leading to a decline in natural gas prices. Additionally, it could have negative implications for the credit outlook and long-term sustainability of independent natural gas producers in the country.

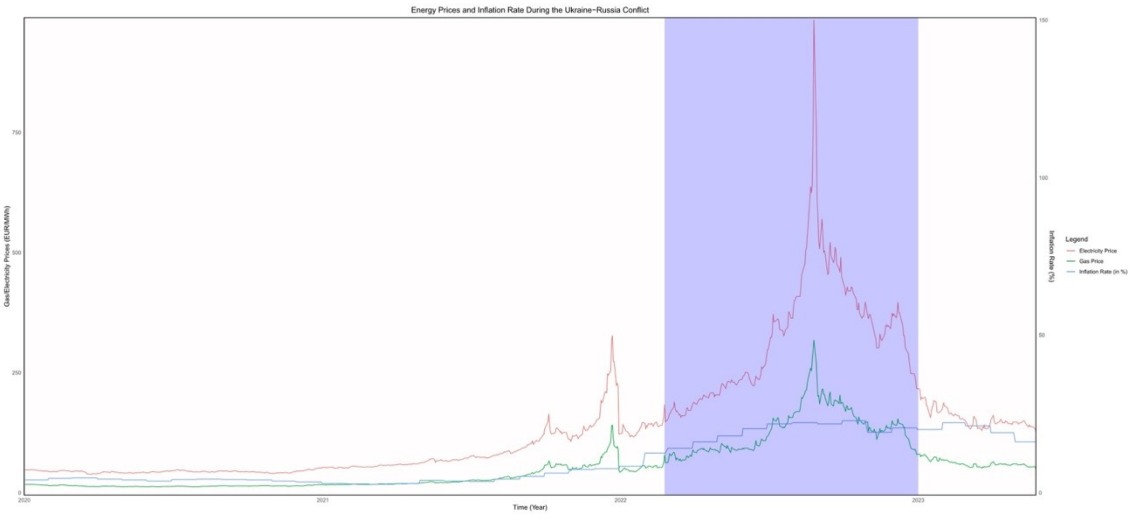
Moreover, the occurrence of the pandemic resulted in a decline in investments and maintenance within various sectors, thereby contributing to an increase in gas prices in the aftermath of the pandemic. The increase in question was partially driven by a decline in investments and maintenance within the sector during the period of the pandemic.

To completely comprehend COVID-19's impact on energy prices, it is critical to evaluate the pandemic's varied consequences on the energy market. This encompasses both the immediate decline in demand and prices as well as the later price increase, particularly for natural gas. This rise was fuelled in part by a drop in sector investments and maintenance during the pandemic. The findings suggest that, to avoid such crises in the future, governments should address pre-COVID-19 issues such as pricing wars, geopolitical conflicts, and structural transformation (Baker Institute, 2020). The COVID-19 epidemic has had a substantial impact on the energy market and is expected to continue for some time. Governments and policymakers must move quickly to address the underlying issues that contributed to the energy market's vulnerability during the pandemic and to control post-pandemic pricing volatility. They can help to make the energy market more resilient to future crises by doing so (IEA, 2020).

3.2. Ukraine – Russian War

The ongoing conflict between Russia and Ukraine has significantly influenced the global energy market, particularly in Europe. Russia has been a crucial energy supplier to Europe, providing 41% of the natural gas imports, 46% of the coal, and 27% of the oil the EU purchases (European Commission, 2021). This dependency on Russia for energy sources has raised concerns about energy security in the region, especially considering the geopolitical and economic implications of the conflict.

Figure 2: Energy Prices and Inflation Rate During Ukraine-Russia Conflict



Source: PXE and Eurostat

The conflict between Russia and Ukraine has resulted in a rise in energy prices, especially for natural gas. Before the invasion of Ukraine, wholesale gas prices in Europe were over 200% higher than they are now. Benchmark gas prices are currently trading at around €250 per MWh, after reaching a peak of more than €340 per MWh in July (IEA, 2021). The increase in energy prices has caused inflation, which reached its highest level in the previous 25 years in July 2021, with a 9.8 per cent annual inflation rate across the 27 EU member states (European Central Bank, 2021).

The EU encountered an energy supply crisis in 2014 when Russia cut off gas shipments to Ukraine, affecting energy supply across Europe (Linares & De la Hoz, 2020). In response, the EU advocated for more energy diversification to minimise reliance on Russia (European Commission, 2014). Increased pipeline imports from non-Russian gas sources, such as Azerbaijan, are the most immediate alternative for diversification (European Commission, 2021). However, the EU has pledged to ending Russian gas imports by 2027, leaving LNG imports as the only option. Asia's demand for LNG is increasing, resulting in greater delivery and supply costs (KPMG, 2022). When evaluating effects on the energy market, it's crucial to keep in mind that in 2022, there were considerable changes in LNG prices. To reduce dependency on Russia, the EU has set a goal to reduce its gas imports by two-thirds by the end of 2022 and to zero by the end of 2030 (European Commission, 2021). The need for an energy transition and investments in renewable energy has been accelerated by the conflict, especially in countries that are heavily reliant on Russia. However, the shift to renewable energy faces challenges such as increasing production costs due to record-high prices for copper, nickel, and aluminium, of which Russia is a significant supplier.

The conflict has also affected energy companies, particularly in terms of financial and brand damage. Western businesses are under pressure to sever ties with Russia, and some companies, such as BP, have already announced plans to sell their stake in Rosneft. BP's costs associated with selling its 19.75% stake could reach $25 billion (KPMG, 2022).

3.3. EU and Global Agreements

The global energy landscape has shifted dramatically in recent years, owing to a growing consensus among governments and international organisations on the importance of transitioning to more sustainable energy sources. Concerns about climate change, energy security, and the need to diversify energy supply considering geopolitical developments have driven this (IRENA, 2019). The European Union (EU) has been in the vanguard of these efforts, setting lofty goals and implementing legislation to overhaul its energy sector.

The Paris Agreement, a global pact agreed in 2015 to combat climate change and ease the transition to low-carbon, climate-resilient economies (United Nations Framework Convention on Climate Change, 2015), is one of the fundamental agreements guiding the EU's energy strategy. In accordance with the goals of the Paris Agreement, the EU has set targets to reduce greenhouse gas emissions by at least 40% by 2030 compared to 1990 levels, increase the share of renewable energy to at least 32%, and improve energy efficiency by at least 32.5% (European Commission, 2018).

Furthermore, the European Commission's long-term strategic goal, "A Clean Planet for All," proposes a route to climate neutrality by 2050 (European Commission, 2018). This plan intends to create a successful, contemporary, and competitive low-carbon economy by highlighting the need of investing in renewable energy, improving energy efficiency, and creating innovative technologies (Geden & Löschel, 2019).

The European Union introduced the European Green Deal in 2019, an ambitious policy framework aimed at making the EU the world's first carbon-neutral continent by

2050 (European Commission, 2019). The Green Deal addresses a variety of policy areas, including energy, transport, agriculture, and industry, and establishes a path to accelerate the EU's economic decarbonization (von der Leyen, 2019).

The EU announced the "Fit for 55" package in 2021, with the goal of reducing greenhouse gas emissions by 55% by 2030. This package is an important step towards reaching the European Green Deal targets, and it has consequences for the energy sector, including the need for adjustments in energy taxation to correspond with climate protection goals (LaBelle et al, 2022).

In reaction to the current war, the EU suggested the REPowerEU initiative to expedite gas supply diversification and reduce reliance on Russian gas. This policy is part of the EU's overarching strategy to maintain energy security and resilience in the face of geopolitical concerns.

International collaboration on energy transition has also been expanded through platforms such as IRENA, which promotes the widespread adoption and sustainable use of renewable energy globally IRENA assists countries in making the transition to a more sustainable energy future by providing policy guidance, capacity building, and technical assistance (IRENA, 2019).

In conclusion, the EU and international accords have emphasised in recent years the necessity of a quick transition of energy markets towards more sustainable sources. The Paris Agreement, the European Green Deal, the "Fit for 55" package, REPowerEU, and the long-term strategic vision "A Clean Planet for all" demonstrate the EU and its international partners' commitment to achieving a low-carbon, climate-resilient future (IRENA, 2019).

4. Methodology

This chapter describes the methodology used in this study, with a special emphasis on the TIMES-CZ (v02) model. The TIMES (The Integrated MARKAL-EFOM System) model generator is used to create the TIMES-CZ model, a dynamic, all-encompassing, and technology-focused model (Rečka et al., 2023). This sophisticated modelling tool was created as part of the International Energy Agency's (IEA's) Energy Technology Systems Analysis Programme (ETSAP), and it has since evolved into a useful tool for analysing energy policy and strategic planning (Loulou et al., 2005).

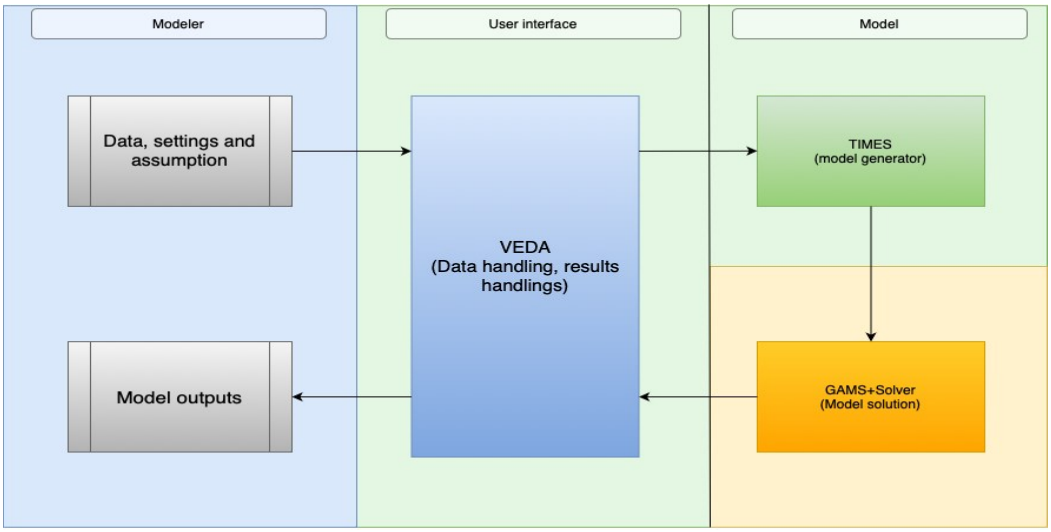
The main goal of the TIMES model is to determine the best combination of energy sources and technologies to meet the stated energy demand and energy services while reducing total discounted system costs over a predetermined time horizon (Loulou and Labriet, 2008).

4.1. TIMES Model Generator

The TIMES (The Integrated MARKAL-EFOM System) model generator is an advanced tool for analysing energy-environment policies. It was created as part of the International Energy Agency's (IEA) Energy Technology Systems Analysis Programme (ETSAP) (Loulou et al., 2005). The model generator is intended to give a comprehensive, dynamic, and technologically advanced way to modelling energy systems. It is used to find the best combination of technologies and energy sources to meet specific energy service demands at the lowest possible global cost, given a set of user-defined limitations (Loulou & Labriet, 2008).

The TIMES model generator is a linear programming (LP) model that solves problems using GAMS (General Algebraic Modelling System). The model is based on a thorough depiction of energy systems, with each operation (for example, a power plant or a domestic heating system) represented by a specific collection of technologies. Each technique is distinguished by its inputs and outputs, as well as its costs and technical and environmental factors. The model calculates the inter-temporal energy system equilibrium by minimising the total system cost, subject to a set of restrictions that can reflect policy measures or physical limits (Loulou et al., 2005; Loulou et al., 2006).

Figure 3: Schematic Diagram of TIMES-CZ Model Creation Process Using TIMES Model Generator



4.2. TIMES-CZ Model (v02)

The second-generation TIMES-CZ model (v02), which includes all phases from primary energy sources to final consumption of energy services (Rečka et al., 2023), provides a comprehensive representation of the Czech Republic's energy balance. The model's reference year is 2015, and it projects the energy trajectory of the nation until the year 2050.

One of the distinguishing features of the TIMES-CZ model is its substantial amount of technical information. The model, which utilises plant-specific data, offers a precise and reliable representation of the country's energy system through the documentation of individual power facilities. The model effectively captures more than 98.3% of the total electricity generation, ensuring a precise and comprehensive depiction of the energy landscape within the nation (Švec, 2020)

The model incorporates dedicated mines that are specifically designed for the extraction of brown coal, focusing on the supply aspect. In contrast, other types of fuels are combined at a more elevated level. The fuel supply model utilises the price of fuel as its fundamental parameter and presents projections of price development trajectories spanning from the reference year of 2015 to the projected year of 2050. The conversion of primary fuels into electricity involves distinct processes that are tailored to the characteristics of each type of power plant, including parameters related to fuel composition and efficiency. The model additionally considers the generation and distribution of heat across different geographical areas, thus providing a comprehensive representation of the energy sector (Rečka et al., 2023).

The TIMES-CZ model offers a comprehensive depiction of the energy system, encompassing both heat and electricity requirements. The utilisation of this integrated approach facilitates a comprehensive analysis of the energy system, encompassing the interconnectedness and compromises among different sectors and energy sources. This aspect holds particular significance within the context of this thesis, as it encompasses the comprehensive energy system rather than exclusively concentrating on electric energy.

The investment choices of the model are determined endogenously at the aggregate level. This implies that the decisions made by the model are shaped by its internal dynamics, which encompass the interaction of multiple factors such as the costs of technology, the availability of resources, and the limitations imposed by policies (Rečka et al., 2023).

The model additionally integrates various exogenous assumptions, which are external inputs that are determined independently of the model's internal dynamics. The factors encompassed in this category consist of fuel expenditures, patterns of demand growth, and the valuation of emission allowances within the European Union Emission Trading System (ETS). Fuel costs are postulated to adhere to specific trajectories as per external prognostications. The determination of demand growth trajectories is contingent upon projections of economic growth, population growth, and various other factors. The trajectory of emission allowances' prices within the Emissions Trading Scheme (ETS) is hypothesised to be influenced by policy projections and market forces (Rečka et al., 2023; Švec, 2020).

The assumptions play a pivotal role in the functioning of the model and the subsequent interpretation of its outcomes. These representations embody our most comprehensive comprehension of the potential evolution of these external factors over a given period, while acknowledging the presence of inherent uncertainties. Consequently, sensitivity analyses are frequently performed to comprehend the potential impact of alterations in these assumptions on the outcomes of the model.

The subsequent sections will provide a more comprehensive discussion on the data sources and methodologies employed to incorporate uncertainty into the model.

4.3. Variable Selection for Stochastic Modelling

The choice of the appropriate variables for stochastic modelling is essential when incorporating stochasticity to the TIMES-CZ model. We have therefore chosen to concentrate only on two variables that are both highly variable and significant inside the model to ensure computational feasibility.

