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Report on Jan Scherz' Dissertation Thesis
"Weak Solutions to Mathematical Models of the Interaction between Fluids, Solids and Electromagnetic Fields"

Dear Sir or Madam,

Jan Scherz' dissertation thesis is concerned with the mathematical analysis of three nonlinear systems. The first two model the dynamics of insulating rigid bodies immersed in an electrically conducting incompressible (1) and compressible (2) fluid, respectively. The third system (3) models the dynamics of a magnetoelastic structure. In each of these cases, a first existence result is proved.
The analysis presented here combines the challenges of PDE theory and calculus of variation approaches to mathematical fluid dynamics, elastic material deformation and electromagnetic dynamics. The interplay of two or more of these problems creates further modeling and mathematical issues. The results as well as their presentation in this thesis show that Jan has not only mastered essential aspects of these theories and a number of technical difficulties associated to the coupled systems, but is also able to creatively combine and newly develop mathematical tools in this broad context.

Starting point of the thesis is a comprehensive **Introduction**. It has a clear structure, uses illustrative figures, and care is taken to address the state of the art and the mathematical difficulties of projects (1) - (3) in a general language. In particular, Jan explains the approaches he chose to each problem and argues for the necessity to deviate from known techniques. Potential applications as in endoscopic medicine and civil engineering are mentioned and referenced. The assumptions of continuum mechanical modeling used in the derivation of models (1) - (3) are briefly and clearly exposed in Section 1.3.

Chapter 2 is devoted particularly to the electromagnetic part of the equations of evolution of the electric-fluid-rigid interaction. The modeling and the simplification by the magnetohydrodynamic approximation derived for this problem appear to be additional original contributions of the thesis.

In Chapters 3 - 5, the thesis contains the proofs of existence of weak solutions for models (1) - (3), respectively.

In the case of the insulating rigid bodies in an incompressible electrically conducting fluid treated in **Chapter 3**, the solutions are of Leray-type concerning the fluid and the techniques involve a Brinkmann penalization for dealing with the presence of the rigid bodies and the Rothe method for solving the induction equations. The tricky part is the strong coupling of these components. In particular, this leads to solution-dependent spaces of test functions that can only be found iteratively. In order to deal with this by a partial decoupling, Jan first constructs and solves a (mostly) time-discretized system. Time-dependence is kept in the characteristic function that provides the rigid bodies' positions. The reason is that in this way, the property of being a characteristic function is transported and thus retained. As a small gap to the best possible result for this model, the magnetic induction is assumed to be equal for both fluid and rigid bodies, in order to improve the regularity of the magnetic field.

The extension of the first existence result to the case of a compressible electrically conducting fluid is shown in **Chapter 4**. Here, the ideas from Chapter 3 are combined with the techniques developed by Feireisl [43] for compressible fluid-rigid interaction. The Brinkman penalization is replaced by an infinite-viscosity limit for re-discovering rigid bodies after solving a fluid equation on the full domain. Even compared to the results of Chapter 3, this work is again original. The strong electric-fluid-rigid coupling has to be dealt with in a very specific manner, and there is an informed and clear exposition of how and why this is done (Section 4.2). The proof is technically involved, including five different approximation levels, but it is written in a way that exposes the key ideas. There is also a recommendable recollection of the main ideas generally used for compressible fluid flow (Sections 4.7 and 4.8).

The results in **Chapter 5** concern the evolution of a magnetoelastic material structure. The modeling as a generalized standard material via appropriate energy and dissipation potentials for elastic deformation and magnetization is taken from recent previous work by Benesovà, Forster, Liu and Schlömerkemper [6, 48]. It alternates between Lagrangian (elasticity) and Eulerian (magnetization) configurations. Solutions are constructed via a De Giorgi minimizing movement scheme — roughly speaking, in each time step, the descent in energy is optimized against the dissipation potential. In particular, there is a time-discrete version of the energy-energy dissipation balance. The deformation determinant is modelled and proven to remain everywhere positive via a Ciarlet-Necas condition. As far as I understand, the existence result remains local in particular in the sense that the deformation is not sufficiently spatially regular at the boundary for providing the full elliptic regularization on the magnetization that would be needed to close the loop. More generally, this seems to be one of the most difficult problems in coupling elasticity with other effects. It is well known in the fluid-elastic literature but it is far from clear how corresponding strategies would work here. Inertia effects are not yet included in the modelling so that the dynamics are quasi-static. It will be exciting to see both issues dealt with in future work. The thorough discussion, new methods and the result presented here are an important starting point.

While auxiliary or complementary results are outsourced into the **Appendix, Chapter 6** contains a short summary, evaluation and an outlook on a number of follow-up projects for the thesis. In particular, treating the full version of a fluid coupled to conducting and elastically deformable objects seems like a natural extension, but also a very challenging problem. Already there are several applications of the present results for the three subsystems and more generally for the mathematical methods developed in the thesis. In particular, there are many situations of fluid-structure interaction coupled to the dynamics of additional tracers or additional mechanisms. The approximation schemes and corresponding estimates obtained in this thesis can then be considered, modified and applied to provide existence and stability theorems. Neighbouring mathematical fields of research that will be impacted are numerical analysis and control theory.

To summarize, the thesis contains pioneering results on the modeling and analysis of electro-fluid-solid interaction and magnetoelasticity. The scope of the thesis is impressive: Jan has contributed to modelling, discretization, variational and PDE analysis of these models. He has also treated the difficulties of strongly interacting systems that need to be decoupled as much as possible (by analysis/approximation tricks), but not further. The techniques developed in this context may be adapted to a range of further applications.
Jan has also provided a thorough and extensive presentation of his findings. He has added instructive text to help the reader through the many technicalities of the subject and it seems he has massaged these technicalities for a while not only so as to give the best possible results but also to improve their presentation.

Yours sincerely,

Prof. Dr. Karoline Disser