Weak Solutions to Mathematical Models of the Interaction between Fluids, Solids and Electromagnetic Fields

Abstract

We analyze the mathematical models of two classes of physical phenomena. The first class of phenomena we consider is the interaction between one or more insulating rigid bodies and an electrically conducting fluid, inside of which the bodies are contained, as well as the electromagnetic fields trespassing both of the materials. We take into account both the cases of the fluid being incompressible and the fluid being compressible. In both cases our main result yields the existence of weak solutions to the associated system of partial differential equations, respectively. The proofs of these results are built upon hybrid discrete-continuous approximation schemes: Parts of the systems are discretized with respect to time in order to deal with the solution-dependent test functions in the induction equation. The remaining parts are treated as continuous equations on the small intervals between consecutive discrete time points, allowing us to employ techniques which do not transfer to the discretized setting. Moreover, the solution-dependent test functions in the momentum equation are handled via the use of classical penalization methods.

The second class of phenomena we consider is the evolution of a magnetoelastic material. Here too, our main result proves the existence of weak solutions to the corresponding system of partial differential equations. Its proof is based on De Giorgi's minimizing movements method, in which the system is discretized in time and, at each discrete time point, a minimization problem is solved, the associated Euler-Lagrange equations of which constitute a suitable approximation of the original equation of motion and magnetic force balance. The construction of such a minimization problem is made possible by the realization that, already on the continuous level, both of these equations can be written in terms of the same energy and dissipation potentials. The functional for the discrete minimization problem can then be constructed on the basis of these potentials