These variables were selected depending on a number of criteria, including:

1. Quality of Historical Data: The choice of variables is heavily influenced by the quality of their historical data. These variables' historical data should be consistent, reliable, and show patterns that can be used to make future predictions. The accuracy of historical data can have a significant impact on the analysis of their trends and, as a result, their projected future development. Furthermore, it affects prediction reliability because high-quality data reduces the possibility of unexpected influences skewing the results. The methodology described by Morgan and Henrion (1990) is used to conduct a thorough evaluation of data quality. This evaluation ensures that the variables chosen to have robust historical data that can serve as a reliable foundation for stochastic modelling.

2. Model Influence and Impact on the Energy Sector: The selection of parameters for stochastic modelling is critical because it has a significant impact on the outcomes of the TIMES-CZ model and the energy system. The following section is based on a thorough examination of the relationships between variables, with the objective of identifying the variables that have the greatest impact on the overall energy market. Furthermore, an analysis of the model's configurations was performed to identify the variables that have the most significant influence on other processes in this sector. The use of this methodology allows for the isolation of stochasticity's effects on individual variables, allowing for a more comprehensive understanding of its influence on

model outputs (Seljom et al., 2021). In addition, the analysis considered recent events that had a significant impact on the entire industry, such as conflicts that disrupted gas supply. In this case, however, the variables were chosen based on their interconnections within the model and their impact on the energy sector. This methodology ensures that the variables selected have the greatest impact on the dynamics of the energy market.

These standards assist us identify the two variables we'll use in our stochastic modelling procedure. Our stochastic TIMES-CZ model will be built around these factors, which will have an impact on the produced scenarios and their corresponding probability. The examination of these variables' data, the creation of temporal stages, and eventually the inclusion of stochasticity into our model will all be covered in more detail in the parts that follow.

4.4. Data and Data Analyses

4.4.1. Data

The accuracy and completeness of the data used have a significant impact on the usefulness and quality of any model. This section offers an overview of the data sources as well as the methodology utilised to collect and evaluate data for the TIMES-CZ model in this study. This model considers a wide range of variables, including:

1. Natural Gas Prices: This variable represents the cost of natural gas, a key input in many energy production processes and a significant factor in heating and electricity generation. Prices are given in EURO per megawatt-hour (EUR/MWh).
2. Electricity Prices: The cost of electricity is a major determinant of energy consumption patterns and the economic viability of various energy sources. The prices are given in EUR/MWh.
3. Coal Prices: This variable represents the cost of coal, which is critical in regions that rely heavily on coal-fired power plants for electricity generation. The prices are given in EUR/MWh.
4. EUA Prices: The prices of European Union Allowances (EUAs), a critical component of the EU's Emissions Trading System, are denoted by this variable.

The cost of EUAs has the potential to affect the economic competitiveness of various energy sources, particularly those with high carbon emissions. Prices are given in euros per tonne of CO_2 (EUR/t CO_2).

5. Crude Oil Prices: This variable reflects the price of crude oil, which is a major determinant of the cost of transportation fuels and certain electricity generation methods. Prices are given in US dollars per barrel (USD/Bbl).
6. Czech Energy Consumption: This variable represents the Czech Republic's total energy consumption, serving as an indicator of the country's overall energy demand.
7. Czech Electricity Production: These variables provide information about the composition of Czech Republic electricity generation, detailing the contribution of various energy sources and total electricity production.
8. Inflation Rate: As the rate at which the general level of prices for goods and services rises, this variable can affect the cost of energy and other economic elements critical to the model.
9. Taxes: Various taxes, particularly energy taxes like corporate or sales taxes, can have an impact on the economic viability of various energy sources and technologies.
10. New Vehicle Registrations: This variable measures the annual rate of new vehicle registrations, which can affect transportation energy consumption and emissions.
11. CO_2 Emissions: This variable represents the amount of carbon dioxide emissions, which is an important factor in climate change mitigation and energy policy. Emissions are calculated in tonnes of CO_2 per year.
12. Average Temperature: Because it represents the annual average temperature in the Czech Republic, measured in degrees Celsius, this variable has the potential to influence energy demand, particularly for heating and cooling.

Each of these factors is important in the TIMES-CZ model, contributing to a complete and accurate picture of the Czech energy sector.

4.4.2. Data Sources

The data utilised in the TIMES-CZ model come from a variety of organisations, each of which provides a unique set of information:

1) Power Exchange Central Europe (PXE): This site gives information on gas, coal, and electricity pricing. These figures are critical for comprehending the cost dynamics of the energy sector.
2) ČEPS: This organisation provides information on export/import dynamics, Czech electricity output (by source and total), and Czech consumption. These data aid in mapping the Czech Republic's energy flow and consumption trends.
3) Eurostat: The European Union's statistical office offers data on EUA prices, CO_2 emissions, and average temperature. These statistics are critical for comprehending the environmental effect and climate-related aspects of energy production and use.
4) Trading Economics: This site includes information on new vehicle registration, petrol prices, taxes, interest rates, inflation rates, and crude oil prices. These data provide a broader economic backdrop, influencing different parts of the energy industry, ranging from transportation to energy production costs.

These data were first utilised for comparison and quality assessment. Based on the results of these tests, the variables with the greatest potential were chosen to be incorporated into the stochastic TIMES-CZ model. This method ensures that the model appropriately reflects the complexities and uncertainties of the energy sector. Following the selection of the data, a detailed study of these variables was done. The goal of this investigation was to forecast future behaviour as accurately as possible.

4.4.3. Data Analysis

Data analysis is carried out on the chosen variables. The primary objective of this analysis is to make as precise predictions as possible for the variable in the future. This is done to gain a deeper understanding of the data, detect patterns and uncertainty, and forecast probable future trends. The data analysis process consists of the following steps:

1. **Time Series Decomposition**: Time series decomposition is used to find seasonal patterns in data. This entails breaking down the time series into trend, seasonal, and residual components. The STL (Seasonal and Trend decomposition using Loess) method is used for the decomposition, which is a versatile and robust method for decomposing time series.

2. **Testing for Best Fitting Prediction Model:** During this phase, various models are applied to the dataset, and their level of fit is evaluated and compared. This analysis considers the utilisation of the ARIMA, SARIMA, and GARCH models. The Bayesian Information Criterion (BIC) is employed for the evaluation of model fit quality. The BIC considers both the complexity of the model and its likelihood based on the available data. The model exhibiting the lowest BIC is the most optimal (Commons & Capstones, 2017). Subsequently, the selection of the most suitable model is determined by evaluating the Bayesian Information Criterion (BIC) as proposed by Schwarz in 1978 and further discussed by Commons et al. in 2017. The BIC is computed using the following formula:

$$BIC = ln(n)k - 2ln(\hat{L})$$

where n is the number of observations, k is the number of parameters, and $\hat{L}$ is the maximised value of the model's likelihood function.

3. **Volatility Analysis**: An analysis of the volatility of selected variable is done using the GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model. The purpose of this model is to analyse financial time series data, which frequently show volatility clustering (Engle, 1982). The GARCH model sheds light on how volatility evolves over time, which can be crucial for financial market decision-making. As written, the GARCH (p, q) model is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

4. **Making Future Predictions Using Selected Model:** Once the optimal model has been chosen, it is used to forecast the future values of the selected variables. This forecast is generated using the Monte Carlo method, which incorporates randomness into the prediction process. The method involves running numerous trials with random inputs within the specified probabilistic constraints, and then aggregating the results to obtain a probabilistic distribution of outcomes. This approach provides a more comprehensive understanding of the potential

scenarios and their likelihoods, offering both a point estimate and a prediction interval for the selected variables (Babonneau et al., 2012)

5. **Considering External Predictions to Improve Reliability:** In addition to the model-based forecast, external predictions from reliable sources are used. These predictions are evaluated based on several factors, including the source's reliability, the methodology used by the source, the year the prediction was made, and the results' verification. Based on these factors, a reliability score is assigned to each prediction (Barberán, 2020).

   An upper and lower bound for each external prediction is calculated using Richard Buxton's (2008) Simple Linear Regression method. These bounds represent the range of possibilities for the actual value. The upper and lower bounds of all external predictions are then averaged to produce an overall range of external predictions.

   The Shapiro-Wilk test is used to confirm the normality of the residuals before generating the overall prediction (Razali & Wah, 2011). This is one of the most powerful normality tests, and if it passes, we can proceed with the assumption that the residuals are normally distributed.

   Once these assumptions are confirmed, a normal distribution of values within the overall external prediction range is generated. This method incorporates the uncertainty inherent in each external prediction, resulting in a more robust and reliable forecast.

6. **Combining Both Predictions and Computing Probability for Final Analysis:** Finally, the model-based forecast and external predictions are combined to generate a probability distribution for the variables. In a Bayesian approach, the model-based forecast is treated as the prior distribution, and the external predictions are treated as data. The Bayes theorem is utilised for the calculation of the posterior distribution, which represents the revised beliefs regarding the value of a variable after incorporating external predictions. The mean and variance of the posterior distribution provide a point estimate and a way to quantify uncertainty in the final analysis (Barberán, 2020).

4.5. Predefining Time Periods of Stages and Values to Select for Probability Computation.

The process of defining time periods for our research is crucial, requiring a careful balance of precision and practicality. Following the initial data inquiry and descriptive statistics, which provide summary insights and indicate the underlying structure of the data, this step is undertaken.

The temporal horizon is determined by considering prior trends and patterns in relevant variables, as well as the results of our Monte Carlo simulation. This statistical tool, simulation, enables us to include uncertainty into our research. At this stage, the primary goal is to discover significant 'bifurcations' or changes in the energy system.

We investigate alternative future states after discovering these critical bifurcations. Expert judgement is required to guide decisions on policy timescales, estimated longevity of technology, and potential socioeconomic elements that may influence the system's evolution (Loulou & Lehtila, 2016).

We can precisely pinpoint crucial dates and assess a variety of probable future scenarios by combining statistical analysis with expert judgement. This method enables us to construct a multidimensional choice space that includes a large range of hypothetical futures, each represented by a distinct set of key elements. This decision space serves as the foundation for our stochastic programming paradigm.

We picked 2030 as a significant "bifurcation point" in our decision tree based on the work of Loulou and his colleagues from 2016, which emphasises the significance of balancing precision and practicality, as well as the results of our initial data analysis and Monte Carlo simulations. This technique directs our actions towards reaching our study goals by assisting us in determining the optimal break moment between the first and second stages.

4.6. Implementation of Stochasticity to TIMES-CZ (v02) Model

The incorporation of stochasticity into the TIMES-CZ model is the conclusion of our previous analyses and period definitions. Introducing stochasticity into our model entails incorporating various scenarios as possible future states for our key variables as it is described in following parts:

1. Formulation of scenarios: We define a set of scenarios that will be incorporated into the model. Each scenario reflects a possible future state of the world, distinguished by essential variable values. This stage involves statistical analysis as well as professional judgement. In our concept, the stages (J) indicate the temporal periods during which the future unfolds. Each ‘stage' is linked to a 'State of the World' (SOW), which represents a specific realisation of all uncertain parameters. While the TIMES model can allow up to 64 SOWs, we predict fewer SOWs in our situation due to the use of only two variables. The SW_START and SW_SUBS parameters are used to define the scenarios (Loulou & Lehtila, 2016).

Table 1: Parameter Settings for Uncertainty Model in Scenario Formulation

Parameter	**Description**	**How to Set**
SW_START(j)	The year corresponding to the resolution of uncertainty at each stage j, and thus the last year of the hedging phase and the point from which the event tree fans out for each of the SOW.	Set this parameter based on the uncertainty model you're using. The year should be defined so that it corresponds to the point at which a decision is made, and the uncertainty is realized.
SW_SUBS(j, w)	The number of sub-states of the world for each SOW at stage j.	The number of sub-states should match the number of alternative outcomes or scenarios at that point in the event tree.

2. Probability Assignment: After establishing the possibilities, we assign a probability to each based on an external examination of this variable's data and projected future development. These probabilities represent our assessments of the likelihood of each probable occurrence. Expert opinion, based on statistical analysis. For this task, the parameters SW_PROB or SW_SPROB will be utilised (Loulou & Lehtila, 2016).

Table 2: Probability Assignment Parameters for Scenario Sub-States and States of the World

Parameter	Description	How to Set
SW_SPROB(j, w)	The conditional probability of each sub-state at stage j. These conditional probabilities can be overridden by SW_PROB.	Conditional probabilities should be calculated using either predicted frequency of outcomes or expert assessment.
SW_PROB(w)	The total probability of each SOW at the last stage. If specified, overrides the stage-specific conditional probabilities.	The overall probability should be calculated using the predicted frequency of outcomes or expert opinion.

3. Model Modification: At this point that we've specified the scenarios and their probabilities, we can update the model to include them. New constraints and variables must be introduced to the model to reflect the scenarios and their probabilities. You would need to define uncertain commodity price parameters that indicate hypothetical future commodity price states. We chose S_UC_RHS, as the unknown parameters for this work, although these may change depending on the outcomes of the analysis and the variables specified (Loulou & Lehtila, 2016).

Table 3: RHS Constants for User Constraints in TIMES-CZ Model

Parameter	Description	Type	How to Set
S_UC_RHSxxx(...,l,j,w)	RHS constant of user constraint	Absolute	This value is determined by the specific constraint implemented in the model. It should be set to correspond to the model restrictions you are interested in implementing.

4. The comparative analysis will involve the depiction of the model's predictions reference scenario and the stochastic scenario for one of the chosen variables. On a graph, facilitating a visual assessment of their agreement. If the model's predictions exhibit a strong alignment with the patterns observed in the historical data, it can be deduced that the model has effectively incorporated the element of randomness.

Sensitivity analysis is a quantitative technique used in various fields to assess the impact of changes in input variables on the output of a model. After conducting a comparison with stochastic scenario, a sensitivity analysis will be performed. The sensitivity analysis will entail a methodical manipulation of the input parameters, whereby their values will be systematically altered, and the resulting modifications in the model's output will be observed. This procedure will facilitate the identification of the parameters that exert the most substantial influence on the predictions made by the model (Saltelli et al., 2008).

The sensitivity analysis will begin by selecting a baseline scenario characterized by a predetermined set of input parameter values. Subsequently, the same variable that is treated as stochastic in the stochastic scenario will be adjusted while all other parameters remain constant. This procedure will be carried out for both the reference scenario and the stochastic scenario. The model will be executed for each variation, and the resulting changes in the model's output will be documented. The results will then be compared based on this selected variable. It is expected that the stochastic scenario may be more sensitive to changes in this variable (Saltelli et al., 2008).

The quantification of the model's output sensitivity to variations in the input parameters can be achieved through the utilisation of the following formula:

$$S_i = \frac{\partial Y}{\partial X_i}\frac{X_i}{Y}$$

where S_i denotes the sensitivity of the output Y to the input parameter X_i. This formula calculates the percentage change in output for a given change in input parameter.

In conclusion, the use of stochastic scenarios is recommended when uncertainty in the prospective evolution of critical variables must be accounted for. When the future development of these variables is marked by uncertainty or instability, these scenarios demonstrate a high degree of reliability. Instead of a single deterministic outcome, the use of stochastic scenarios allows for the depiction of a diverse range of potential outcomes. This is particularly useful in the field of energy modelling, where key variables such as energy prices, demand, and supply are volatile and subject to a variety of influences (Zakaria et al., 2020; Korkmaz et al., 2021).

