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Report for "Two-phase scheduling with unknown speeds"

Dear Prof. Klazar,
Dear Colleagues,

This is a report for "Two-phase scheduling with unknown speeds", the thesis submitted by Josef Minařík for obtaining his Master's degree. The thesis focuses on a scheduling problem with unknown machine speeds, where an algorithm has to partition a set of jobs into b groups (or bags) without knowing the speeds of m related machines. After the machine speeds are revealed, each group has to be assigned to one machine. The objective is to minimize the makespan, the latest machine completion time, and the performance of an algorithm is evaluated by calculating the robustness factor, the worst-case ratio between its makespan and that of an optimal algorithm that assigns jobs directly.

Sand: There are infinitely many infinitesimal (identical) jobs that need to be scheduled. Previously, a $\frac{m^m}{m^m-(m-1)^m}$-robust algorithm was known for the case $m = b$, i.e., the number of machines is equal to the number of groups the algorithm generates. Making a clever observation on the relation between group sizes, the robustness factor, and a greedy bag-to-machine assignment, the previous algorithm and its analysis are generalized, simplified and strengthened to give a $\frac{m^b}{m^b-(m-1)^b}$-robust algorithm for $b \geq m$. Moreover, by giving a matching lower bound, it is shown that the algorithm is best possible from an information-theoretic point of view.

Pebbles: Here, the size of each job is at most p-times the average machine load. In contrast to the previous chapter, the job sizes can be different. This particular problem variant has not been studied before, but it is a natural intermediate step between the setting with infinitely many infinitesimal jobs and the general setting. Building on the ideas and results of the previous chapter, a $\left(\frac{m^b}{m^b-(m-1)^b} + p\right)$-robust algorithm is developed and analyzed.

Bricks: Here, the input consists of a set of n identical jobs, whose sizes do not have to be bound by p-times the average machine load. Since the average machine load is $\frac{m}{n}$, the previous chapter on pebbles immediately implies a $\left(\frac{m^b}{m^b-(m-1)^b}+\frac{m}{n}\right)$-robust algorithm. Observing that also the hindsight optimum has to assign an integral number of jobs to each machine, an algorithm for small average load is developed for the special case $m=b$. Combining this algorithm with the one for pebbles, a 1.6-robust algorithm is given when $m=b$, which improves upon the best previously known algorithm with robustness factor 1.8.

Rocks: This chapter considers the most general problem variant with jobs of arbitrary sizes. Previously, a $\left(1+\frac{m-1}{m}\right)$-robust algorithm was known if $m=b$. Generalizing this result, a $\left(1+\frac{m-1}{b}\right)$-robust algorithm is developed and analyzed.

Evaluation: This thesis reports strong results for a particular scheduling problem. New and clever insights are presented and combined with techniques of a previous paper to generalize and strengthen known algorithms and their analyses as well as design and analyze new algorithms. The results are publishable and of great interest to the scientific community as they (partially) answer open questions from previous publications. While the results are strong and the proofs are correct or can be easily fixed when incorrect, the writing is lacking at some places, in particular in some proofs: In the proof of Lemma 2.7, the calculations on top of Page 15 are not correct, and further, the proof is missing an argumentation on why the shown inequalities prove the lemma statement. The proof of Claim 4.15 does not explain what the relative brick surplus is useful for, and the proof is unfortunately split between the main part and the appendix, making it unnecessarily difficult to verify. Additionally, there are several avoidable typos, e.g., technieques and existance (twice) on Page 36 or the usage of both $\bar{\rho}(m,b)$ and $\bar{\rho}(b,m)$ on Page 8. Overall, I gladly recommend accepting this thesis.

Please let me know if I can be of further help.

Sincerely

[redacted]

(Franziska Eberle)