DEPARTMENT OF
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SCIENCE

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Report on the doctoral thesis of Kristina Asimi entitled *Promises in Satisfaction Problems*

The doctoral thesis of Kristina Asimi deals with promise variants of the Constraint Satisfaction Problem (CSP). The CSP is a fundamental topic in theoretical computer science, with interesting connections to various branches of mathematics. The promise CSP (PCSP) is one of the most exciting generalisations of the CPS.

Summary

The thesis consists of a brief introduction to CSPs and basic notions used in the study of the computational complexity of CSPs, and three contributed chapters.

The first contributed chapter deals with so-called finite tractable CSPs, which have connections to universal algebra. In my view, this is the most significant contribution of the thesis. The work extends an important result of Barto and introduces a technique for establishing that PCSPs are not finitely tractable. In detail, a PCSP template (A,B) is finitely tractable if there is a finite structure C such that C can be sandwich homomorphically between A and B, and CSP with the template C is solvable in polynomial time. The introduced technique (of ``doubly cyclic polymorphisms'') is then used to classify the basic types of Boolean symmetric PCSPs with disequalities with respect to finite tractability.

The second contributed chapter deals with the problem of promise model checking, which has connections to logic. The chapter builds on previous work in the non-promise setting. In addition to promise CSPs and promise quantified CSPs, there are two other interesting fragments of first order logic with respect to the considered problem. The chapter gives a complexity classification for one of them and initiates an exploration of the second. The results here are not terribly complicated but are not trivial either, requiring a generalisation of techniques used in the non-promise setting. Moreover, this chapter has opened a new, interesting line of work on the complexity of the equality-free promise model checking problem, which I expect to be explored further by the research community.

The third contributed chapter deals with promise CSPs parameterised by the class of left-hand side structures, also called promise CSPs seen from the other side. This problem has connections to graph theory. After reviewing a celebrated result of Grohe, which established a fixed-parameter reduction from the problem of k-Clique in graphs to CSPs of unbounded treewidth modulo homomorphic equivalence, the chapter gives a simple but clever generalisation of Grohe's reduction. Thus, it provides a condition that can be used to establish hardness of (not only but most importantly) promise CSPs parameterised by the class of left-hand side pairs of structures. The chapter is finished with a discussion on the relationship of the studied problem, which remains open, and the problem of approximating k-Clique, a well-known problem in the approximation community. While the full resolution of the

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computational complexity of the studied problem appears to require more substantial work, the presented chapter gives solid contribution to this problem.

Presentation

The thesis is written very well with essentially no grammatical or other typographical issues. I have not identified any mathematical problems with the proofs and believe that the results are correct.

Minor corrections

I have only a few small comments on the presentation, mostly nonessential things that could be changed for the final version (if such a thing is possible).

- There is a repetition of the basic definitions and notions in the Introduction, then in Section 1.1 (Introduction to Chapter 2), and then again in Section 1.2 (Preliminaries to Chapter 2). I understand that Chapter 2 was published as a self-contained paper, but it would be more appropriate to remove some of these repetitions. Similarly, the first half of Section 2.2 is covered in Chapter 1.

- The references should include the final, published versions of the important papers. For instance, two seminal papers in this area (BG'18, and BKO'19 + B'19) have published journal versions (BG SICOMP'21 and BBKO JACM'21) and those should be used and cited. Similarly, the references of Martin and company cited in Chapter 2 might benefit from a consolidation.

- The role of = should be mentioned (or a forward reference given) after Definition 3 on p.4 (second half of p.6 is too late). There is also a small inconsistency later, namely talking about PCSP as a model checking with \exists, \wedge and = in p.39 but only with \exists and \wedge on p.41.

- "in this paper" should be "in this chapter" on p.14, p.30, p.38 (twice), p.39, and p.40.

- A brief remark on why in Definition 30 on p.41 it suffices to take a superset of S and a subset of T would be helpful.

- The correct opening quotation marks are not used on several occasions.

- ``polynomial-time solvability'' would be more precise and in my view better than ``standard tractability'' on p.11.

- The last paragraph on p.13 could be made a bit clearer.

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- The proof of Lemma 4 on p.14 could be more than sketched given this is a thesis (with no page limit).

Evaluation

The thesis contains novel and interesting results presented in a scholarly manner.

The author of the thesis proved to have an ability to perform research and to achieve scientific results. I recommend that the thesis should be accepted as a PhD thesis.

Oxford, 21st July 2023 Stanislav Živný