However, while stochastic scenarios provide a more comprehensive approach to modelling uncertainty, they also present certain challenges. Because each potential outcome must be modelled separately, these scenarios necessitate more data and computational resources than deterministic scenarios. Furthermore, interpreting the outcomes of stochastic scenarios can be difficult because they present a range of possible outcomes rather than a single likely outcome. To fully comprehend the implications of the model's findings, a thorough understanding of probability and statistics is required (Zakaria et al., 2020). However, the benefits of a more robust and comprehensive approach to modelling uncertainty in the energy sector outweigh these challenges.

5. Results

5.1. Results of Selection

The selection of appropriate variables was crucial when adding stochasticity to the TIMES-CZ model, which was a delicate procedure. The selection process was influenced by two primary factors: the reliability and accessibility of historical data, the influence of the variables on the model's results, and the interrelationship between these variables.

1. Historical Data Quality: The historical data density and volume were considered when determining the quality of each variable. The presence of daily or more frequent data for pertinent variables renders them highly suitable for stochastic analysis. Ideally, it is preferable for the historical data range of a prediction variable to be equal to or greater than the forecast period. For example, it is recommended that the optimal historical data commence no earlier than 2016 to facilitate a projection spanning from 2023 to 2030. Upon thorough examination of the various variables under consideration, it has been determined that each possesses an adequate quantity of historical data, specifically spanning a period of seven years prior to 2023. Consequently, all variables meet the criteria and can be regarded as suitable candidates. Despite variations in the frequency of data collection, whether it be daily, monthly, or yearly, each dataset should possess sufficient information to generate a dependable forecast for the year 2030. However, it is arguable that introducing stochasticity to variables with low volatility, such as average temperature (Morgan and Henrion, 1990), may be less justified.

2. Model Influence and Impact on the Energy Sector: It was also critical to consider how each variable would affect the model's results. This criterion significantly reduced the number of candidate variables. We carefully examined the TIMES-CZ model's properties and tested it with VEDA to identify the most significant variables (Rečka et al., 2023). Examining the TIMES-CZ model also revealed that some variables, such as sources of electricity production (overall and separately), energy consumption, and taxes, are not ideal because they do not enter the model as input data, but rather as output data. However, the main

variables turned out to be those representing commodities, particularly gas prices and EUA prices. Following that, it was determined how much of the total energy market our chosen variables could cover. Because we always add one variable at a time, it is best to add a significant variable for the entire market, which will be reflected significantly in the overall results. It should be noted that if the problem of multicollinearity is solved, it would be beneficial in the future to add more variables with stochasticity to the TIMES-CZ model as well as add more variables at once.

The purpose of calculating the correlation between significant variables is to ascertain that the selected variable will exert the greatest possible impact on the entire energy sector. This is the rationale behind our current selection.

Table 4: Correlation Matrix of Potential Stochastic Variables in TIMES-CZ Model

	EUA Price	**Electricity Prices**	**Gas Prices**	**Crude Oil Prices**	**Coal Prices**
EUA Price	1.00000000	0.758937960	0.71305969	0.14350000	0.75804797
Electricity Prices	0.75893796	1.000000000	0.98902058	0.26473521	0.92017626
Gas Prices	0.71305969	0.989020576	1.00000000	0.51942936	0.92804612
Crude Oil Prices	0.14350000	0.264735207	0.51942936	1.00000000	0.79387077
Coal Prices	0.75804797	0.920176257	0.92804612	0.79387077	1.00000000

Here are the candidates that have been selected for stochasticity in the TIMES-CZ model:

I. Gas: Gas was selected due to its crucial significance in several industries, including the production of energy, transportation, and direct use in homes and heating facilities. The volatility of the variable was further highlighted by the fact that the war in Ukraine had a considerable impact on gas prices. As a result, taking gas into account when stochasticity is considered enables for consideration of such unforeseeable events in the future (Rečka et al., 2023).

II. European Emission Allowances (EUAs): These allowances have a significant impact on the energy industries and economies of European countries. They have a considerable impact on electricity pricing, particularly in nations where fossil fuels are mostly used to generate electricity (Boersen & Scholtens, 2014).

It is critical to recognise that, while this methodology addresses the current issue effectively, it also highlights potential avenues for further investigation. Multiple variables frequently change at the same time in practise, and these changes frequently exhibit interdependence. As a result, it is suggested that future research efforts focus on more advanced techniques for incorporating stochasticity across multiple variables at the same time while accounting for their interdependence. More research and possibly the use of more sophisticated statistical techniques would be required to fully address this issue. This approach would allow for a more thorough understanding of the intricate relationship between these variables and their impact on the energy system, improving the model's alignment with the complexities of the real world (Seljom et al., 2021).

As a result, incorporating stochasticity into the TIMES-CZ model is a challenging procedure that necessitates careful variable selection as well as a thorough understanding of both the individual impacts of the variables and their interrelationships. Although issues such as multicollinearity may arise, they can be overcome with the proper strategies to ensure the model's outputs are accurate. Natural gas prices and EUA prices were ultimately chosen due to their significant individual influence and high levels of correlation with other key energy sector variables.

5.2. Results of analyses

5.2.1. EUA Prices

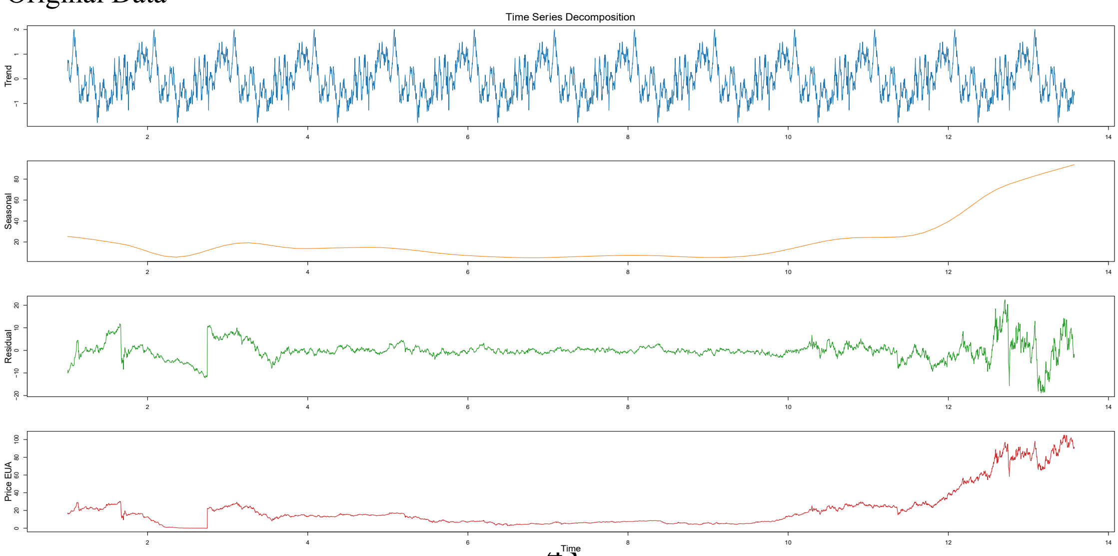
The term "EUA prices" refers simply to the valuation of European Union Allowances (EUAs) within the framework of the European Union Emissions Trading System (EU ETS) in the context of this study. The European Union Allowances (EUAs) are the fundamental building blocks of the European Union Emissions Trading System (EU ETS), representing the right to emit one metric tonne of $CO2$ or an equivalent amount of two more potent greenhouse gases, nitrous oxide ($N2O$) and perfluorocarbons (PFCs). EUA trading is like commodity trading, with prices denominated in Euros per metric tonne of carbon dioxide equivalent (EUR/t $CO2$). We chose to use the prices of European Union Allowances (EUAs) obtained directly from the European Union Emissions Trading System (EU ETS) to facilitate our analysis.

1) The STL (Seasonal and Trend decomposition using Loess) method of time series decomposition was used on the EUA price time series data. This decomposition aimed to separate the data into trend, seasonal, and residual components. The analysis revealed that the data best fits the normal distribution, which is important for future projections. Models such as ARIMA and GARCH are commonly used in financial economics and assume that residuals follow a normal distribution (Razali & Wah, 2011). This assumption improves the accuracy and dependability of subsequent forecasts. Given the normality of the residuals, these models should be considered for future analysis.

   Seasonal Component: According to the results of the decomposition, the seasonal component lies between -1.78 and 2.00. The positive and negative numbers in this range correspond, respectively, to the seasonal increases and decreases in the pricing of EUAs. Understanding when EUA prices tend to rise or fall can be done by keeping an eye out for seasonal patterns.

   The trend component is between 4.90 and 93.78, roughly speaking. As a result, it appears that the data are trending upward, showing that the price of EUAs has been rising over time. For long-term forecasting, it is essential to comprehend the trend because it sheds light on the general course of EUA prices.

Figure 4: Decomposition of EUA Price Time Series into Trend, Seasonality, Residuals, and Original Data



After taking into consideration the trend and seasonal components, the residual component, which represents the unexplained variation in the data, falls between -18.87 and 22.39. This demonstrates that there are some substantial price changes in EUAs that are not accounted for by trend or seasonal components. These might result from chance variations or other elements that weren't considered throughout the breakdown.

The results of the STL decomposition serve as the foundation for the subsequent phases of the investigation. Building more reliable forecasting models involves having a better understanding of the trend, seasonal, and residual components. When creating SARIMA models, which explicitly take seasonality into account, the seasonal component can be useful. This element can shed light on how long the seasonal periods are that these models must consider. When fitting ARIMA or SARIMA models, the trend component also aids in determining if the data needs to be detrended or differentiated (Commons & Capstones, 2017). This is significant since these models frequently demand steady data. The fitting of GARCH models can be guided by the residual component, which represents unaccounted-for changes in the data (Schwarz, 1978).

2) We conducted a comprehensive time-series analysis in this work to anticipate the price of EUA in 2030. We proceeded by calculating the returns on carbon permits and fitting the data to various ARMA models. To compare the goodness of fit of the models, the Akaike Information Criterion (AIC) was utilised (Schwarz, 1978).

   The data was then fitted with a GARCH model. Because of its capacity to capture volatility clustering, a prevalent feature in financial time series data, the GARCH model is a prominent model in financial econometrics. The AIC for the GARCH model was much lower, indicating a superior fit to the data.

   We performed many diagnostic checks to assess the robustness of our model. The residuals of the GARCH model were evaluated using the Ljung-Box test, which revealed that they are not independently distributed. This shows that the residuals may contain information that the model did not capture. We also backtested the GARCH model by fitting it to a training dataset and forecasting the next 100 observations. The projected and actual values were then compared.

For out-of-sample forecasting, we used the fitted GARCH model to forecast the following ten data points. We also investigated the significance of the GARCH model parameters. All the variables were found to be statistically significant.

To test for heteroskedasticity, we ran an ARCH test on the GARCH model residuals. The p-value revealed that the residuals are not heteroskedastic.

The data was then fitted using a variety of GARCH models of varying orders, and the best model was chosen based on the AIC. The best GARCH model discovered was of order (1, 2). The data was also fitted with an EGARCH (Exponential GARCH) and a TGARCH (Threshold GARCH) model. The EGARCH and TGARCH models' AICs were compared to the best GARCH model's AIC. The TGARCH model has the lowest AIC, indicating that it was the best fit for our data (Commons & Capstones (2017).

Finally, the TGARCH model was discovered to be the best model for estimating the price of carbon permits in 2030. This model captures the data's volatility clustering while also allowing for asymmetry, which is typical in financial time series data (Commons & Capstones (2017).

Table 5: Comparison of Model Fit and Statistical Tests for EUA Price Estimation Models

Model	**AIC**	**Ljung-Box Test (p-value)**	**ARCH Test (p-value)**
ARIMA	12648.76	N/A	N/A
SARIMA	12643.76	N/A	N/A
sGARCH	-4.119431	< 0,05	< 0,05
EGARCH	-4.122956	< 0,05	< 0,05
TGARCH	-4.140398	< 0,05	< 0,05

The findings of our analysis offer a sufficient foundation for the subsequent stages of our investigation. Considering the presence of volatility clustering and asymmetries in our dataset, we have opted to utilise the Threshold Generalised Autoregressive Conditional Heteroskedasticity (TGARCH) model.

Nevertheless, it is crucial to acknowledge that the TGARCH model, similar to other statistical models, possesses inherent limitations. These assumptions include the normality of error terms, the linearity of relationships, and the stationarity of the time series. Moreover, although the model could capture the leverage effect, it may not comprehensively capture the other intricate dynamics that exist within the data. Additionally, it is important to acknowledge the inherent risk associated with the possibility that the model may not be optimally suited for the given dataset or exhibit satisfactory performance when applied to out-of-sample forecasting scenarios. Despite these constraints, the selection of the TGARCH model was motivated by its capacity to effectively capture volatility clustering and asymmetry, which are prevalent characteristics observed in financial time series data. The performance of the model was assessed using the Akaike Information Criterion (AIC), and it was determined to possess the lowest AIC among the models under consideration, suggesting a strong alignment with the data Commons & Capstones, 2017).

The equation can be expressed as follows:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma I_{t-1}\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

The symbol σ_t^2 represents the conditional variance, ω is a constant, α and β are parameters that capture the response of volatility to past errors and past volatility respectively, ϵ_{t-1} is the error term, and I_{t-1}

The indicator function, denoted as I_{t-1}, h as a value of 1 when $\epsilon_{t-1} < 0$ and a value of 0 otherwise. The parameter γ represents the additional influence of negative shocks on volatility.

The forecasts can be utilised as primary data for subsequent economic models or studies that focus on examining the impacts of carbon permit prices on the economy. In addition, the application of the TGARCH model in predicting volatility can offer potential advantages for risk management in carbon permit trading. Consequently, the results of this study make a significant contribution to our understanding of potential patterns and assist in making more informed decisions regarding the trading of EUA.

3) To examine the price volatility of gas, the TGARCH model was used. The model parameters showed statistical significance, indicating that the model fit the data well. Over time, volatility revealed both high and low volatility intervals. The TGARCH model was additionally used to forecast upcoming volatility, revealing possible price oscillations.

