The main aim of this text is to present and investigate some basic arithmetical functions and relations with regard to being recursive in a countable non-standard model of Peano arithmetic, PA for short, or some weaker fragment, like $I\Delta_0$ or $I\Sigma_1$, of PA.

In PART I, we present a known result called Tennenbaum's theorem. It states that every non-standard model M of PA with domain $\mathbb{N}$ can have neither $+^M$ nor $\times^M$ recursive. Moreover, we present the case for + in a strengthened version for $I\Delta_0$, which is due to K. McAloon. To show that not everything is lost, we also present a well know result stating that $<$ and the successor function can be simultaneously recursive in some non-standard model of PA with domain $\mathbb{N}$.

In PART II, we make our own investigation into the questions related to whether there can be a non-standard model of PA s.t. $x \operatorname{div} y$, the quotient function, and $x \bmod y$, the remainder function, are recursive in it. Furthermore, we often restrict y to some *standard number n*. To give a *non-exhaustive* list of problems we have solved, we showed that there can be no non-standard model of PA with both $x \operatorname{div} n$ and $x \bmod n$ recursive. Furthermore, there can be no non-standard model of $I\Sigma_1$ with $x \operatorname{div} y$ recursive. On the other hand, $x \operatorname{div} n$ and $x \bmod n$ can be separately recursive in a non-standard model of PA.