The present thesis is focused on the study of properties of function spaces containing measurable functions, and operators acting on them. It consists of four papers.

In the first paper, we establish a new characterization of the set of Sobolev functions with zero traces via the distance function from the boundary of a domain. This characterization is innovative in that it is based on the space $L^{1,\infty}_a$ of functions having absolutely continuous quasinorms in $L^{1,\infty}$.

In the second paper, we investigate properties of certain new scale of spaces governed by a functional involving the maximal nonincreasing rearrangement and powers. Motivation for studying such structures stems from a recent research of sharp Sobolev embeddings into spaces furnished with Ahlfors measures.

In the third paper, we extend discretization techniques for Lorentz norms by eliminating nondegeneracy restrictions on weights. We apply the method to characterize general embeddings between classical Lorentz spaces.

In the fourth paper, we characterize triples of weights for which an inequality involving the superposition of two integral operators holds. We apply results from the third paper to avoid duality and to obtain thereby a general result.