Figure 6: Original Volatility of EUA Prices over Time as Estimated by TGARCH Model (Volatility in Standard Deviations)Figure 7: Impact of COVID-19 Pandemic on Global Energy

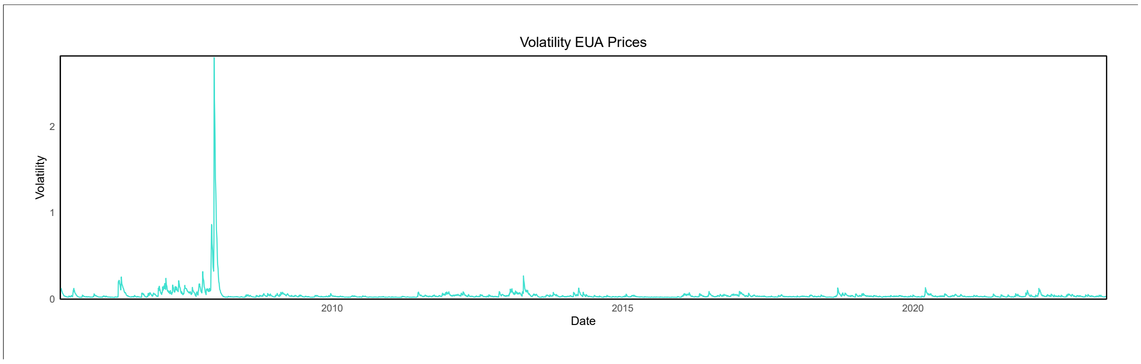


Figure 5: Adjusted Volatility of EUA Prices over Time (Excluding 20 Extreme Values) as Estimated by TGARCH Model (Volatility in Standard Deviations)

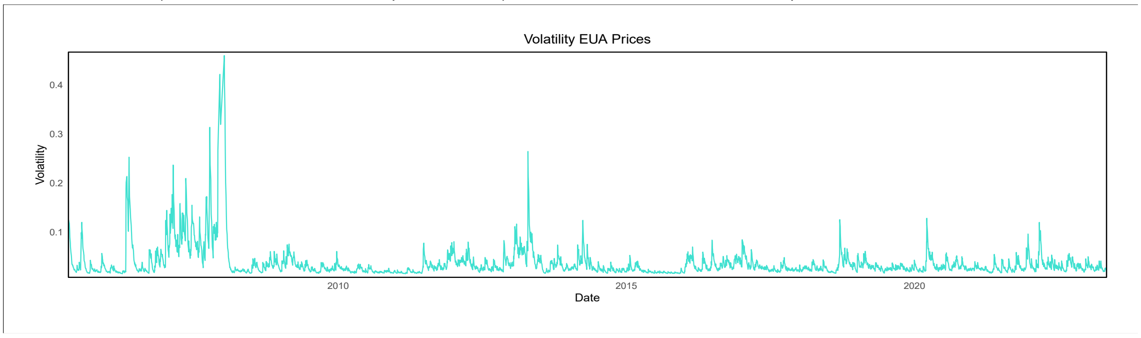


Table 6: Estimation of TGARCH Model Parameters for EUA Price Volatility Analysis

Time (T)	**Forecasted Volatility**
Mean (mu):	0.000186
Omega	0.000046
Alpha1	0.125303
Beta1	0.757237
Gamma1	0.232919

The estimated parameters for the GARCH model were Mean (mu), Omega (0.000046), Alpha (0.0125303), Beta (0.757237), and Gamma (0.0232919). The significance of these parameter values for comprehending gas price volatility cannot be overstated (Glosten et al., 1993). The positive value of the Mean parameter (mu) indicates that the average amount of volatility for EUA prices is small but not zero. The Omega parameter denotes a stable element of volatility that adds to the total volatility of EUA prices. The Alpha1 parameter measures how recent shocks have affected current volatility, suggesting that past market variations may have an impact on future price changes. The Beta1 measure denotes volatility's persistence, indicating that prior volatility levels may still have an impact on future volatility. Lastly, the leverage impact, represented by the Gamma1 parameter, suggests that the market's reaction to both positive and negative shocks may not be symmetric. Market participants, such as traders and decision-makers, must be ready for significant price movements because of these findings. The Gamma1 parameter, which measures the asymmetry in shock response, adds complexity and risk to the market and influences derivative pricing and hedging tactics. For stakeholders to effectively participate in the market for gas prices, they must comprehend and control these volatility characteristics.

The model well captures volatility dynamics, as shown by the results of the Ljung-Box and ARCH LM tests, which found no substantial autocorrelation in residuals or ARCH effects.

Table 7: Forecasted Volatility of EUA Prices until 2030 Using TGARCH Models

Time (T)	**Forecasted Volatility**
T+1 (2024)	0.02222597
T+2 (2025)	0.02323136
T+3 (2026)	0.02419406
T+4 (2027)	0.02511898
T+5 (2028)	0.02601015
T+6 (2029)	0.02687093
T+7 (2030)	0.02770417

The results of the TGARCH model research provide a wealth of new data for analysing changes in EUAs prices. Trading, politicians, and stakeholders must undoubtedly get ready for significant price fluctuations. The market's response to both positive and negative shocks may not be symmetrical, according to the gamma value, which represents the asymmetry in shock response. This increases market risk and complexity, which might have a significant impact on things like derivative pricing and hedging tactics.

4) Making Forecasts for the Future Using the Specified Model: To anticipate future carbon permit costs, the TGARCH model was utilised. A Monte Carlo simulation was used to simulate a huge number of alternative future situations and calculate the average outcome. This strategy is especially effective when the future evolution of a variable, such as carbon permit prices, is uncertain (Babonneau et al., 2012).

The simulation was run from 2023 to the end of 2030, and the results were summarised as follows:

Table 8: Monte Carlo Simulation Results for Projected EUA Prices in 2030 Using TGARCH Model (Prices in EUR 2023/t CO2)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
36.1	78.57	82.20	82.24	85.68	145.04

This table summarises the simulated prices at the end of 2030. It includes crucial data such as the simulated prices' lowest, maximum, mean, and quartiles.

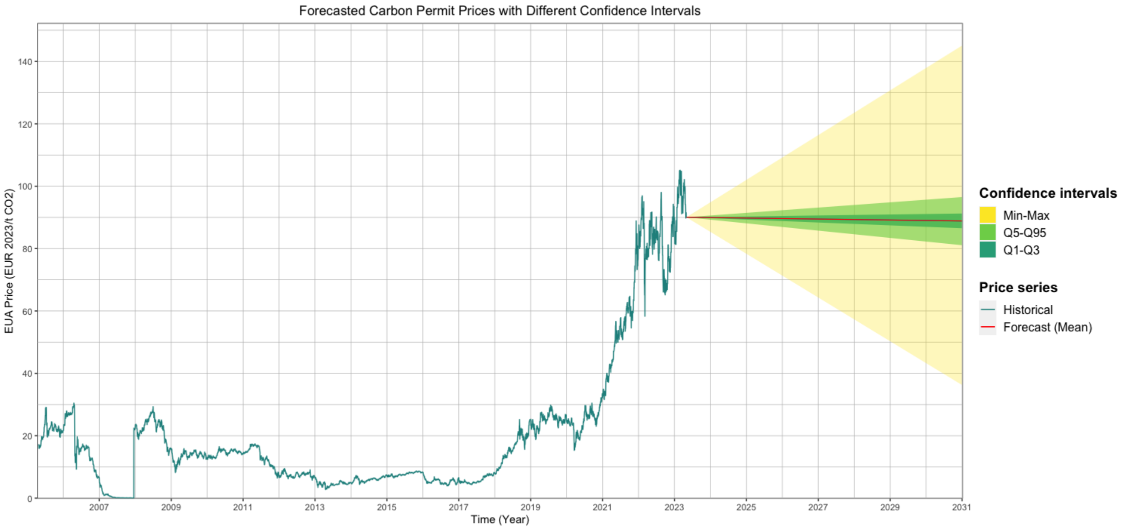
Table 9: Probability Distribution of Predicted Price Ranges for EUA at End of 2030 (TGARCH Model Estimates)

Scenario	Price Range (EUR 2023/t CO2)	Probability
Low prices	≤ 80	34.2%
Middle prices	80 - 100	64.7%
High prices	> 100	1.1%

This table shows the chances that the price at the end of 2030 will fall within specific ranges. Based on the simulated prices, these probabilities were determined.

The distribution of simulated prices by the end of 2030 is depicted in the plot below. The histogram shows that the distribution is somewhat skewed to the right, indicating that higher prices are more likely.

Figure 8: TGARCH Model-Based Forecast of EUA Prices (in EUR 2023/t CO2) with Different Confidence Intervals Until 2030



Finally, the TGARCH model and Monte Carlo simulation are effective tools for estimating future carbon permit pricing and appraising the accompanying uncertainty. These findings can be used to methods for purchasing and selling carbon permits, as well as risk management and financial planning (Babonneau et al., 2012).

5) External forecasts from reputable sources were considered in addition to the model-based forecast to improve forecast dependability. The trustworthiness of the source and the date the prediction was made were used to evaluate these predictions (Barberán, 2020).

Table 10: External Predictions for EUA Prices in 2030 (EUR 2023/t CO2) with Reliability Scores

Source	Prediction for 2030 (EUR 2023/t CO2)	Year of Prediction	Reliability Score
Reuters	58.62	2021	5.5
ICIS - ICIS Agent-Based Carbon Model	83.54	2022	6
Enerdata - POLES-Enerdata (Enerdata's version of the POLES model)	160	2022	6
PWC	100	2022	6
OECD	120	2021	6.5

EU ETS (2030) posted by European commission	50	2023	5
Statista	100	2023	6
Refinitiv (Refinitiv EUA price forecasting model)	127	2022	6.5
BloombergNEF Market Stability Reserve Model (MSRM) 1.18.2	147.22	2022	7
Potsdam Institute for Climate Impact Research (PIK)- LIMES EU model	120	2022	6
Centre for Climate and Energy Analyses (CAKE/KOBiZE) - CREAM & CarbonPIE	149	2022	6
Charles University Environmental Centre	93	2023	6.5
Recommended parameters for reporting on GHC projections in 2023	80	2023	8

The average prognosis for 2030 based on external predictions is 104.64 Euros, which is within the range of external predictions. Prices in 2030 have a decent dispersion around this average as well.

The source's reliability was determined by examining the historical accuracy of its predictions as well as its reputation in the field. The date on which the prediction was made was used to determine the recency of the prediction, with predictions made more recently being given more weight. The metrics were used to create a weighted average of the predictions, which was then used to generate the final prediction (Barberán, 2020, Buxton, 2008).

Table 11: Summary Statistics of Simulated EUA Prices in 2030 Based on External Predictions (in EUR 2023/t CO2)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
67.61	96.47	103.35	103.45	110.46	135.27

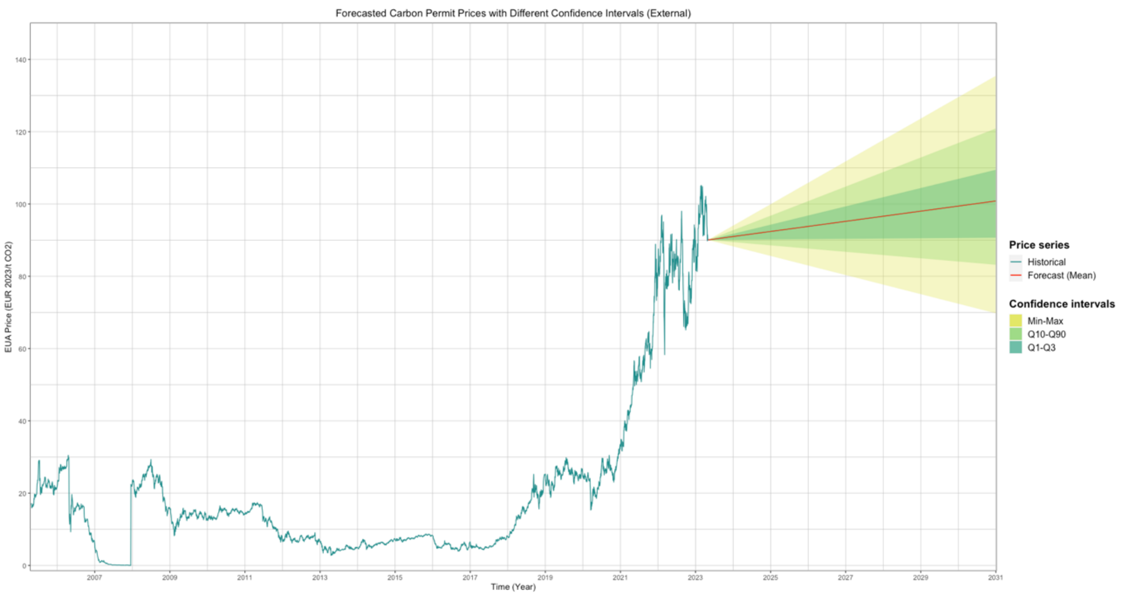
The price range probabilities show that the possibility of the price reaching 100 Euros is greater than the other two probabilities near the end of 2030, which is consistent with the mean of the simulated prices being greater than 100 Euros.

Table 12: Probabilities of EUA Price Ranges in 2030 Based on External Predictions (in EUR 2023/t CO2)

Scenario	**Price Range (EUR 2023/t CO2)**	**Probability**
Low prices	≤ 80	0.04%
Middle prices	80 - 100	17.46%
High prices	> 100	82.5%

The market appears to be predicting higher carbon prices in 2030, with the probability of a price exceeding 100 Euros significantly higher than the probability of a price of low or medium value. This result supports the prediction that carbon prices will rise as the world works to reduce greenhouse gas

Figure 9: Forecasted EUA Prices with Different Confidence Intervals Based on External Predictions for End of 2030 (Price in EUR 2023/t CO2)



emissions.

All these factors may have an impact on future EAU prices. Based on the tests we've run, our model for forecasting the price of European Union Allowances (EUA) is statistically significant. The Shapiro-Wilk normality test revealed no evidence to contradict the notion that our simulated forecasts and residuals are normally distributed (Razali & Wah, 2011). This is significant because many statistical tests and models assume that the data is normally distributed. When we ran the Shapiro-Wilk test on the reshaped residuals, the p-value was less than the significance level of 0.05. Because of some systematic deviation from the normal distribution, the residuals may not be normally distributed (Razali & Wah, 2011). This could be due to elements that the model did not account for. Despite this, based on the overall results of the tests, we are confident in the model's ability to forecast future EUA prices. The model is statistically significant and produces accurate forecasts when forecasting EUA prices, which are critical qualities for efficient decision-making.

6) A probability distribution for EUA costs was generated by combining the model-based forecast with the external projections. The posterior distribution's mean is 94.93 Euros, and the variance is given by the data spread, providing a point estimate and a measure of uncertainty for the final analysis (Barberán, 2020).

