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A report on the thesis of Jan Scherz

Dear Prof. Latoschik, dear Prof. Rokyta, dear members of the committee,

With this letter, I would like to report on the thesis of Jan Scherz that was submitted to be accepted as a Ph.D.-thesis at the University of Würzburg and the Charles University, Prague within a *cotutelle de thèse*-agreement as well as his performance within the PhD.-study.

The thesis of Jan Scherz is concerned with the mathematical analysis of several systems of PDEs in fluid-structure interaction combined with electromagnetic effects. As I will report later in more detail, I believe that the thesis contains several new and interesting results that Jan achieved both in collaboration and independently.

The thesis consists of six chapters and an appendix. After a clear introduction overviewing the main results, chapter 2 is devoted to mathematical modelling while the bulk of the mathematical analysis is concentrated to chapters 3-5.

In chapter 2 of the thesis, Jan first introduces the modelling of the movement of a rigid body, that is an insulator, in an electrically conducting fluid. Here, Jan starts from the (compressible) Navier-Stokes equation for the fluid, the balance of linear and angular momentum for the solid and the full system of Maxwell's equations for both the fluid and the solid. Additionally, an equality of velocities and tractions is assumed at the fluid-solid boundary and the Lorentz force is included as a driving force in the Navier-Stokes equation. From this general model, Jan derives the one which will later enter the mathematical analysis: He first, under the assumption of sufficient smoothness, deduces appropriate interface conditions for the magnetic fields on the fluid-solid boundary and later (heuristically) performs the magnetohydrodamic approximation in which he assumes that the typical velocities achieved by the fluids are much smaller than the speed of light. I appreciate this work since it gives the system for analysis into a broader context a proves Jan's ability to work even on topics on the boundary of physics and mathematics. Though these results have not been published yet, as far as I know, Jan and Anja Schlömerkemper are preparing them for a submission.

In chapter 3 Jan then proves existence of weak solutions to the system of equations derived in the previous chapter, under the additional assumption of incompressibility, up until collision. This assumption is then dropped in chapter 4, where also existence of weak global solutions is shown. While in the absence of electromagnetic effects the existence of weak solutions for fluid-structure interaction with a rigid body is by now standard, the inclusion of electromagnetic effects is new and interesting also from an analytical point of view. Namely, the electromagnetic fields are in principal global and "exist" in the whole space, thus particularly are able to travel through the solid material. However, the assumption that the solid in an insulator forces the addition assumption that B is curl-free on the solid domain. The restriction is then reflected in the choice of test functions that are also curl-free on the solid domain and thus dependent on the solution itself. This is a standard scenario in fluid-structure interaction, but here it is carried across several weakly formulated equations. Due to the restrictions at hand, the proof is then performed by introducing several levels of approximation with the fore-most one being a backward Euler time-discretization by the Rothe method. Introducing this time-discretization allows to control the position of the solid body at any time while also de-coupling the system. The limit passage is then based on a series of careful a-priori estimates and a suitable choice of an approximating sequence. This result has been published as a joint publication of Jan and his advisors Anja Schlömerkemper, Šárka Nečasová and myself in *SIAM J. Math. Anal.*. While I am sure that during this work Jan has strongly profited from the advise obtained from Šárka Nečasová, who is a distinguished expert on fluid-structure interaction problems, and Anja Schlömerkemper with her rich expertise on modelling of electromagnetic effects, I can tell that Jan was the main driving force in carrying out the formal argument of the proof.

In chapter 4 then, Jan generalizes the obtained results to the compressible case. This however comes with an additional difficulty, as now the restrictions on the magnetic induction would call for a time-discretization while the proof of the positivity of the density would rather call for a time-continuous scheme. Jan solves this problem by introducing a hybrid approximation that can cope with both these issues at the same time. This problem and its solutions have been proposed and solved completely independently by Jan and resulted in a his single-authored publication in *Math. Nachr.*

Lastly, in chapter 5 Jan tackles a new and independent, albeit related, problem to the previous two. Namely, he goes on to propose and analyse a model in micromagnetism for a magnetic viscoelastic solid. He works in the so-called *Langangian-Eulerian* regime, where the deformation of the solid is Lagrangian while the magnetic variables are Eulerian. Such models have been proposed and analyzed in the literature already, but, in my opinion, Jan's approach is still very interesting and, in particular, paves the road to be combined with a fluid-structure interaction problem as well as the dynamic case under minimal addition of regularizing terms. In particular, Jan designs a time-stepping variational scheme based on the DeGiorgi time-incremental minimization that minimizes the overall energy plus dissipation at the same time. Care, however, is at place when designing the cost function as the resulting Euler-Lagrange equations should be a suitable approximation of the balance of momentum as well as transport of the magnetization. I believe that the construction proposed in Jan's work is a nice piece of work that could be used also in numerics and is also interesting from a modelling point of view. In my impression, Jan has carried out this work quite independently and in a very short time.

Overall, I rate the thesis of Jan to be of a very high quality, I also appreciate the clear presentation and the clarity and detail of the arguments included. Overall, in my impression, it shows Jan's abilities as a mathematician and his particular attention to detail and correctness. His understanding of the matter is not just superficial but he truly "digs" into the details, which then allows him to argue accurately. This deep understanding is evident in the thesis at many places.

Having co-advised Jan over the past four years together with Šárka Nečasová and Anja Schlömerkemper, I can comment that I had the same impression from the collaboration with him. Due to the restrictions of the pandemic (and the cotutelle arrangement), our joint meetings have been mostly virtual, where Jan always showed progress from meeting to meeting and prepared and asked questions that demonstrated his understanding of the material. He clearly is a passionate mathematician with a particularly good understanding of all the technical difficulties.

I believe that he also highly profited from the embedding into the institutions at which he was able to learn and study. Namely, in Prague he became part of the working group of Šárka Nečasová and profited from the seminars and possibility of discussions there, as this group is really one of the world-leading ones in fluid-structure interaction. Within this group. Jan was successful to work on topics that may be related to the ones in the thesis but reach beyond its scope. Here, I would like to highlight the paper by Šárka Nečasová, Justyna Ogorzaly and Jan on the compressible Navier-Stokes equation with slip boundary conditions published in *Z. Angew. Math. Phys.*. Also, I believe, that without the background of this group his single-authored work on the fluid structure interaction of an insulator with a compressible electrically conducting fluid would not be possible.

In Würzburg, on the other hand, he also had the chance to discuss and work with not only Anja Schlömerkemper herself but also her working group that has worked very successfully on problems of magnetodynamics over the past years when producing many well-received results.

The Charles University, then, offers a wide selection of seminars and lectures within its PhD study program on mathematical analysis and beyond that Jan visited and I have been able witness that his knowledge of partial differential equations and particularly also calculus of variations has substantially improved over the years.

I believe that all the contributions in the thesis, the fact that Jan is able to suggest, carry out and also successfully publish mathematical work both in collaboration but also by himself, both within the work on his thesis but also beyond more than illustrate his capability of independent scientific work. Thus, I would like to recommend his thesis to be accepted for defence without reservation. In view of the obtained results I would like to propose the grade *magna cum laude*.

Respectfully yours,

Barbora Benešová