Short Abstract

This thesis focuses on the complexity of the promise version of Constraint Satisfaction Problem (CSP) and its variants.

The first study concerns the Promise Constraint Satisfaction Problem (PCSP), which extends the traditional CSP to include approximation variants of satisfiability and graph coloring. A specific PCSP, referred to as finding a valid Not-All-Equal solution to a 1-in-3-SAT instance, has been shown by Barto [LICS '19] to lack finite tractability. While it can be reduced to a tractable CSP, the latter is necessarily over an infinite domain (unless P=NP). We say that such a PCSP is not finitely tractable and we initiate a systematic study of this phenomenon by giving a general necessary condition for finite tractability. Additionally, we characterize finite tractability within a class of templates.

In the second study, we focus on the CSP in the context of first-order logic. The fixed-template CSP can be seen as the problem of deciding whether a given primitive positive first-order sentence is true in a fixed structure (also called model). We study a class of problems that generalizes the CSP simultaneously in two directions: we fix a set $\mathcal{L}$ of quantifiers and Boolean connectives, and we specify two versions of each constraint, one strong and one weak (making the promise version). Given a sentence which only uses symbols from $\mathcal{L}$, the task is to distinguish whether the sentence is true in the strong sense, or it is false even in the weak sense. We call these problems Promise Model Checking Problems, and they are a generalization of Model Checking Problems. We classify the computational complexity of Promise Model Checking Problems for the existential positive equality-free fragment of first-order logic, i.e., $\mathcal{L} = \{\exists, \wedge, \vee\}$, and we prove some upper and lower bounds for the positive equality-free fragment, $\mathcal{L} = \{\exists, \forall, \wedge, \vee\}$.

In addition to the aforementioned studies, we introduce the framework of the Left-Hand Side Restricted PCSP (a generalization of the Left-Hand Side Restricted CSP) and study its complexity.