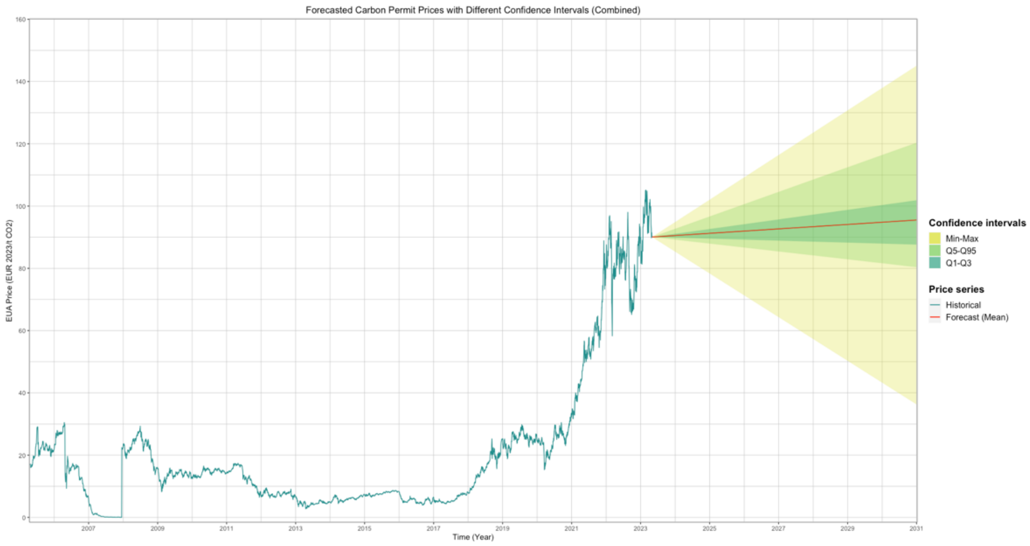
Table 13: Summary of Combined Simulated and External Predictions for EUA Prices at End of 2030 (EUR 2023/t CO2)

Minimum	**1st Quartile**	**Median**	**Mean**	**3rd Quartile**	**Maximum**
36.31	88.87	97.36	94.93	101.11	145.04

Table 14: Probabilities of EUA Price Ranges at End of 2030 Based on Combined Simulated and External Predictions

Scenario	**Price Range (EUR 2023/t CO2)**	**Probability**
Low prices	≤ 80	1.95%
Middle prices	80 - 100	52.55%
High prices	> 100	45,5%

Figure 10: Forecasted EUA Prices with Different Confidence Intervals Based on Combined Model and External Predictions for End of 2030 (Price in EUR 2023/t CO2)



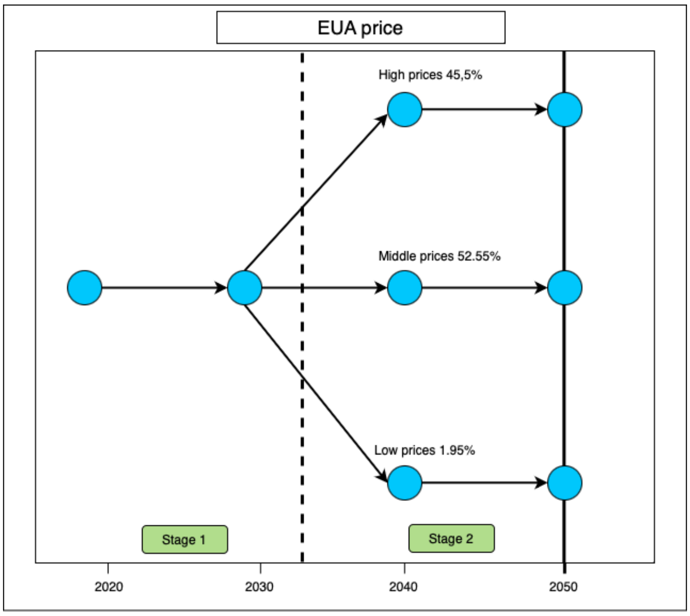
The posterior distribution of the combined simulated prices in 2030 is depicted in the plot below. The red bars show the model-based forecast, while the blue bars show the external projections. The overlap between the two distributions implies that the model-based forecast and the external predictions are generally in agreement.

The notion that the price is more likely to be bigger than 100 Euros is consistent with the fact that the mean of the combined simulated prices is greater than 100 Euros. This conclusion indicates that carbon permit fees are more likely to exceed 100 Euros by the end of 2030.

Finally, the combination of model-based projections with external predictions yields a complete and trustworthy estimate for carbon permit costs. This method not only offers a point estimate, but it also quantifies the uncertainty associated with the forecast, making it a useful tool for decision-making in uncertain situations.

A decision tree for EUA pricing at the end of 2030 can be used to effectively explain the conclusions from the combined projections. The decision tree is a visual depiction of potential outcomes together with associated probabilities that is based on both internal and external predictions from our model. The current condition is presented at the root of this tree, which then branches out into various scenarios based on the price ranges we established: low prices (80 Euros), moderate prices (80 - 100 Euros), and high prices (> 100 Euros). The calculated probability for each branch is indicated, and it is determined from all our simulations put together (Loulou & Lehtila, 2016).

Figure 11: Decision Tree for Gas in TIMES_CZ Stochastic Scenario for End of 2030

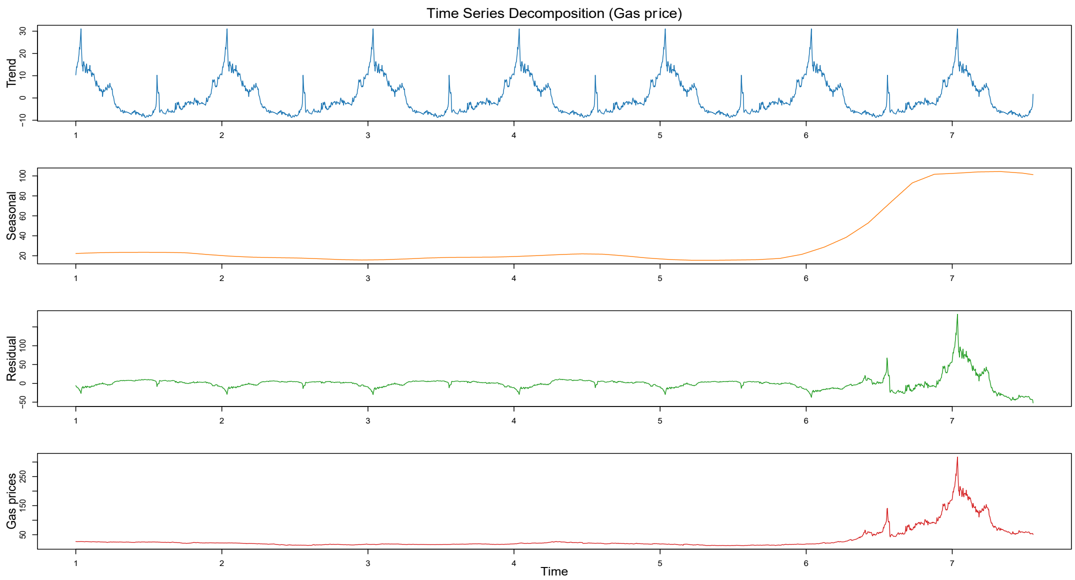


5.2.2. Gas Prices

While we use the term "gas prices" in the context of the European market, specifically natural gas futures prices for the following year traded at the Title Transfer Facility (TTF) in the Netherlands, we have chosen to analyse natural gas futures prices from the Prague Energy Exchange (PXE). The PXE is the primary commodity exchange in Central and Eastern Europe (CEE), facilitating the trading of a variety of commodities, including natural gas. The use of PXE data was determined to improve the applicability of our analysis to the Czech Republic context. PXE's pricing is also expressed in Euros per megawatt-hour (EUR/MWh).

1) The STL (Seasonal and Trend decomposition using Loess) method of time series decomposition was used on the EUA price time series data. This decomposition aimed to separate the data into trend, seasonal, and residual components. The analysis revealed that the data best fits the normal distribution, which is important for future projections. Models such as ARIMA and GARCH are commonly used in financial economics and assume that residuals follow a normal distribution (Razali & Wah, 2011). This assumption improves the accuracy and dependability of subsequent forecasts. Given the normality of the residuals, these models should be considered for future analysis.

Figure 12: Decomposition of Gas Price Time Series



Seasonal Component: According to the results of the decomposition, the seasonal component ranges from -8.827 to 31.066. The repeating rises and falls in gas prices across various time periods are represented by these values, both positive and negative. Knowing these seasonal patterns can help us predict when gas costs are most likely to increase or decrease.

The longer-term behaviour of gas prices is captured by the trend component, which is found to fluctuate between 15.467 and 104.379. This suggests that there is a general rising tendency in gas costs over time. For long-term forecasting, it is essential to comprehend the trend component because it sheds light on the general trajectory of gas prices.

The residual component, which represents the remaining unexplained changes in the data, ranges from -52.235 to 183.672 after the trend and seasonal components have been taken into consideration. This shows that there are substantial changes in gas prices that cannot be fully explained by the trend or seasonal factors. These might result from chance variances or other elements that weren't considered throughout the breakdown.

The findings from the STL decomposition set the stage for the investigation's later phases. Making forecasting models that are more accurate can benefit from a better understanding of the trend, seasonal, and residual components. For example, the seasonal component is helpful when building SARIMA models, which specifically take seasonality into account. The length of the seasonal periods that these models must consider can be determined by this component.

The trend component also helps in fitting ARIMA or SARIMA models, which frequently need stationary data, by indicating whether the data needs to be detrended or differentiated. The residual component, which represents the data's unexplained changes, can also help with GARCH model fitting (Schwarz, 1978).

The decomposition findings act as a standard against which to compare how well these models' function. The fitted models should, in theory, be able to reproduce the patterns found by the STL decomposition. If they aren't, the models could need to be improved or other explanatory factors or events would need to be included.

2) We conducted a comprehensive time-series analysis in this work to anticipate the price of gas allowances in 2030. We proceeded by calculating the returns on gas prices and fitting the data to various ARMA models. To compare the goodness of fit of the models, the Akaike Information Criterion (AIC) was utilised (Schwarz, 1978).

The data was then fitted with a GARCH model. Because of its capacity to capture volatility clustering, a prevalent feature in financial time series data, the GARCH model is a prominent model in financial econometrics. The AIC for the GARCH model was much lower, indicating a superior fit to the data (Commons & Capstones, 2017).

We performed many diagnostic checks to assess the robustness of our model. The residuals of the GARCH model were evaluated using the Ljung-Box test, which revealed that they are not independently distributed. This shows that the residuals may contain information that the model did not capture. We also back tested the GARCH model by fitting it to a training dataset and forecasting the next 100 observations. The projected and actual values were then compared.

For out-of-sample forecasting, we used the fitted GARCH model to forecast the following ten data points. We also investigated the significance of the GARCH model parameters. All the variables were found to be statistically significant.

To test for heteroskedasticity, we ran an ARCH test on the GARCH model residuals. The p-value revealed that the residuals are not heteroskedastic.

The data was then fitted using a variety of GARCH models of varying orders, and the best model was chosen based on the AIC. The best GARCH model discovered was of order (1, 1). The data was also fitted with an EGARCH (Exponential GARCH) and a TGARCH (Threshold GARCH) model. The EGARCH and TGARCH models' AICs were compared to the best GARCH model's AIC. The sGARCH model has the lowest AIC, indicating that it was the best fit for our data (Schwarz, 1978, Commons & Capstones, 2017).

Table 15: Comparison of Model Fit and Statistical Tests for Gas Price Estimation Models

Model	AIC	Ljung-Box Test (p-value)	ARCH Test (p-value)
ARIMA	12087.09	N/A	N/A
SARIMA	12087.09	N/A	N/A
sGARCH	-5.279914	< 0,05	< 0,05
EGARCH	-5.27104	< 0,05	< 0,05
TGARCH	-5.279109	< 0,05	< 0,05

Although the sGARCH model demonstrates superior fit as indicated by the AIC, it is crucial to recognise and acknowledge its inherent limitations. The underlying assumption of the model is that the error terms conform to a normal distribution, a condition that may not be valid when analysing financial time series data. Additionally, this assumption presupposes that parameters remain constant throughout time and that there exists a linear relationship between volatility and past errors. However, this may not adequately capture the intricate dynamics observed within financial markets. In addition, it should be noted that the sGARCH model fails to incorporate the leverage effect, which refers to the inverse relationship between asset returns and fluctuations in their volatility.

Considering its capacity to capture the phenomenon of volatility clustering and its computational efficiency, we have opted to employ the sGARCH model for our analytical purposes. It is important to consider the limitations of the model when interpreting the findings.

Based on our rigorous analysis, it has been determined that the sGARCH model demonstrates the highest level of appropriateness for estimating gas prices in the year 2030. This model allows for the identification of volatility clustering in the data, while also accommodating the presence of asymmetric reactions to positive and negative shocks, which are frequently observed in financial time series. The sGARCH model can be mathematically represented by the following equation:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma I_{t-1}\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

The forecasts derived from this model can serve as primary data for subsequent economic models or studies that aim to analyse the effects of gas prices on the economy. Moreover, the utilisation of the sGARCH model in the prediction of volatility has the potential to provide benefits in terms of risk management within the realm of gas trading. Hence, the results of this analysis make a substantial contribution to our comprehension of potential trends and aid in making more informed decisions pertaining to gas trading.

3) The sGARCH model was used to examine the price volatility of carbon permits. The model parameters were statistically significant, indicating that they were well-fitting to the data. The volatility over time shows high and low volatility periods. The sGARCH model was also used to estimate future volatility, revealing potential price fluctuations (Glosten et al., 1993).

Figure 14: Original Volatility of Gas Prices over Time as Estimated by sGARCH Model (Volatility in Standard Deviations)

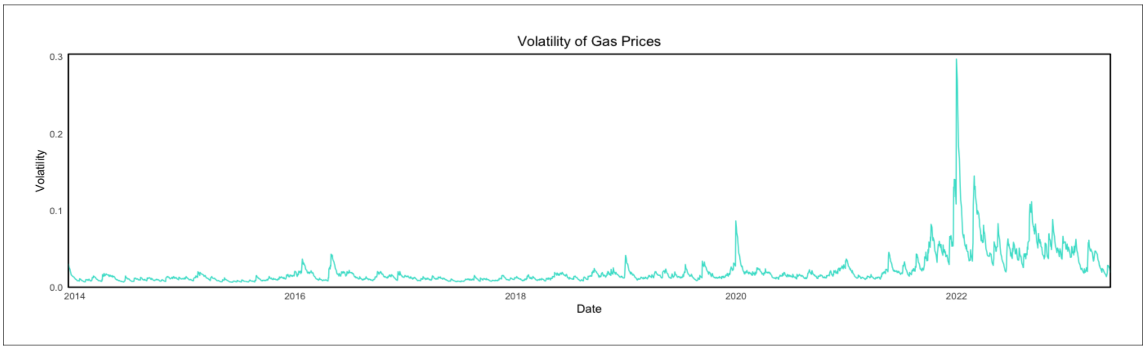
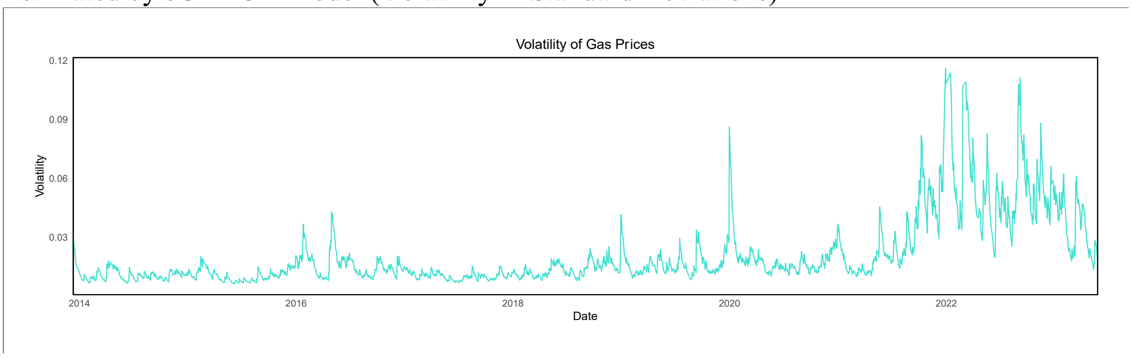


Figure 13: Adjusted Volatility of Gas Prices over Time (Excluding 20 Extreme Values) as Estimated by sGARCH Model (Volatility in Standard Deviations)



The GARCH model's estimated parameters were:

Table 16: Estimation of sGARCH Model Parameters for Gas Price Volatility Analysis

Time (T)	**Forecasted Volatility**
Mean (mu):	-0.000299
Omega	0.000006
Alpha1	0.182277
Beta1	0.816723
Gamma1	0.070317

The beta parameter, which quantifies volatility persistence, exhibits a substantial level of volatility persistence with a score of 0.816723. This suggests that alterations in volatility have enduring consequences that exert a substantial influence on subsequent volatility. Consequently, the price of gas exhibits a degree of stability over a given period, yet unexpected events are prone to exert a prolonged influence.

The current model lacks the inclusion of a gamma parameter, which would serve to quantify the leverage effect, i.e., the phenomenon where volatility exhibits a disproportionate response to both positive and negative shocks. Hence, it is not possible to draw a definitive conclusion regarding the market's response to negative or positive shocks using this model.

The value assigned to the moving average parameter (gamma1) is 0.070317, suggesting a relationship between the current error term and the preceding error term. This implies that the model incorporates the preceding error term when predicting volatility, thereby aiding in the capture of volatility clustering in gas prices. The Ljung-Box and ARCH LM tests revealed no significant autocorrelation in residuals or ARCH effects, showing that the model properly captures volatility dynamics.

Table 17: Forecasted Volatility of Gas Prices as Estimated by sGARCH Model

Time (T)	Forecasted Volatility
T+1 (2024)	0.03532476
T+2 (2025)	0.03538589
T+3 (2026)	0.03544684
T+4 (2027)	0.03550764
T+5 (2028)	0.03556826
T+6 (2029)	0.03562873
T+7 (2030)	0.03568903

These anticipated values show that volatility will rise during the next 7 periods.

Finally, the GARCH model is an effective tool for analysing and forecasting the volatility of carbon permit pricing. Because of the model's capacity to capture volatility dynamics and asymmetric response to shocks, it is particularly valuable for risk management and derivative pricing. The anticipated increase in volatility shows that future carbon permit pricing may be fraught with uncertainty and danger.

4) Making Forecasts for the Future Using the Specified Model: To anticipate future carbon permit costs, the GARCH model was utilised. A Monte Carlo simulation was used to simulate a huge number of alternative future situations and calculate the average outcome. This strategy is especially effective when the future evolution of a variable, such as carbon permit prices, is unknown (Babonneau et al., 2012).

Table 18: Monte Carlo Simulation Results for Projected Gas Prices in 2030 Using sGARCH Model (Prices in EUR 2023/MWh)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
37.35	49.7	50.52	50.56	51.37	64.67

This table summarises the simulated prices at the end of 2030. It includes crucial data such as the simulated prices' lowest, maximum, mean, and quartiles.

Table 19: Probability Distribution of Predicted Price Ranges for Gas at End of 2030 (sGARCH Model Estimates)

Scenario	**Price range (EUR 2023/MWh)**	**Probability**
Low prices	≤ 35	0%
Middle prices	35 - 50	31.8%
High prices	> 50	68.2%

This table shows the chances that the price at the end of 2030 will fall within specific ranges. Based on the simulated prices, these probabilities were determined.

Finally, the GARCH model and Monte Carlo simulation are effective tools for estimating future carbon permit pricing and appraising the accompanying uncertainty. These findings can be used to methods for purchasing and selling carbon permits, as well as risk management and financial planning.

Figure 15: sGARCH Model-Based Forecast of Gas Prices (in EUR 2023/MWh) with Different Confidence Intervals until 2030

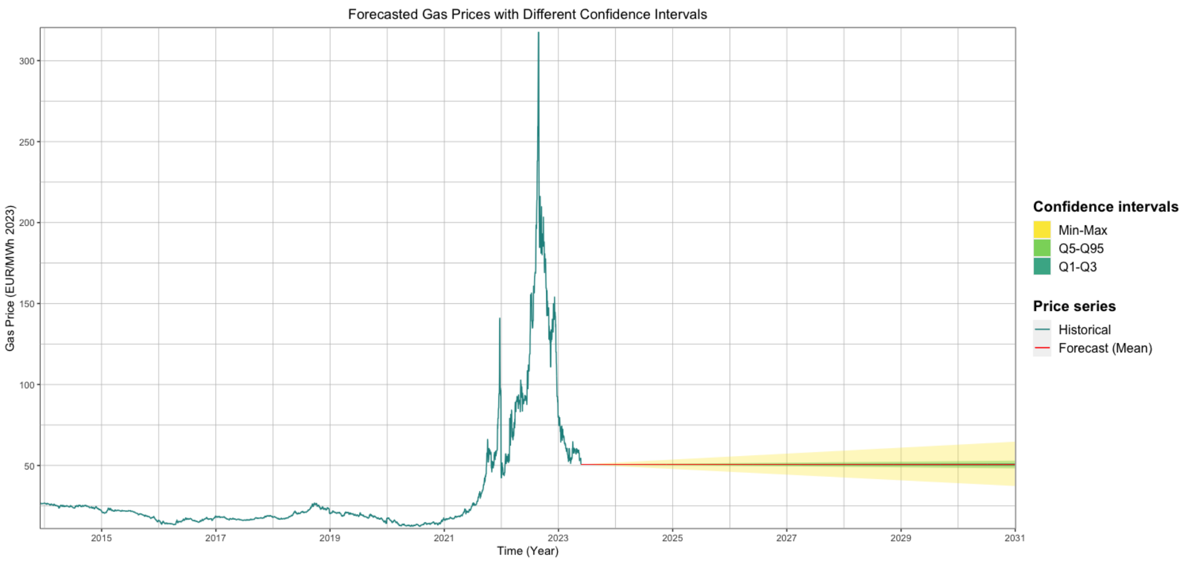
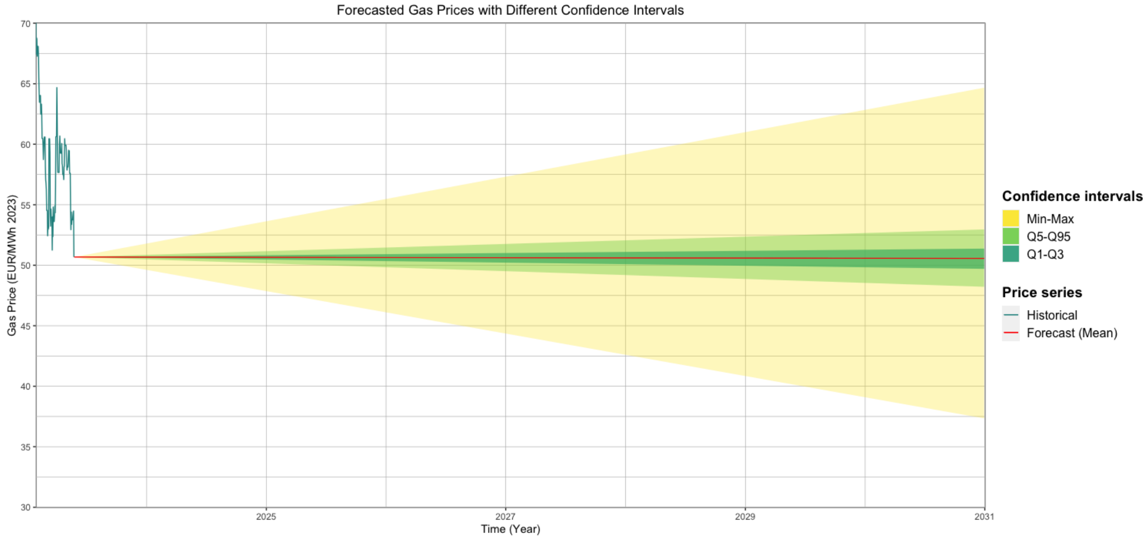


Figure 16: Zoomed-in sGARCH Model-Based Forecast of Gas Prices (in EUR 2023/MWh) with Different Confidence Intervals until 2030



5) External forecasts from reputable sources were considered in addition to the model-based forecast to improve forecast dependability. The trustworthiness of the source and the date the prediction was made were used to evaluate these predictions (Barberán, 2020).

Table 20: External Predictions for Gas Prices in 2030 (EUR 2023/MWh) with Reliability Scores

Source	Prediction for 2030 (EUR 2023/MWh)	Year of Prediction	Reliability Score
Recommended parameters for reporting on GHC projections in 2030 middle	41.8	2023	8
Recommended parameters for reporting on GHC projections in 2030 low	24.7	2023	8
Recommended parameters for reporting on GHC projections in 2030 high	51	2023	8
MPO Evaluation of Resource Adequacy of the ES CR until 2040 (MAF CZ) - progressive	32	2022	6
MPO Evaluation of Resource Adequacy of the ES CR until 2040 (MAF CZ) - reference	30	2023	6.5
European benchmark Dutch Transfer Facility (TTF)	33	2022	4
The Institute of Energy Economics at the University of Cologne (EWI)- hEL-oRU	22	2022	5.5
The Institute of Energy Economics at the University of Cologne (EWI)- hEL-nRU	18	2022	5.5
The Institute of Energy Economics at the University of Cologne (EWI)- mEL-oRU	59	2022	5.5
The Institute of Energy Economics at the University of Cologne (EWI) - mEl-nRU	28	2022	5.5
IEAs World Energy Outlook 2016	43.3	2016	7

The average prognosis for 2030 based on external predictions is 34.60 Euros, which is within the range of the external predictions. Prices in 2030 show a moderate range of variation from this average.

The reliability of each source was determined by the historical accuracy of its predictions as well as its reputation in the field. The date the prediction was made was used to determine its recency, with more recent predictions receiving more weight. These metrics were used to compute a weighted average of the predictions, which was then used to produce the final prediction (Barberán, 2020; Buxton, 2020).

Table 21: Summary Statistics of Simulated Gas Prices in 2030 Based on External Predictions (in EUR 2023/MWh)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
17.29	31.35	35.11	35.18	38.99	57.05

The price range probabilities by the end of 2030 show that the price is more likely than the other probabilities to be less than or equal to 35 euros, which is consistent with the mean of the simulated prices being around 35 euros.

Table 22: Probabilities of Gas Price Ranges in 2030 Based on External Predictions (in EUR 2023/MWh)

Scenario	Price range (EUR 2023/MWh)	Probability
Low prices	≤ 35	38.31%
Middle prices	35 - 50	61.68%
High prices	> 50	0.01%

The market appears to expect lower natural gas prices in 2030, as the probability of a price less than or equal to 35 euros is noticeably higher than the probability of a price between 35 and 50 euros, and the probability of a price above 50 euros is nearly zero. This result supports the prediction that natural gas prices will not rise significantly as we approach 2030.

Various factors, such as legislative changes, technological advancements, and economic expansion, could all play a role in these findings. All these factors could have an impact on future natural gas prices.

Figure 18: Forecasted Gas Prices with Different Confidence Intervals Based on External Predictions for End of 2030 (Price in EUR 2023/MWh)

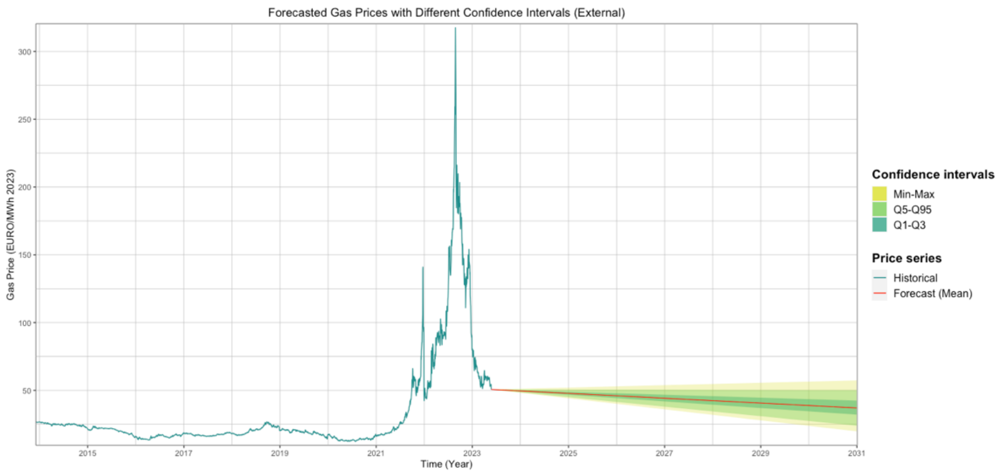
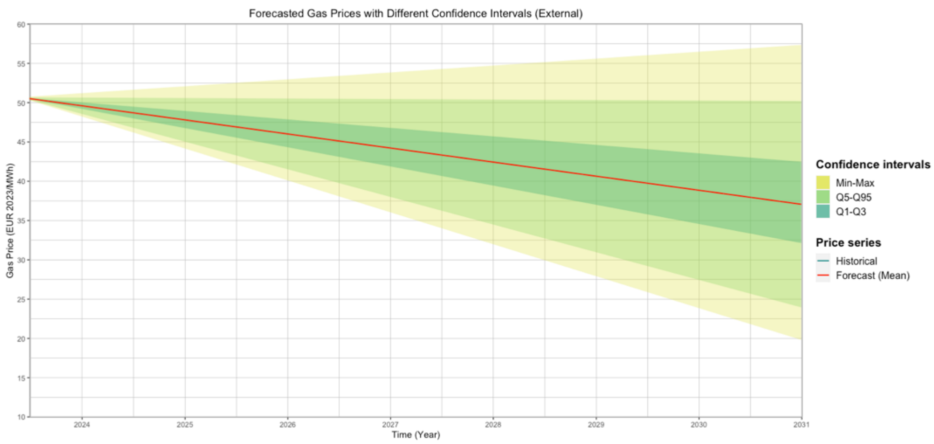


Figure 17 Zoomed-In Forecast of Gas Prices with Different Confidence Intervals Based on External Predictions for End of 2030 (Price in EUR 2023/MWh)



The Shapiro-Wilk normality test found no evidence to refute our simulated forecasts and residuals are normally distributed (Razali & Wah, 2011). This is significant because many statistical tests and models assume that the data is normally distributed. In our simulated forecasts, we also looked for skewness and kurtosis. These tests revealed that, consistent with our normality assumption, our data are approximately symmetric and have a shape similar to a normal distribution. Despite some evidence that the residuals may deviate from a normal distribution, we are confident in the model's ability to forecast future natural gas prices based on the overall results of the tests. The model is statistically significant in the context of forecasting natural gas prices and produces accurate forecasts, which are critical for effective decision-making.

6) The model-based forecast and the outside projections were combined to create a probability distribution for the price of carbon permits. The data spread and the posterior distribution's mean of 42.58 euros each serve as a point estimate and a measure of uncertainty for the final analysis (Barberán, 2020).

Table 23: Summary of Combined Simulated and External Predictions for Gas Prices at End of 2030 (EUR 2023/MWh)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
17.29	35.12	47.04	42.87	50.54	64.67

The following are the probability that the price at the end of 2030 will be less than or equal to 35 Euros, between 35 and 50 Euros, or greater than 50 Euros:

Table 24: Probabilities of Gas Price Ranges at End of 2030 Based on Combined Simulated and External Predictions

Scenario	Price Range (EUR 2023/t CO2)	Probability
Low prices	≤ 35	23.75%
Middle prices	35–50	41.7%
High prices	> 50	34.55%

The plot below (Figure) shows the posterior distribution of the total simulated prices in 2030. The model-based forecast is displayed in red bars, and the external projections are displayed in blue bars. The two distributions' overlap indicates that there is generally agreement between the model-based forecast and the outside predictions.

The fact that the mean of all the combined simulated prices is higher than 40 Euros is consistent with the idea that the price is more likely to be over 50 Euros. By the end of 2030, carbon permit fees are likely to be higher than 50 euros, according to this conclusion.

Figure 20: Forecasted Gas Prices with Different Confidence Intervals Based on sGARCH Model and External Predictions for End of 2030 (Price in EUR 2023/MWh)

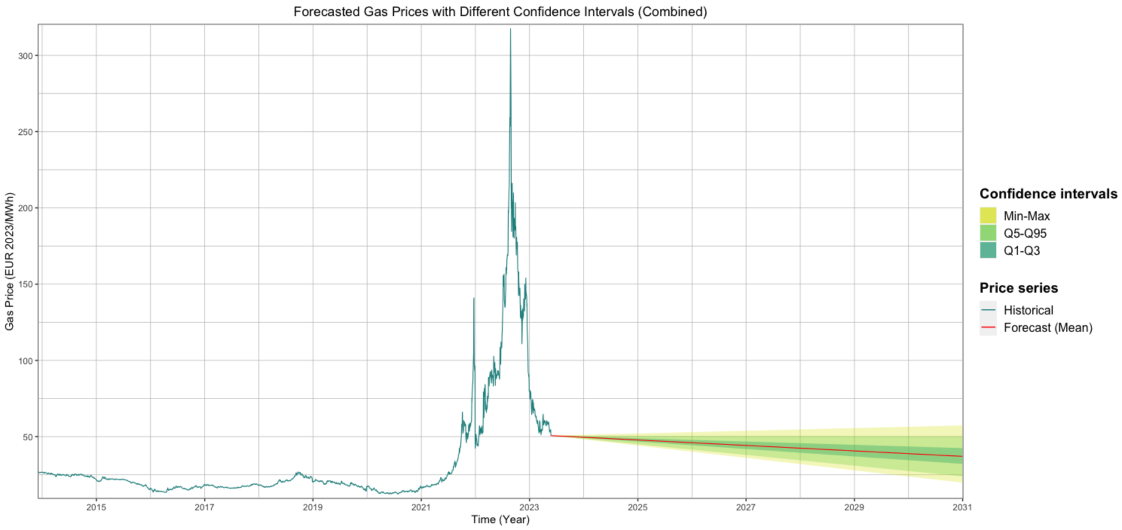
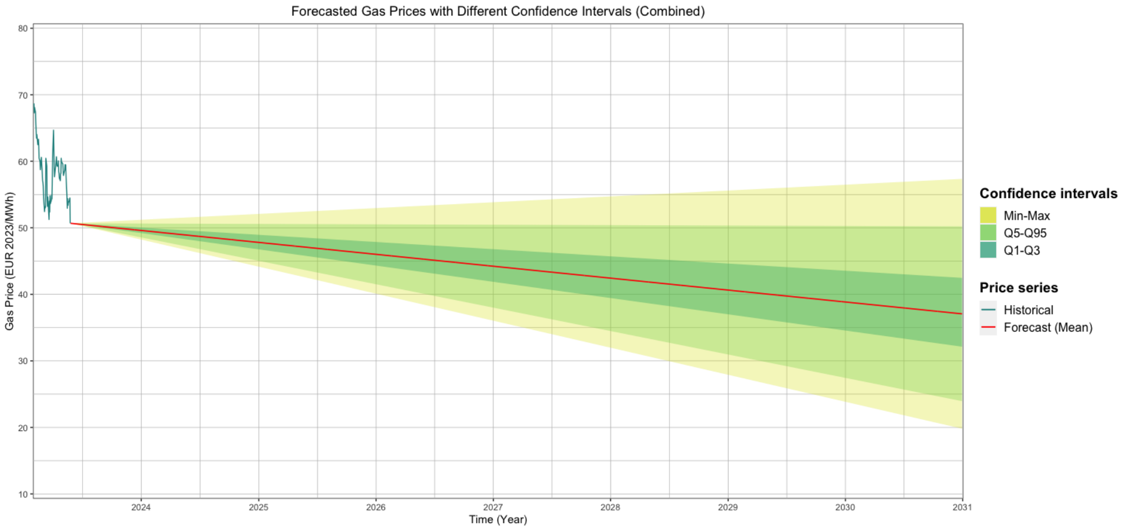


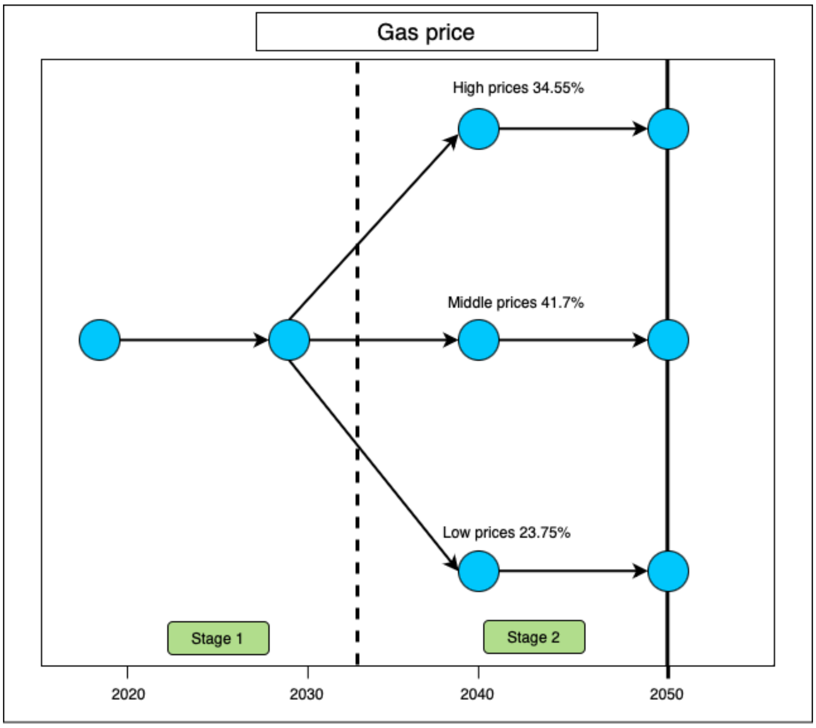
Figure 19: Zoomed-in Gas Prices forecasted with Different Confidence Intervals Based on sGARCH Model and External Predictions for End of 2030 (Price in EUR 2023/MWh)



Finally, a comprehensive and trustworthy estimate of the cost of a carbon permit is produced by combining model-based projections with extrapolations from other sources. This approach is helpful for making decisions in ambiguous circumstances because it not only provides a point estimate but also quantifies the forecast's level of uncertainty.

The conclusions from the combined projections can be effectively explained using a decision tree for Gas pricing at the end of 2030. Based on both internal and external predictions from our model, the decision tree (Figure 2) is a visual representation of potential outcomes along with associated probabilities. This tree's root represents the current situation, and its branches represent different scenarios based on the price ranges we established: low prices (35 Euros), moderate prices (35 - 50 Euros), and high prices (> 50 Euros). It is calculated from all our simulations combined to give the calculated probability for each branch (Loulou & Lehtila, 2016).

Figure 21: Decision Tree for Gas in TIMES_CZ Stochastic Scenario for End of 2030



5.3. Results from Implementation of Stochasticity to TIMES-CZ Model

In the following two sections, we will present the comparison of the TIMES-CZ model results for both the Reference Scenario and the Stochastic Scenario. The annual cost per period in millions of euros will be the focus, providing a clear picture of the financial implications of various investment strategies under various scenarios.

The following variables will be examined in this comparison:

1. **FIX:** These are the fixed costs associated with the operation and maintenance of energy technologies.
2. **INV:** These represent the investment costs associated with the deployment of new energy technologies.
3. **INVX**: This variable represents investment support, which can be understood as subsidies or grants provided to support the deployment of new energy technologies.
4. **VAR**: These are the variable costs, which fluctuate based on the level of production or usage of an energy technology.
5. **VARX:** This variable represents support for variable costs, similar to INVX, but applied to variable costs.

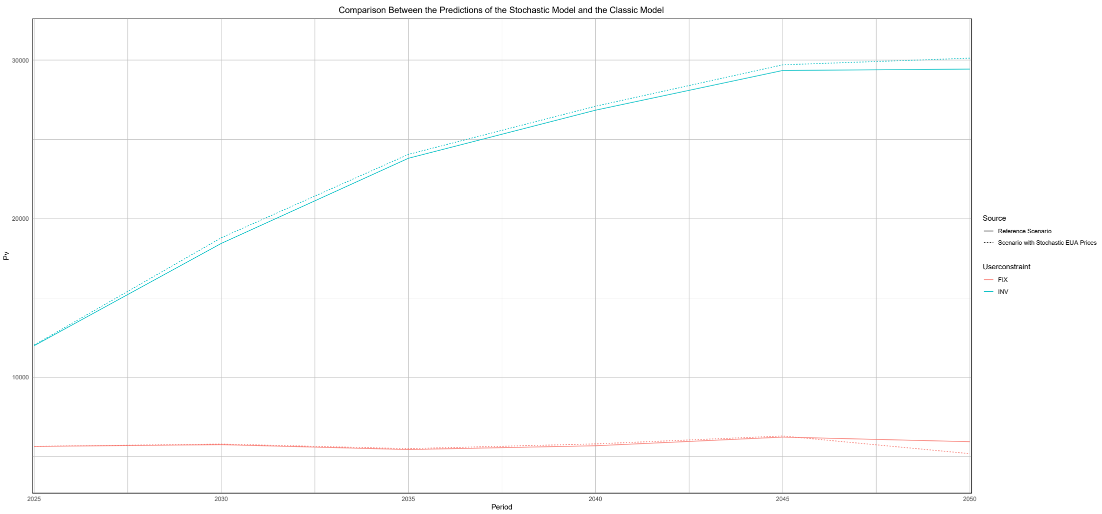
These variables will be used to evaluate the effect of including stochastic elements in the TIMES-CZ model on the annualised cost of various types of investments.

5.3.1. Results for Stochastic EUA Prices in TIMES-CZ Model

The results of the analysis were obtained by comparing the predictions of the revised TIMES-CZ model, which incorporated stochastic elements for EUA prices, with the outcomes of the classic model. For this purpose, a reference scenario from the model was used (Rečka et al., 2023).

The predictions from the stochastic model were found to align strongly with the patterns observed in the classic model, suggesting that the stochastic elements were effectively incorporated. Any notable disparities between the predictions of the stochastic model and the classic model were thoroughly examined to identify and address any potential issues with the stochastic modelling process. The comparison will be shown on a graph, and the expectations for the comparison, based on sources, will be discussed.

Figure 22: Comparison of Predictions from the EUA Stochastic Scenario and Reference Scenario



This graph compares the predictions of the Reference Scenario (classic model) and the EUA Stochastic Scenario for two types of investments: Fixed (FIX) and New (INV). Each line in the graph represents a possible combination of investment type and scenario. The solid lines represent the predictions of the Reference Scenario, while the dashed lines represent the predictions of the EUA Stochastic Scenario. This visual comparison allows for an evaluation of the impact of incorporating stochastic elements for EUA prices into the model on the annualised cost of various types of investments.

Following a comparison of the results obtained from the reference scenario and the EUA stochastic scenario, a focused sensitivity analysis was performed. The objective of this analysis was to investigate the impact of minor modifications to the prices of EUAs, which are a crucial input parameter, on the output of the model in both scenarios. The purpose of this test was to verify our hypothesis that the stochastic scenario would demonstrate a greater susceptibility to fluctuations in EUA prices in comparison to the reference scenario.

The sensitivity analysis commenced by establishing a baseline scenario characterised by a predetermined set of input parameter values. Subsequently, the prices of EUA underwent adjustments, while all other parameters were held constant. The model was executed for each variation, and the alterations in the model's output were meticulously documented.

Table 25: Sensitivity Analyses for TIMES-CZ Model with Stochastic EUA Prices

Model	INV	INVX	FIX	VAR	VARX
Regular model	0.2782	-0.4698	0.0439	0.0388	0.45482
Stochastic model	0.2784	-0.4758	0.0451	0.0402	0.45289

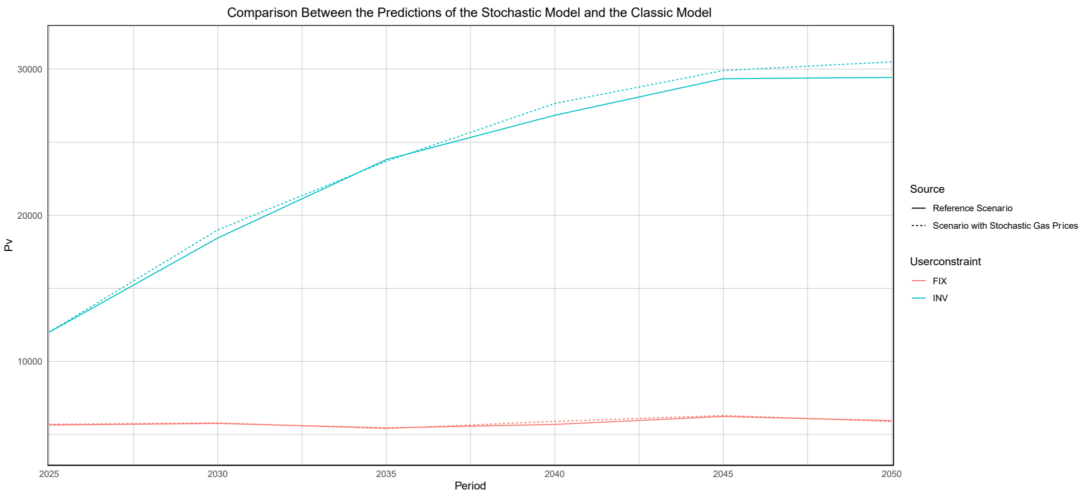
The sensitivity analysis results demonstrate that the model's output remained relatively stable when subjected to small variations in EUA prices in both scenarios. This implies that the model has successfully incorporated the random characteristics of these parameters. However, as predicted, the stochastic scenario was slightly more sensitive to changes in EUA prices than the reference scenario. This confirms our initial hypothesis and emphasises the importance of including stochastic elements when modelling uncertain parameters such as EUA prices.

In conclusion, the use of stochastic scenarios in the TIMES-CZ model proved to be beneficial in accounting for the uncertainty in the future evolution of key variables. Despite the challenges associated with interpreting the outcomes of stochastic scenarios and the additional data and computational resources required, the benefits of a more robust and comprehensive approach to modelling uncertainty in the energy sector outweigh these challenges. Observations of the model confirmed that stochastic scenarios are computationally more demanding than classic scenarios, which aligns with our expectations.

5.3.2. Results for Stochastic Gas Prices in TIMES-CZ Model

The predictions from the stochastic model were found to align strongly with the patterns observed in the classic model, suggesting that the stochastic elements were effectively incorporated. Any notable disparities between the predictions of the stochastic model and the classic model were thoroughly examined to identify and address any potential issues with the stochastic modelling process. The comparison will be shown on a graph, and the expectations for the comparison, based on sources, will be discussed.

Figure 23: Comparison of Predictions from Gas Stochastic Scenario and Reference Scenario



The graph above depicts the annualised costs for fixed (FIX) and new (INV) investments under two scenarios: the Reference Scenario and the Stochastic Gas Prices Scenario. The comparison emphasises the effect of stochastic petrol prices on model predictions.

A sensitivity analysis was performed after comparing the results from the reference scenario and the stochastic scenario. The objective of this analysis was to see how minor changes in gas prices, a critical input parameter, affected the model's output in both scenarios. This test was created specifically to validate our assumption that the stochastic scenario would be more sensitive to changes in gas prices than the reference scenario.

Table 26: Sensitivity Analyses for TIMES-CZ Model with Stochastic Gas Prices

Model	INV	INVX	FIX	VAR	VARX
Regular model	0.2781	-0.4695	0.0437	0.04	0.4534
Stochastic model	0.2783	-0.4716	0.044	0.0394	0.4521

The sensitivity analysis findings demonstrate that the model's output exhibited a relatively stable behaviour when exposed to slight variations in petrol prices, both in the reference and stochastic scenarios. This implies that the model has successfully integrated the probabilistic characteristics of these parameters. As expected, the stochastic scenario exhibited a slightly greater degree of sensitivity to fluctuations in petrol prices when compared to the reference scenario. The observation validates our initial hypothesis and emphasises the significance of incorporating stochastic elements in the modelling of uncertain parameters, such as petrol prices.

In summary, the utilisation of stochastic scenarios within the TIMES-CZ model has demonstrated its benefits in addressing the inherent uncertainty associated with the future trajectory of crucial variables. Notwithstanding the difficulties inherent in interpreting the results of stochastic scenarios and the need for additional data and computational resources, the advantages of adopting a more robust and comprehensive methodology for modelling uncertainty in the energy sector surpass these challenges. The observations made in the model provide confirmation that stochastic scenarios require more computational resources compared to classic scenarios, which is consistent with our initial expectations.

6. Conclusion

The main objective of this thesis was to supplement the TIMES-CZ model with stochastic scenarios relating to the war period and other recent extreme events, which brought a large number of unanticipated occurrences that significantly impacted not only the energy sector but also the entire economy (KPMG, 2022). Two stochastic variables were added to the model, and the results were observed.

It is important to mention that, while not directly used by policymakers or other institutions, the TIMES-CZ model is frequently utilised by the Environmental Centre at Charles University. This centre generates forecasts using this model for various purposes, which are then provided to ministries and other institutions. The objective of this thesis was to introduce a new feature to the TIMES-CZ model that could enhance its ability to account for future uncertainties. The benefit of this feature is that it could extend the model's accuracy over a longer period compared to a deterministic model, which may require updates in line with the evolving state of the energy and economic sectors.

Subsequently, a sensitivity analysis was conducted, which showed that the model's output remained relatively stable when subjected to minor variations in EUA and gas prices in both scenarios. This suggests that the model has successfully incorporated the random characteristics of these parameters. As predicted, the stochastic scenario was slightly more sensitive to changes in EUA and gas prices than the reference scenario.

The TIMES-CZ model, while not directly employed by policymakers and other institutions, is extensively utilized by the Environmental Center at Charles University. This center generates forecasts using this model for various purposes, which are then provided to ministries and other institutions. Building upon the work of Rečka, Máca, & Ščasný (2023), the objective of this thesis was to introduce a new feature to the TIMES-CZ model that could enhance its ability to account for future uncertainties. This enhancement has the potential to assist Czech decision-makers by providing them with a wider variety of potential scenarios and outcomes for their strategic decision-making processes regarding the nation's energy future. The benefit of this feature is that it could extend the model's accuracy over a longer period compared to a deterministic model, which may require updates in line with the evolving state of the energy and economic sectors. This strategy can enhance the energy system's resilience and adaptability, enabling it to respond effectively to changing conditions and unpredictability.

Nevertheless, it is crucial to recognise the constraints of this study. The inclusion of stochastic elements to the TIMES-CZ model increases its computational complexity significantly. The inclusion of stochastic elements to the TIMES-CZ model increases its computational complexity significantly. Nevertheless, stochasticity integration can assist to represent uncertainties, it cannot eliminate them. Unexpected events and developments can have an impact on the energy system if they are not adequately accounted for in the model (Loulou, & Lehtila, 2016).

A significant limitation stems from the TIMES-CZ model's inability to simultaneously incorporate stochastic elements for gas prices and EUA prices, primarily due to their strong correlation. However, in this work, both variables were added to the model separately (for separate scenario), so this is not a problem in this case. In the future, however, this provides an opportunity to solve this problem and add them simultaneously for even better prediction with uncertainty. In addition, other variables with stochasticity can be added in the future (Zakaria et al., 2021).

As a result, it is critical to continuously evaluate and improve the model using current data. This study's external analysis provided valuable insights into the potential trajectory of gas prices and carbon permit costs. The study conducted a comprehensive analysis that reveals the majority of these two variables' potential paths until 2030. The study chose to conduct an external analysis outside of the TIMES-CZ model to ensure that the results were not influenced by its settings. This method ensures that the stochasticity introduced into the model accurately reflects real-world uncertainties, rather than being shaped by the model's inherent assumptions and constraints.

Future research could expand on this study by exploring alternative methods for incorporating uncertainty into energy system models, such as robust optimisation or scenario analysis (Pflug & Pichler, 2016). Further research could investigate the impact of various sources of uncertainty on the model's outcomes, such as policy changes or technological advancements (Kotzur et al., 2018).

Finally, this study adds to our understanding of the role of stochasticity in the TIMES-CZ model and its potential impact on Czech energy planning and policy. It emphasises the significance of incorporating uncertainty into energy system modelling and sets the foundations for future research in this area. This research has shown that introducing stochasticity into the TIMES-CZ model can influence the model's results and thus the decision-making process in Czech energy planning and policy.

List of References

1. *Czech Ministry of Industry and Trade. (2019). National Energy and Climate Plan.*
2. *Eurostat. (2018). Energy statistics - an overview.*
3. *Després, J., Hadjsaid, N., Criqui, P., & Noirot, I. (2015). Modelling the impacts of variable renewable sources on the power sector: Reconsidering the typology of energy modelling tools. Energy, 80.*
4. *Pfenninger, S., Hawkes, A., & Keirstead, J. (2014). Energy systems modeling for twenty-first century energy challenges. Renewable and Sustainable Energy Reviews, 33.*
5. *Pflug, Georg & Pichler, Alois. (2016). Multistage Stochastic Optimization. 10.1007/978-3-319-08843-3.*
6. *Kotzur, L., Markewitz, P., Robinius, M., & Stolten, D. (2018). Impact of different time series aggregation methods on optimal energy system design. Renewable Energy, 117.*
7. *Pfenninger, S. (2017). Dealing with multiple decades of hourly wind and PV time series in energy models: A comparison of methods to reduce time resolution and the planning implications of inter-annual variability. Applied Energy, 197.*
8. *Rečka, L., & Ščasný, M. (2016b). Impacts of carbon pricing, brown coal availability and gas cost on Czech energy system up to 2050. Energy. https://doi.org/10.1016/j.energy.2015.12.003*
9. *Koutroulis, A. G., & Sarno, L. (2021). The COVID-19 pandemic and exchange rate forecasting. Journal of International Money and Finance, 115, 102381.*
10. *International Energy Agency (IEA). (2020). Global Energy Review 2020. IEA, Paris.*
11. *Dmytrów, K., Landmesser, J., & Bieszk-Stolorz, B. (2021). The impact of COVID-19 on the natural gas market in Poland. Energies, 14(4), 1038.*
12. *Baker Institute. (2020). The Impact of COVID-19 on Energy Markets. Baker Institute for Public Policy.*
13. *International Energy Agency (IEA). (2021). Gas Market Report - Q4 2021.*
14. *European Central Bank. (2021). Economic Bulletin Issue 6 / 2021.*
15. *Linares, P., & De la Hoz, J. (2020). How 100% renewable energy will use much less of the world's land than today. Energy Policy, 134, 111132.*
16. *European Commission. (2014). European Energy Security Strategy.*

17. *KPMG. (2022). Russia-Ukraine conflict: Impact on the energy sector.*
18. *International Renewable Energy Agency (IRENA). (2019). Global Energy Transformation: A Roadmap to 2050.*
19. *United Nations Framework Convention on Climate Change. (2015). Paris Agreement.*
20. *European Commission. (2018). A Clean Planet for all: A European strategic long-term vision for a prosperous, modern, competitive, and climate-neutral economy.*
21. *European Commission. (2019). The European Green Deal.*
22. *von der Leyen, U. (2019). A Union that strives for more: My agenda for Europe. Political guidelines for the next European Commission 2019-2024.*
23. *LaBelle, M., Tóth, T., & Szép, B. (2022). The Fit for 55 package: Implications for Central and Eastern Europe. Energy Policy, 161, 112620.*
24. *Goldstein, G. A., Loulou, R., Kanudia, A., Lehtilä, A., & Remme, U. (2021). Documentation for the TIMES Model: Part I. Energy Technology Systems Analysis Programme.*
25. *Loulou, R., & Lehtila, A. (2016). Stochastic Programming and Tradeoff Analysis in TIMES. KANLO Consultants, France; VTT, Finland.*
26. *Morgan, M. G., & Henrion, M. (1990). Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis. Cambridge University Press.*
27. *Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., ... & Tarantola, S. (2008). Global Sensitivity Analysis: The Primer. John Wiley & Sons.*
28. *Kanudia, A., & Loulou, R. (1998). Robust responses to climate change via stochastic MARKAL: The case of Quebec. European Journal of Operational Research, 106(1).*
29. *Loulou, R., & Kanudia, A. (1999). Minimax regret strategies for greenhouse gas abatement: Methodology and application. Operations Research Letters, 25(5).*
30. *Loulou, R., & Labriet, M. (2008). ETSAP-TIAM: the TIMES integrated assessment model Part I: Model structure. Computational Management Science, 5(1-2)..*
31. *Kanudia, A., & Loulou, R. (1999). Robust responses to climate change via stochastic MARKAL: the case of Quebec. European Journal of Operational Research, 106(1).*
32. *Dantzig, G. B. (1955). Linear programming under uncertainty. Management science, 1(3-4).*

33. *Razali, N. M., & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. Journal of statistical modeling and analytics, 2(1).*
34. *Schwarz, G. (1978). Estimating the dimension of a model. The annals of statistics, 6(2).*
35. *Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica: Journal of the Econometric Society.*
36. *Korkmaz, P., Schmid, D., & Fahl, U. (2021). Incorporating uncertainties towards a sustainable European energy system: A stochastic approach for decarbonization paths focusing on the transport sector. Energy Strategy Reviews, 38. https://doi.org/10.1016/j.esr.2021.100707*
37. *Loulou, R., Goldstein, G., Kanudia, A., Lettila, A., Remme, U. (2016). Documentation for the TIMES Model - Part III. Reviewed by Wright, E., Giannakidis, G., Noble, K. Energy Technology Systems Analysis Programme.*
38. *Vera Barberán, José. María. (2020). Adding external factors in Time Series Forecasting: Case study: Ethereum price forecasting. Degree Project in Information and Communication Technology (2nd cycle, 30 credits). Stockholm, Sweden.*
39. *Babonneau, F., Haurie, A., Loulou, R., & Vielle, M. (2012). Combining Stochastic Optimization and Monte Carlo Simulation to Deal with Uncertainties in Climate Policy Assessment. Environmental Modeling & Assessment, 17(1), https://doi.org/10.1007/s10666-011-9275-1*
40. *Ioannou, A., Fuzuli, G., Brennan, F., Yudha, S., & Angus, A. (2019). Multi-stage stochastic optimization framework for power generation system planning integrating hybrid uncertainty modelling. Energy Economics, 80.*
41. *Rečka, L., Máca, V., & Ščasný, M. (2023). Green Deal and Carbon Neutrality Assessment of Czechia. Energies, 16, 2152. https://doi.org/10.3390/en16052152*
42. *Commons, H. Capstones, H. (2017). Application of Time Series Models (ARIMA, GARCH, and ARMA-GARCH*
43. *Boersen, A., & Scholtens, B. (2014). The relation between energy and emission markets: An empirical analysis. Energy, 74(C). https://doi.org/10.1016/j.energy.2014.07.024*

44. *Seljom, P., Kvalbein, L. & Ortiz, M. M. (2021). Stochastic modelling of variable renewables in long-term energy models: Dataset, scenario generation & quality of results. Energy.*
45. *Buxton, R. (2008). Simple Linear Regression: Reliability of predictions.*
46. *Zakari, A., Ismail, F. B., Lipu, M. S. H., & Hannan, M. A. Uncertainty models for stochastic optimization in renewable energy applications.*
47. *Švec, J. (2020). The Impact of increased temporal detail in long-term dynamic energy system model with an increasing share of volatile renewable energy sources. Bachelor thesis. Charles University, Faculty of Social Sciences, Institute of Economic Studies. Supervisor: Mgr. Lukáš Rečka, Ph.D.*

List of Data Sources

1. *Power Exchange Central Europe (PXE). Gas, coal and power pricing. Available at: https://pxe.cz/cs/komoditni-trh*
2. *ČEPS. Export/import dynamics, Czech electricity output (by source and total), and Czech consumption. Available at: https://www.ceps.cz/cs/data*
3. *Eurostat. EUA prices, CO2 emissions, and average temperature. Available at: https://ec.europa.eu/eurostat/web/main/data/database*
4. *Trading Economics. New vehicle registration, petrol prices, taxes, interest rates, inflation rates, and crude oil prices. Available at: https://tradingeconomics.com/czech-republic/indicators-eurostat-data.html*